केन्द्रीय विद्यालय संगठन क्षेत्रीय कार्यालय एर्नाकुलम

KENDRIYA VIDYALAYA SANGATHAN
ERNAKULAM REGION

CLASS – X

STUDENT SUPPORT MATERIAL

MATHEMATICS

SESSION: 2023-24

Our Patrons

SHRI. N. SANTHOSH KUMAR
HON'BLE DEPUTY COMMISSIONER
KVS, ERNAKULAM REGION

SMT. DEEPTI NAIR
ASSISTANT COMMISSIONER
KVS, ERNAKULAM REGION

SHRI. PATIL RAMESH ABHIMANYU
ASSISTANT COMMISSIONER
KVS, ERNAKULAM REGION

SHRI. GANESH SURYAKANT INDRALE
ASSISTANT COMMISSIONER
KVS, ERNAKULAM REGION

SHRI. JOY JOSEPH
ASSISTANT COMMISSIONER
KVS, ERNAKULAM REGION

COORDINATED BY
Smt. BINDU LEKSHMY P L

PRINCIPAL

K V CHENNEERKARA

केन्द्रीय विद्यालय संगठन, क्षेत्रीय कार्यालय, एरणाकुलम
KENDRIYA VIDYALAYA SANGATHAN REGIONAL OFFICE, ERNAKULAM
(An Autonomous Body Under MoE, Govt. of India)
कडवन्त्रा पी.ओ. कोच्ची - २०
Kadavanthra P.O. Kochi - 20
Phone No.0484- 2205111(DC), 2203091(Fax) 2202116(AC), 2203066 (Admn& Accts)
E-mail: dcernakulamregion@gmail.com

F. 31390/KVS/RO/EKM/2023-24/Acad./CBSE

Date: 25.07.2023

MESSAGE FROM DEPUTY COMMISSIONER

I am delighted to announce the release of Support Material/Study material for the students of class X. In our relentless pursuit of academic excellence, we have been constantly revising and upgrading our teaching methodologies and resources. And one of the important resources is the support material which is prepared by Kendriya Vidyalayas for the students of class X. This document is in two parts: Part A is the core concepts/topics of all subjects which aims minimum level of learning for each student and Part B is a detailed topics/ lessons with practice questions which aims to foster a deep understanding of subjects, stimulate critical thinking and helps in achieving better score in CBSE Examination. Part-A is planned to be printed and distributed amongst all students whereas Part-B is larger one which is planned to disseminate through electronic media /blogs.

I hope this support material will greatly benefit the academic journey of students in pursuit of good results in CBSE Exam as well as helpful for various Entrance Examinations.

Let's march ahead with dedicated minds and relentless endeavours for a better future through better education

With Best Wishes,

(SANTHOSH KUMAR N.)
DEPUTY COMMISSIONER
KVS, ERNAKULAM REGION

Preface

This Study Material is an in-house academic exercise undertaken by the Maths teachers of KVS Ernakulam Region under the supervision of a subject expert, Smt. Bindu Lekshmy P L, Principal, KV Chenneerkara, to provide the students a comprehensive, yet concise, support tool for consolidation of learning.

It consists of curriculum, deleted topics and questions from all chapters. This material is developed keeping in mind the latest CBSE curriculum and pattern of the question paper. It will definitely provide the students a valuable window on precise information and it covers all essential components that are required for effective revision of the subject.

Hoping this material will prove to be a helpful tool for quick revision and will serve the purpose of enhancing students' confidence level to help them perform better.

Best of Luck.

KENDRIYA VIDYALAYA SANGATHAN

ERNAKULAM REGION

TEAM OF TEACHERS WHO CONTRIBUTED

NAME OF THE TEACHER & KV (Mrs./ Ms./ Mr.)

1. **Ragini (Contractual), K V NO. 1 CPCRI, KASARAGOD**
2. **NISHA B C (Contractual), K V NO. 2 KASARAGOD**
3. **Hima B, K V KANHANGAD**
4. **Santosh Kannan, K V EZHIMALA**
5. **Ramseena K P, K V CRPF PERINGOME**
6. **Sabitha A K, PGT, K V PAYYANNUR**
7. **Veena M, K V KELTRON NAGAR**
8. **Remya Gopi E P, K V KANNUR**
9. **Jyothi Raghavan, K V THALASSERY**
10. **Seena P R , K V NO.1 CALICUT**
11. **Asha P , K V NO.1 CALICUT**
12. **Swapna sethu, K V NO.1 CALICUT**
13. **Vijimol., K V MALAPPURAM**
14. **Valsamma Thomas , K V NO. 1 PALAKKAD**
15. **Jyothi K , K V KANJIKODE**
16. **Shalini K, K V KANJIKODE**
17. **Agnes. P. Varghese, K V ERNAKULAM**
18. **Elsy Mathew., K V ERNAKULAM**
19. **Mithun Basheer., K V ERNAKULAM**
20. **Manoj Kumar.D., K V ERNAKULAM**
21. **Roopitha EK, PURANATTUKARA, K V THRISSUR**
22. **Jayalalitha MR, PURANATTUKARA, K V THRISSUR**
23. **Nazeema PM, PURANATTUKARA, K V THRISSUR**
24. **Lisi Mookan, PURANATTUKARA, K V THRISSUR**

