FIRST TERM EXAMINATION 2023

Mathematics (Code-041)

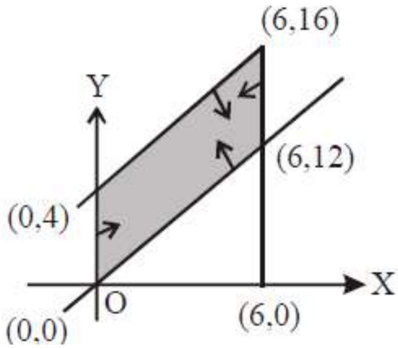
Class 12 **Time Allotted: 3 Hours**

13-09-2023 **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based question of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment(4 marks each) with sub parts.

SECTION A Multiple Choice Questions of 1 mark each		
1	If $\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the values of a and b, respectively, are a) -2, -3 (b) -2, 3 (c) 2, -3 (d) -3, 3	1
2	If A$(adjA)$=$\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then $\|adjA\|$ is: (a) 8 (b) 64 (c) 512 (d) 16	1
3	The matrix $\begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 0 & 7 \end{bmatrix}$ is not invertible for (a) λ = −1 (b) λ = 0 (c) λ = 1 (d) λ = R − {1}	1
4	If $f(x) = \begin{cases} kx+1, x \leq 5 \\ 3x-5, x > 5 \end{cases}$ is continuous at x = 5, then value of k is (a) $\frac{6}{7}$ (b) $\frac{7}{6}$ (c) $\frac{9}{5}$ (d) $\frac{5}{9}$	1

5	Which of the following functions from Z into Z are bijective? (a) $f(x) = x^3$ (b) $f(x) = x + 2$ (c) $f(x) = 2x + 1$ (d) $f(x) = x^2 + 1$	1
6	Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b ∀ a, b ∈ T. Then R is a) reflexive but not transitive (b) transitive but not symmetric (c) equivalence (d) None of these	1
7	The feasible region for LPP is shown shaded in the figure. Let Z = 3 x − 4 y be the objective function, then the maximum value of Z is:  (a) 12 (b) 8 (c) 0 (d) −18	1
8	$\cos^{-1}(x) + \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, find x a) ½ b) 1 c) -½ d) $\sqrt{3}/2$	1
9	f(x) = $\begin{cases} ax^2 + 1\ ,\ x > 1 \\ x + a\ , x \leq 1 \end{cases}$ is differentiable at x = 1, then find the value of a. a) 2 b) 1 c) 0 d) $\frac{1}{2}$	1
10	If A is a square matrix such that $A^2 = A$, then the simplified value of $(I - A)^3 + A$ is equal to (a) A (b) A^2 (c) I (d) A^3	1
11	Consider the following system of linear inequalities: $2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ If the corner points of the feasible region are (0, 0), (5, 0), (3, 4) and (0, 5). Let Z = p x +q y, where p, q > 0. Condition on p and q, so that the maximum value of Z occurs at both (3, 4) and (0, 5) is: (a) p = q (b) p =2 q (c) p = 3q (d) q = 3p	1

12	If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, the possible value(s) of x are (a) 2 (b) 3 (c) 2, 3 (d) -2	1
13	If A is a square matrix of order 3 and |A| = 5, then $\|adjA\|$ = (a) 5 (b) 25 (c) 125 (d) $\frac{1}{5}$	1
14	Let A and B are two events such that P(A) = 0.6, P(B) = 0.2 and P(A/B) = 0.5, then P(A'/B') = (a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$	1
15	The interval in which $y = x^2e^{-x}$ is increasing is $a)$ $(-\infty,\infty)$ b) $(-2,0)$ c) $(2,\infty)$ d) $(0,2)$	1
16	Derivative of $e^{logx}\ is$ (a) e^{logx} (b) $\log x$ (c) 0 (d) 1	1
17	A pair of dice is thrown 3 times. If getting a doublet is considered a success, find the probability of three successes. a) $\frac{1}{6}$ b) $\frac{1}{36}$ c) $\frac{1}{216}$ d) $\frac{125}{216}$	1
18	If area of triangle is 35 sq.units with vertices (2,-6) , (5 , 4) and (k,4) .Then k is (a) 12 (b) -2 (c) -12, -2 (d) 12,-2	1
19 20	**ASSERTION-REASON BASED QUESTIONS** In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true. Assertion (A): $tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ $= tan^{-1}\left(\tan(\pi - \frac{\pi}{3})\right) = tan^{-1}(-\tan\frac{\pi}{3}) = -tan^{-1}\left(\tan\frac{\pi}{3}\right) = -\frac{\pi}{3}$ Reason (R): The principal range of $\tan^{-1}x$ is (- $\frac{\pi}{2}$,$\frac{\pi}{2}$) Assertion: if A and B are any two independent events then P(AUB) = P(A).P(B)	1 1