25. **Ceena John (Contractual), K V RAMAVARMAPURAM**
26. **Mereena George , K V ,NAD, ALUVA**
27. **Kunjumol. P.X., K V ,NAD, ALUVA**
28. **Saju. P.K., K V NO. 1 NAVAL BASE, KOCHI**
29. **Tessy Jimmy, K V NO. 1 NAVAL BASE, KOCHI**
30. **Beenakumari. , K V NO. 1 NAVAL BASE, KOCHI**
31. **Leena Manoj., K V NO. 1 NAVAL BASE, KOCHI**
32. **Preethy KR, K V NO. 2 NAVAL BASE, KOCHI**
33. **Prajisha , KV NO 2 NAVAL BASE, KOCHI**
34. **Jancy Mathew, K V PORT TRUST, KOCHI**
35. **Deepa S, K V PORT TRUST, KOCHI**
36. **Sheeja Rajan, K V INS DRONACHARYA**
37. **Tiny Binu(contr), K V INS DRONACHARYA**
38. **Alphin Antony(contr),K V INS DRONACHARYA**
39. **Kuldeep Godara, K V , IDUKKI**
40. **Latesh Rani, K V IDUKKI**
41. **Deepti. K.Thampy, K V RUBBER BOARD KOTTAYAM**
42. **Dillis Rani Joseph, K V RUBBER BOARD KOTTAYAM**
43. **Salini PC(contrl), K V KADUTHURUTHY**
44. **Geetha.S, K V NTPC KAYAMKULAM**
45. **Umadevi N, K V ADOOR SHIFT - 1**
46. **Rajasree S, K V ADOOR SHIFT – 1**
47. **Prasanna Kumari K P, K V ADOOR SHIFT – 2**
48. **Kavitha G S, K V CRPF, PALLIPURAM**
49. **Sajitha T M, K V CRPF, PALLIPURAM**
50. **Indu.K.S, K V PATTOM SHIFT– 1**
51. **Sreeja. S, K V PATTOM SHIFT– 1**
52. **Thomas Varghese , K V PATTOM SHIFT– 1**
53. **Adeline M Jaqueline, K V PATTOM SHIFT– 1**
54. **Neha Singh , K V PATTOM SHIFT– 2**
55. **Sandhya CS, K V PATTOM SHIFT– 2**

56. **Alice George, K V AFS AKKULAM**
57. **Aagya Pathak, K V AFS AKKULAM**
58. **Manju. S. Nair. , K V PANGODE**
59. **Priyaja Snehajan., K V PANGODE**
60. **Amitha.J., K V PANGODE**
61. **Dhanya M G, K V SAP, PEROORKADA**
62. **Nunnu Michael, K V SAP, PEROORKADA**
63. **Reema Singh , K V SAP, PEROORKADA**
64. **Sunarlal Chikla., K V SAP, PEROORKADA**

REVIEW AND COMPILATION TEAM

65. **R. P. Usha, K V NO 1 CALICUT (COMPILATION)**
66. **Mini Shekhar, KV PALAKKAD (COMPILATION)**
67. **K. Sathi, K V OTTAPALAM (COMPILATION)**
68. **Anisha.N, KV NO 2 CALICUT (COMPILATION)**
69. **Akhil. K, KV NO 2 CALICUT (COMPILATION)**

CONTENT INDEX

MATHEMATICS (CODE NO. 041)

COURSE STRUCTURE CLASS –X

UNITS	UNIT NAME	MARKS
I	NUMBER SYSTEM	6
II	ALGEBRA	20
III	CO-ORDINATE GEOMETRY	6
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILITY	11
	TOTAL	80
	INTERNAL ASSESSMENT	20
	TOTAL	100

INTERNAL ASSESSMENT

INTERNAL ASSESSMENT	Marks	TOTAL MARKS
Pen Paper Test and Multiple Assessment (5+5)	10marks	20 marks
Portfolio	05 marks	
Lab Practical (Lab activities to be done from the prescribed books)	05 marks	

UNIT I: NUMBER SYSTEMS

1. REAL NUMBER (15) periods
   Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$

UNIT II: ALGEBRA

1. POLYNOMIALS (8) periods

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES (15) Periods

Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency. Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS (15) Periods

Standard form of a quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots. Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS (10) Periods

Motivation for studying Arithmetic Progression. Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III: CO-ORDINATE GEOMETRY

Co-ordinate Geometry (15) Periods

Review: Concepts of co-ordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division).

UNIT IV: GEOMETRY

1. TRIANGLES (15) Periods

   Definitions, examples, counter examples of similar triangles.

   1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
   2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
   3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
   4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
   5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES (10) Periods

   Tangent to a circle at, point of contact

   1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
   2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

UNIT V: TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY (10) Periods

   Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0^o and 90^o. Values of the trigonometric ratios of 30^0 , 45^0 and 60^0 . Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES (15) Periods

   Proof and applications of the identity $\sin^2A + \cos^2A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES: Angle of elevation, Angle of Depression. (10) Periods

   Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, and 60°.

UNIT VI: MENSURATION

1. AREAS RELATED TO CIRCLES (12) Periods

   Area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only.

2. SURFACE AREAS AND VOLUMES (12) Periods

   Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones.

UNIT VII: STATISTICS AND PROBABILITY

1. STATISTICS (18) Periods

   Mean, median and mode of grouped data (bimodal situation to be avoided).

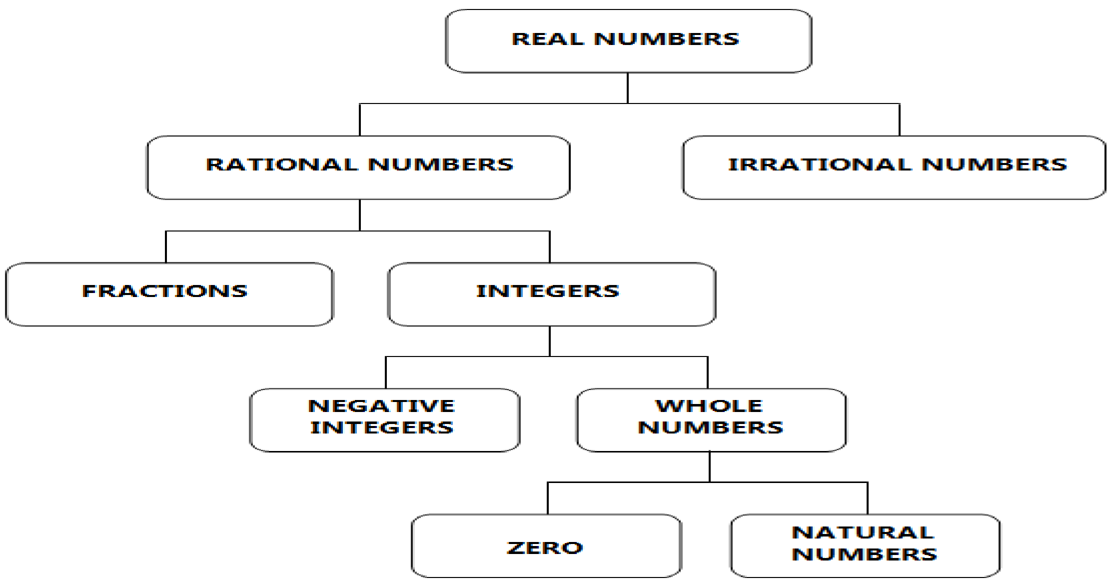
2. PROBABILITY (10) Periods

   Classical definition of probability. Simple problems on finding the probability of an event.

UNIT - 1

REAL NUMBERS

IMPORTANT FORMULAS & CONCEPTS



The Fundamental Theorem of Arithmetic

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

Prime and Composite numbers

A prime number is a number which has only two factors i.e. one and itself whereas the composite number is a number which has more than two factors.

HCF and LCM of numbers

HCF is the highest common factor also known as GCD i.e. greatest common divisor.

LCM of two numbers is their least common multiple.

Property of HCF and LCM of two positive integers ‘a’ and ‘b’:

HCF (a, b)·LCM (a, b) = a ·b

HCF and LCM by Prime factorization method

- HCF (a, b) = Product of the smallest power of each common prime factor in the numbers.
- LCM (a, b) = Product of the greatest power of each prime factor, involved in the numbers.

MULTIPLE CHOICE QUESTIONS AND OBJECTIVE QUESTIONS (1 MARK):

SECTION A

LEVEL 1

Q1. The number 'π' is

a) natural number
b) rational number
c) irrational number
d) rational or irrational

Q2. The product of a non-zero number and an irrational number is:

a) always irrational
b) always rational
c) rational or irrational
d) none of the above

Q3. The product of a rational and irrational number is

a) Rational
b) Irrational
c) both of above
d) none of above

Q4. If HCF of two numbers is 1, the two numbers are called relatively ________ or ________.

a) prime, co-prime
b) composite, prime
c) Both (a) and (b)
d) None of these

LEVEL 2

Q5. Express 98 as a product of its primes

a) $2^2 \times 7$ b) $2^2 \times 7^2$ c) 2×7^2 d) $2^3 \times 7$

Q6. HCF of 8, 9, 25 is

a) 8 b) 9 c) 25 d) 1

Q7. L.C.M. of $2^3 \times 3^2$ and $2^2 \times 3^3$ is:

a) 2^3 b) 3^3 c) $2^3 \times 3^3$ d) $2^2 \times 3^2$

Q8. If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a =?

a) 2 b) 3 c) 4 d) 1

LEVEL 3

Q9. If HCF (16, y) = 8 and LCM (16, y) = 48, then the value of y is

a) 24 b) 16 c) 8 d) 48

Q10. The ratio between the LCM and HCF of 5, 15, 20 is:

a) 9 : 1 b) 4 : 3 c) 11 : 1 d) 12 : 1

LEVEL 1

Q11. State fundamental theorem of arithmetic

Q12. Find the LCM of smallest prime and the smallest odd composite natural number

Q13. If p and q are two coprime numbers, then find the HCF and LCM of p and q.