	Reason: for two independent events, P(A/B) = P(A) Provided P(B) ≠ 0, P(B/A) = P(B) Provided P(A) ≠ 0.	
	SECTION B **Very Short Answer Type-Questions (VSA) of 2 marks each**	
21	Find the domain of the function defined by $f(x) = sin^{-1}\sqrt{x-1}$ OR Write in the simplest form $\tan^{-1}\frac{\cos x-\sin x}{\cos x+\sin x}$, $-\frac{\pi}{4} < x < \frac{3\pi}{4}$.	2
22	A man 1.6 m tall walks at the rate of 0.3 m/sec away from a street light that is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening?	2
23	Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let F = 4 x + 6 y be the objective function. Find the corner point in which the minimum value of F occurs.	2
24	If cos y = x cos (a + y), with cos a ≠ ±1 prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$	2
25	Find the equation of the line joining (3,1) and (9,3) using determinants.	2
	SECTION C **Short Answer Type Questions (SA) of 3 marks each**	
26	Determine whether the relation R defined on the set R of all real numbers as $R = \{(a,b): a,b \in R \ and \ a-b+\sqrt{3} \in S,$ $where\ S\ is\ the\ set\ of\ all\ irrational\ numbers\}$ is reflexive ,symmetric and transitive.	3
27	From a lot of 30 bulbs which includes 6 defective, a sample of 4 bulbs is drawn at a random with replacement. Find the probability of getting at least 3 non defective bulbs. OR The probability of A, B and C solving a problem are 1/3,2/7, and 3/8 respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them can solve it.	3
28	Evaluate: $sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$ OR Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left(\frac{3}{4}\right)$	3

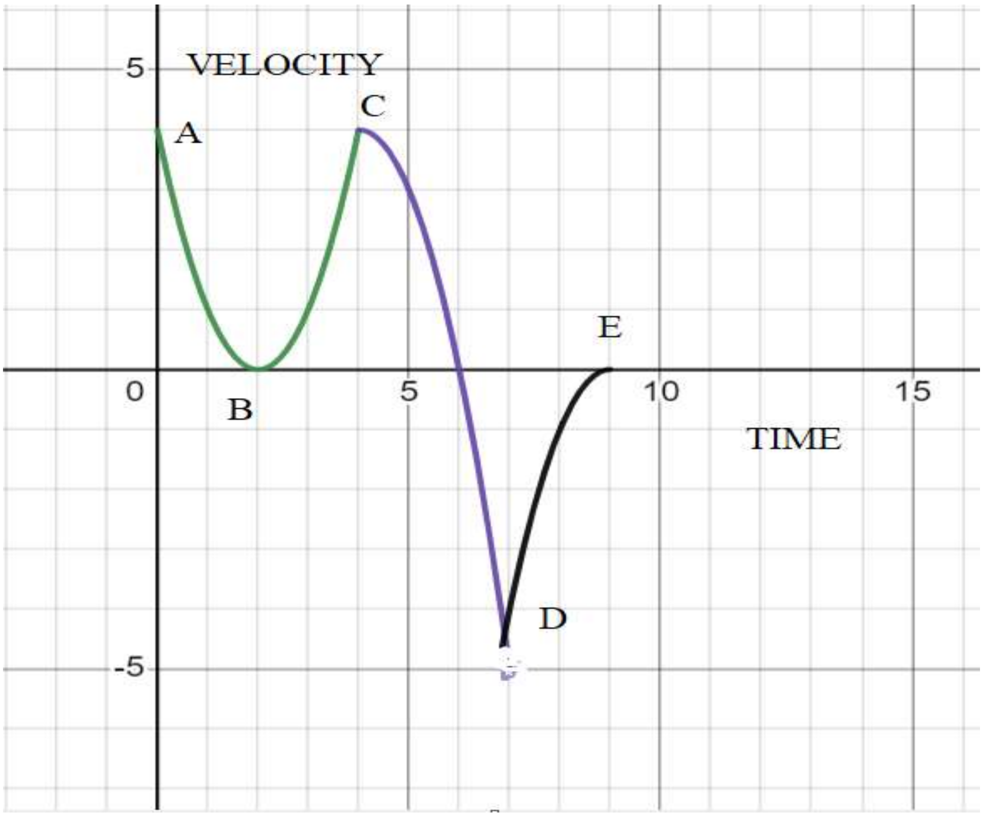
29	The total cost c(x) associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$.Find the marginal cost when 17units are produced. OR Find the intervals in which the functions given below are st.decreasing or st.increasing: $f(x) = x^3 - 12x^3 + 36x + 17$.	3
30	Solve the linear programming problem graphically: Maximize: Z = 8000 x + 12000 y Subject to the constraints: $3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0$, $y \geq 0$.	3
31	Find $\frac{dy}{dx}$ $if,$ $y = x^{sinx} + (sinx)^x$	3
SECTION D **Long Answer-Type Questions (LA) of 5 marks each**		
32	Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.	5
33	Define the relation R in the set $N \times N$ as follows: For (a, b), (c, d) $\in N \times N$, (a, b) R (c, d) iff ad = bc. Prove that R is an equivalence relation in $N \times N$. OR Show that the relation R in the set A=$\{1,2,3,4,5\}$ given by R=$\{(a,b): \|a-b\| is\ even\}$ is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other & all the elements of $\{2,4\}$ are related to each other. But No elements of$\{1,3,5\}$ is related to any element of $\{2,4\}$. Write equivalence class [1] and [2]	5
34	A factory as two machine A and B . Past Record Shows that Machine A Produced 60% of the items of output and machine B produced 40% of the items. Further 2% of the items produced by machine A and 1% of Produced by Machine B is defective. All the items are put into one stockpile and then one item is chosen at a random from this and is found defective. What is the probability that it was produced by machine B.?	5
35	If A= $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Hence,	5