LEVEL 2

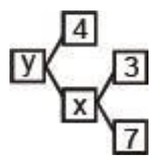
Q14. What is the greatest possible speed at which a man can walk 52 km and 91 km in an exact number of minutes?

Q15. Prime factorization of 120 is …

Q16. If HCF (26, 169) = 13, then LCM (26, 169) is …

Q17. Given that LCM (91, 26) = 182, then HCF (91, 26) is:

Q18. The values of x and y in the given figure are:



LEVEL 3

Q19. If two positive integers a and b are written as $a = p^3q^2$ and $b = pq^3$; p, q are prime numbers, then HCF (a, b) is:

Q20. If two positive integers a, b are written as $a = xy^2$ and $b = x^3 y$, where x, y are prime numbers, then find LCM (a, b).

Q21.

Short Answer Type Questions (2 marks):

SECTION B

LEVEL 1

Q1. Find the prime factorization of 1152

Q2. Determine the prime factorisation of 2057?

Q3. Explain why 3 × 5 × 7 + 7 is a composite number.

Q4. Explain why $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a composite numbers

Q5. Can the number 4^n, n being a natural number, end with the digit 0? Give reasons.

Q6. Show that 12^n cannot end with the digit 0 or 5 for any natural number n

LEVEL 2

Q7. What is the LCM and HCF of smallest odd prime number and smallest odd composite number?

Q8. P and Q are two positive integers such that $P = p^3 q$ and $Q = (pq)^2$, where p and q are prime numbers. What is LCM (P, Q)?

Q9. If $a=2^3\times3$, $b=2\times3\times5$, $c=3^n\times5$ and LCM $[a,b,c] = 2^3\times3^2\times5$ then, n=?

Q10. Show that the product of two numbers 60 and 84 is equal to the product of their HCF and LCM

Q11. Given that LCM (91, 26) = 182, then HCF (91, 26) is:

Q12. The product of two numbers is 228096 and their LCM is 66. Find their HCF.

Q13. Find the sum of exponents of prime factors in the prime factorization of 216?

LEVEL 3

Q14. Find the largest number which divides 245 and 1029 leaving remainder 5 in each case.

Q15. If p and q are two coprime numbers, then p^3 and q^3 are?

Q16. If n is an even prime number then, $2(7^n + 8^n)$ ends with?

Q17. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Q18. A shopkeeper has 120 litres of petrol, 180 litres of diesel and 240 litres of kerosene. He wants to sell oil by filling the three kinds of oils in tins of equal capacity. What should be the greatest capacity of such a tin?

Q19. 144 cartons of coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Q20. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

SHORT ANSWER QUESTIONS (3 MARKS):

SECTION C

LEVEL 1

Q1. Prove that $3\sqrt{2}$ is irrational

Q2. Prove that $7 + 3\sqrt{2}$ is not a rational number.

Q3. Prove that $2 - 3\sqrt{5}$ is irrational number.

Q4. Find the LCM and HCF of the following pairs of positive integers by applying the prime factorization method.

a) 225, 240 b) 52 ,63 ,162

LEVEL 2

Q5. The LCM of two numbers is 64699, their HCF is 97 and one of the numbers is 2231.Find the other.

Q6. Find HCF and LCM of 135 and 225 and verify the that HCF x LCM = Product of the two given numbers

Q7. Find HCF and LCM of 867 and 255 and verify the that HCF x LCM = Product of the two given numbers.

LEVEL 3

Q8. If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF then, find the product of two numbers

Q9. The LCM of two number is 14 times their HCF. The sum of LCM and HCF is 600.If one numberis 280, then find the other number.

Q10. Find the largest number that will divide 398 , 436 and 542 leaving reminders 7,11 and 15respectively.

Q11. Find the largest number which divides 70 and 125 leaving reminder 5 and 8 respectively.

Q12. Can two numbers have 15 as their HCF and 175 as their LCM? Give reasons.

Q13. Two bells toll at intervals of 24 minutes and 36 minutes respectively. If they toll together at 9am, after how many minutes do they toll together again, at the earliest?

Q14. There are 44 boys and 32 girls in a class. These students arranged in rows for a prayer in such a way that each row consists of only either boys or girls, and every row contains an equal number of students. Find the minimum number of rows in which all students can bearranged.

Q15. Prove that $\sqrt{n}$ is not a rational number if n is not a perfect square

Long Answer Type Questions (4 marks):
SECTION D

LEVEL 1

Q1. Prove that $\sqrt{5}$ is an irrational number

Q2. Find HCF and LCM of 378, 180 and 420 by prime factorization method. Is HCF X LCM of these numbers equal to the product of given three numbers?

LEVEL 2

Q3. The sum of LCM and HCF of two numbers is 7380.If the LCM of these numbers is 7340 more than their HCF. Find the product of the two numbers

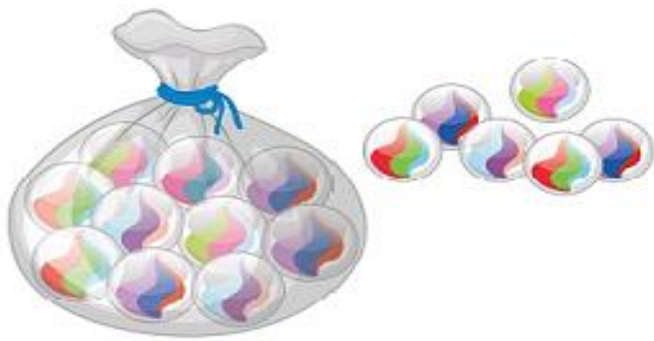
Q4. A charitable trust donates 28 different books of Maths,16 different books of science and 12 different books of Social Science to the poor students. Each student is given maximum number of books of only one subject of his interest and each student got equal number of books

i. Find the number of books each student got.

ii. Find the total number of students who got books.

CASE BASED QUESTIONS

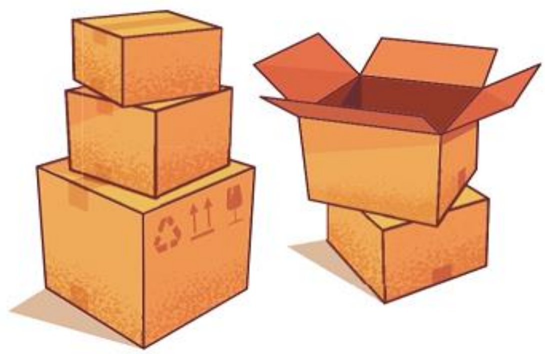
Q1. When the marbles in a bag are divided evenly between two friends, there is one marble left over When the same marbles are divided evenly among three friends, there is one marble left over .When the marbles are divided evenly among five friends, there is one marble left over.



i. What is the least possible number of marbles in the bag?

ii. What is another possible number of marbles in the bag?

Q2. Flipkart is an Indian e-commerce company, headquartered in Bangalore, Karnataka and incorporated in Singapore as a private limited company. The company initially focussed on online book sales before expanding into other product categories such as consumer electronics fashion, home essentials groceries and lifestyle products.



Flipkart sells 10 types of items which are packed into various sizes of cartons which are packed into various size of cartons which are given below

Carton type	Inner Dimension (l X b) cm^2
Small	6 x 8
Medium	12 x 24
Large	24 x 36
Extra Large	36 x 48
XXL	48 x 96

Flipkart places supporting thermocol sheets inside every package along the edges. The company thought of buying same sized sheets for all type of cartons

i. What should be the maximum size of the sheet that fits into all type of cartons?

ii. What should have been size of semi large (which is larger than medium carton but smaller than large carton) so that the maximum sized sheet remains same?

Q3. Kerosene, paraffin, or lamp oil is a combustible hydrocarbon liquid which is derivative from petroleum. Kerosen's uses vary from fuel for oil lamps to cleaning agents , jet fuel , heating oil or fuel for cooking

Two oil tankers contain 825 litres and 675 litres of kerosene oil respectively.

i. Find the maximum capacity of a container which can measure the Kerosene oil of both the tankers when used an exact number of times.

ii. How many times we have to use container for both the tanker to fill?

Q4. Amar, Akbar and Anthony are playing a game. Amar climbs 5 stairs and gets down 2 stairs in one turn .Akbar goes up by 7 stairs and comes down by 2 stairs every time. Anthony goes 10 stairs up and 3 stairs down each time.

During this they have to reach to the nearest point of 100th stairs and they will stop once they find it impossible to go forward. They can not cross 100th stair any way

i. Who reaches the nearest point?

ii. Who takes least number of steps to reach nearest hundred?

Q5. A woman wants to organise her birthday party. She was happy on her birthday but there was a problem that she does not want to serve fast food to her guests because she is very health conscious. She as 15 apples and 40 bananas at home and decided to serve them. She want to distribute fruits among guests. She does not want to discriminate among guests so she decided to distribute equally among all. So

i. How many guests she can invite?

ii. How many apples and banana will each guest get?