37	solve the system of equations: $2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5$ and $x + y - 2z = -3$	

SECTION E

This section comprises of 3 case-study/passage-based questions of First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36 **Case Study 1:** 4



The above graph is the velocity – time graph of a moving particle. (answer any 4)

i) At which point the velocity is minimum?
a) A b) E c) D d) B

ii) In which interval the velocity is decreasing?
a) (2,4) b) (4,7) c) (7,9) d) None

iii) In which interval the velocity is Increasing?
a) (0,2) b) (4,7) c) (7,9) d) None

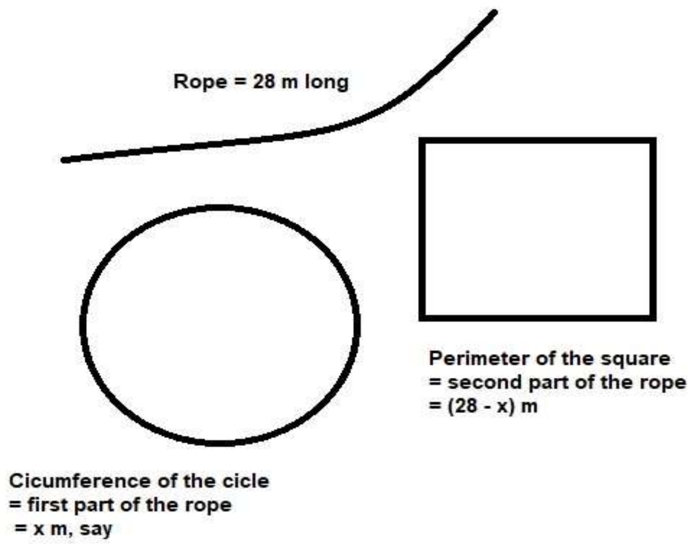
iv) Which of the point is not a critical point?
a) (2,0) b) (6,0) c) (4,4) d) None

v) At which points the velocity is maximum?
a) A, C b) B,E c) B,D d) None

37 **Case-Study 2:** 4

Read the following passage and answer the questions given below.

A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle.



In order to find the lengths of the two pieces, so that the combined area be minimum, two steps are followed:

(i) Find the area of the circle.

(ii) Find the area of the square.

(iii) What should be the lengths of the two pieces so that the combined area of the circle and the square is minimum?

38 **<u>Case-Study 3</u>**: 4

A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.1 and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.

Based on the given information, answer the following questions.

(i) When the doctor arrives late, what is the probability that he comes by metro?

	(ii) When the doctor arrives late, what is the probability that he comes by cab?	

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