A hall has a certain number of chairs. Guests want to sit in different groups like in pairs, triplets, quadruplets, fives and sixes etc. When organiser arranges chairs in such pattern like 2's, 3's ,4's.5's and 6's then 1,2,3,4 and 5 chairs are left respectively. But when he arranges in 11's no chair will be left

i. In the hall how many chairs are available?

   a) 407

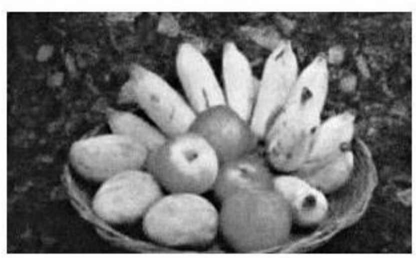
   b) 143

   c) 539

   d) 209

ii. If one chair is added to the total number of chairs, how many chairs will be left when arranged in 11's

Q7. Khushi wants to organize her birthday party. Being health conscious, she decided to serve only fruits in her birthday. She brought 36 apples and 60 bananas and decided to distribute fruits equally among all. (CBSE -2022



Based on the above information, answer the following questions:

i. How many guests Khushi can invite at the most?

ii. How many apples and bananas will each guest get?

iii. If Khushi decides to add 42 Mangoes, how many guests Khushi can invite at the most?

iv. If the cost of 1 dozen of Banana is₹60, the cost of 1 apple is ₹15 and the cost of 1 mango is ₹20, find the total amount spent on fruits?

ANSWER KEY

Q.I Multiple Choice Questions (1 mark):	
Q.No.	**Answer**
1	d) 1 c) Irrational number
2	c) 2×7^2 d) 12 : 1
3	c) 4 b) irrational
4	c) 338 (a) prime, co-prime
5	b) irrational c) 2×7^2
6	a) 24 d) 1
7	c) Irrational number c) $2^3 \times 3^3$
8	d) 12 : 1 c) 4
9	(a) Always irrational a) 24
10	c) $2^3 \times 3^3$ d) 12 : 1
11	LCM of 2 and 4 is 4
12	HCF = 1 and LCM = pq
13	13m / min
14	2^3 x 3 x 5
15	338
16	HCF = 13
17	x = 21 and y = 84
18	b) 2
19	HCF = pq^2
20	x^3y^2
Q.II Short Answer Type Questions (2 marks):	
1	1152= 2^7 x 3^2
2	$2 \times 5 \times 11^2 \times 17$
3	112 is an even number and is therefore a composite number
4	$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ $= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) = 5 \times (1008 + 1)$
5	No
6	$12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$ Since, there is no term containing 5. Therefore, there is no value of n ∈ N for which 12^n ends with the digit zero or five
7	the HCF and LCM of the smallest odd composite number and the smallest odd prime number are 3 and 9 respectively
8	$P^3 \times q^2$
9	2
10	LCM × HCF =420×12=5040 Also, 60×84=5040

11	13
12	36
13	15
14	The HCF of (245 – 5) and (1029 – 5) i.e., 240 and 1024. HCF =16
15	Coprime
16	6
17	LCM of 12 and 18 = 2 × 2 × 3 × 3 = 36 Therefore, Ravi and Sonia will meet together at the starting pointafter 36 minutes
18	HCF of 120,180 and 240 is 60.
19	18
20	75 cm
Q.III Long Answer Type Questions (3 marks):	
1	Refer textbook
2	Refer textbook
3	Refer textbook
4	a)HCF (225, 240) = 15 LCM (225, 240) = 600 b)HCF (52, 6, 162) = 1 LCM (52, 63, 162) = 29484
5	2813
6	LCM (135, 225) = 675, HCF (135, 225) = 45. Verification by showing LHS = RHS i.e., 135 x 225 = 675 x 45
7	LCM (867, 255) = 4335, HCF (867, 255) = 51. Verification by showing LHS = RHS i.e., 867 x 255 = 4335 x 51
8	194400
9	HCF = x LCM = 14 x HCF = 14x LCM + HCF = 600 14x + x = 500 15x = 600 x = 40 HCF = 40 and LCM = 14 x 40 = 560 Since, LCM x HCF = product of the numbers 560 x 40 = 280 x second number Second number = 80.
10	398 – 7 = 391 436 – 11 = 425 542 – 15 = 527 391 = 17 x 23 425 = 5^2 x 17 527 = 17 x 31 HCF = 17

	i.e., 17 is the largest number that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively
11	70 – 5 = 65 125 – 8 = 117 65 = 5 x 13 117 = 3^2 x 13 HCF = 13 i.e., 13 is the largest number that will divide 65 and 117.
12	No, two numbers cannot have 15 as their HCF and 175 as LCM because, HCF of the numbers must be a factor of the LCM. Therefore, LCM = k x HCF (k ∈ N) 175 = k x 15 k = 175/15 = 35/3 ∉ N.
13	24 = 2^3 x 3 36 = 2^2 x 3^2 LCM = 2^3 x 3^2 = 8 x 9 = 72 After 72 minutes = 1 hr 12 minutes they toll together.
14	44 = 2^2 x 11 32 = 2^5 HCF = 2^2 = 4 Therefore, minimum number of rows in which all srudents can be arranged = $\frac{44}{4}$ + $\frac{32}{4}$ = 11 + 8 = 19 rows
15	Let on the contrary say it is rational . Then $\sqrt{n}=p/q,\ q\neq 0$ where p and q are coprime integers. so $n=p2q2$ $p2=nq2$ This shows p divides q which is a contradiction. Hence $\sqrt{n}$ is irrational if n is not a perfect square
Q.IV Very Long Answer Type Questions (4 marks):	
1	Assume that $\sqrt{5}$ is a rational number Therefore $\sqrt{5}$ = $\frac{p}{q}$ p and q are co primes and q ≠ 0 (1) p = $\sqrt{5}\,q$ Squaring both the sides p^2=5q^2 Thus 5 is a factor of p^2 Therefore 5 is a factor of p (1) Let p=5c where c is some integer, then we have p^2=25c^2 Substituting p^2=5q^2 5q^2 = 25c^2

	q^2=5c^2 (1) Thus 5 is a factor of q^2 and also 5 is also a factor of q Thus 5 is a factor of both p and q .But this is a contradiction to the fact that p and q are co primes (1) Thus our assumption is wrong that $\sqrt{5}$ is a rational number Hence $\sqrt{5}$ *is* an irrational number
2	378= 3^3 x 2 x 7 180= 3^2 x 2^2 x 5 (1) 420= 3 x 2^2 x 5 x 7 HCF = 3 X 2 =6 (1) LCM = 3^3 x 2^2 x 5 x 7 =3780 (1) HCF x LCM = 3780 x 6 = 22,680 Product of numbers = 378 x 180 x 420 = 28576800 No HCF x LCM is not equal to product of three numbers (1)
3	LCM + HCF = 7380 LCM – HCF = 7340 2LCM = 14720 LCM = 14720/2 **LCM = 7360** (2) LCM + HCF = 7380 7360+ HCF = 7380 HCF = 7380 – 7360 **HCF = 20** (1) HCF x LCM = product of numbers 20 x 7360= product of numbers 147200 = product of numbers (1)
4	(i) HCF of 28,16 and 12 is 4 Therefore maximum number of books each student get is 4 (2) (ii) Number of maths books 28/4 = 7 Number of science books 16/4 = 4 Number of social science = 12/4 = 3 Total books = 7 + 4 + 3 =14 (2)
5	(i) LCM of 2,3 and 5 = 30 Thus 31 marbles are there in the bag (2) (ii) If we add 1 in multiple of 30 we will get another possible number of marble. These are 61,91,121,… (2)
6	(i) HCF of all length HCF(6,12,24,36,48) = 6 (ii) HCF of all width HCF(8,24,36,48,96) = 4 Thus maximum size of sheet is 6 by 4
7	(i) HCF of 825 and 625 825 = 3 x 5 x 5 x 11 675 = 3 x 3 x 3 x5 x 5 HCF = 3 x 5 x 5 = 75 (2) Maximum capacity reqired is 75 litres (ii) The first tanker will require 875/75 = 11 times to fill

	The second tanker will require 675/75 = 9 times to fill (2)
8	(i)Amar reaches 96 stairs Akbar reaches 95 stairs Anthony reaches 91 stairs Thus Amar will reach nearest point (2) (ii)Amar will take 100/3 = 33.3 Akbar will take 100/5 = 20 Anthony will take 100/7 = 14.22 Anthony will take least step (2)
9	(i)HCF of(15,40) =5 Fruits will be distributed equally among 5 guests (2) (ii)Out of 15 apples each guest will get 15 /5 = 3 apples Out of 40 banana each guest will get 40/5 =8 bananas (2)
10	(i) 539 chairs (2) (ii) if 1 chair is added as 539 is already divisible by 11 ,1 chair will be left (2)
Case based 7	Ans: i) 12 ii) 3 apples, 5 bananas iii) 6 guests

UNIT 2- ALGEBRA

POLYNOMIALS

IMPORTANT CONCEPTS

- **A polynomial is an algebraic expression in which the exponent on any variable is a whole number. / A polynomial is an algebraic expression with variables having positive integral powers only.**
- General Form:

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

- **Degree of a polynomial**

- The highest power of x in p(x) is called the degree of the polynomial p(x).

Name of the polynomial	Degree of the polynomial	Example
Zero polynomial	Not defined	0,5,-3…..
Linear polynomial	1	x-3
Quadratic polynomial	2	$6x^2-3y$

Value of a polynomial:

If p(x) is a polynomial in x, and if k is any real number, then the value obtained byreplacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k).

Q. Find the value of the polynomial $p(x) = x^2 + 4x + 4$ where x = 2.
Given polynomial: $p(x) = x^2 + 4x + 4$.
Value of given polynomial when x = 2 and we get: $p(2) = (2)^2 + 4(2) + 4$
$= 4 + 8 + 4 = 16$
Hence the value of $p(x) = x^2 + 4x + 4$, where x = 2, is 16

Cubic polynomial	3	$4x^3+5y^2$ -1

❖Zero of a polynomial

A real number k is said to be a zero of a polynomial p(x), if p(k) = 0

What is the value of $p(x) = x^2 - 3x - 4$ at $x = -1$?
We have : $p(-1) = (-1)\ 2 - \{3 \times (-1)\} - 4 = 0$
Also, note that $p(4) = 4^2 - (3 \times 4) - 4 = 0$.
As $p(-1) = 0$ and $p(4) = 0$,
-1 and 4 are called the zeroes of the quadratic polynomial $x^2 - 3x - 4$.

RELATIONSHIP BETWEEN ZEROES & COEFFICIENTS OF POLYNOMIALS

Type of Polynomial	General form	No. of zeroes	Relationship between zeroes and coefficients
Linear	$ax + b,\ a \neq 0$	1	$k = -\dfrac{b}{a}$, i.e. $k = -\dfrac{\text{Constant term}}{\text{Coefficient of x}}$
Quadratic	$ax^2 + bx + c,\ a \neq 0$	2	Sum of zeroes $(\alpha + \beta) = -\dfrac{\text{Coefficient of x}}{\text{Coefficient of x}^2} = -\dfrac{b}{a}$ Product of zeroes $(\alpha\beta) = \dfrac{\text{Constant term}}{\text{Coefficient of x}^2} = \dfrac{c}{a}$
Cubic	$ax^3 + bx^2 + cx + d$, $a \neq 0$	3	Sum of zeroes $(\alpha + \beta + \gamma) = -\dfrac{\text{Coefficient of x}^2}{\text{Coefficient of x}^3} = -\dfrac{b}{a}$ Product of sum of zeroes taken two at a time $(\alpha\beta + \beta\gamma + \gamma\alpha) = \dfrac{\text{Coefficient of x}}{\text{Coefficient of x}^3} = \dfrac{c}{a}$ Product of zeroes $(\alpha\beta\gamma) = -\dfrac{\text{Constant term}}{\text{Coefficient of x}^3} = -\dfrac{d}{a}$

MULTIPLE CHOICE QUESTIONS

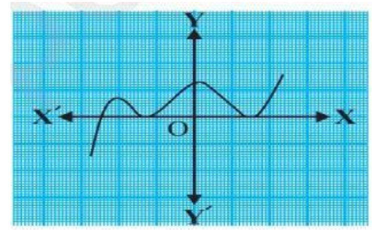
SECTION A

LEVEL 1

Q1. If one zero of the quadratic polynomial x^2+3x+k is 2, then the value of k is
a) 10 b) -10 c) 5 d) -5

Q2. If 2 and 1/2 are the zeros of $px^2 + 5x + r$, then
a) p = r =2
b) p = r = -2
c) p = 2, r = -2
d) p = -2, r = 2

Q3. How many zeros are there for the given polynomial?



a) 0
c) 2
b) 1
d) 3

Q4. A quadratic polynomial, the sum of whose zeros is 2 and one zero is 3 is

a) x^2-9
c) x^2+3
b) x^2+9
d) x^2-3

LEVEL 2

Q5. If one zero of the polynomial f(x)= $(k^2+4)\,x^2+13x+4k$ is the reciprocal of the other, k =

a) 2 b) -2 c) 1 d) -1

Q6. If ∝, β are the zeros of the polynomial $f(x) = x^2 + x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

a) 1
c) 0
b) -1
d) None of these

Q7. The number of polynomials having zeros -2 and 5 is

a) 1
c) 3
b) 2
d) More than 3

LEVEL 3

Q8. If ∝, β are the zeros of the polynomial, $f(x) = x^2 + bx + c$, then $\frac{1}{\propto^2} + \frac{1}{\beta^2}$

a) $\frac{b^2-2ac}{a^2}$
c) $\frac{b^2+2ac}{a^2}$
b) $\frac{b^2-2ac}{c^2}$
d) $\frac{b^2+2ac}{c^2}$

Q9. If ∝, β are the zeros of the polynomial $f(x) = x^2 - p(x+1) - c$ then $(\alpha+1)(\beta+1) =$

a) c-1 b) 1-c c) C d) 1+c

Q10. If ∝, β are the zeros of the polynomial x^2-6x+k and 3∝+ 2β=20, then value of k is

a)-8 b)16 c)-16 d)8

Q11. What should be added to the $x^2 - 5x + 4$, so that 3 is a zero of the resulting polynomials?

a) 1 b) 2 c) 4 d) 5

OBJECTIVE TYPE QUESTIONS (I MARK QUESTIONS)

LEVEL 1

Q1. Check whether -2 is a zero of the polynomials $9x^3 - 18x^2 - x - 2$

Q2. Write the zeros of the polynomial $x^2 - x - 6$

Q3. Write a polynomial whose zeros are $(2 + \sqrt{(3)})$ *and* $(2 - \sqrt{3})$

Q4. If α, β are the zeros of the polynomial, such that $\alpha + \beta = 6$ and $\alpha\beta = 4$, then write the polynomial.

Q5. Find a quadratic polynomial with $-\frac{1}{4}$ *and* $\frac{1}{4}$ as the sum and product of its zeros respectively.

LEVEL 2

Q6. If α and 1/α are the zeros of the polynomial $4x^2 - 2x + (k - 4)$, find the value of k .

Q7. Find the zeros of the polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

Q8. For what value of k is 3 a zero of the polynomial $2x^2 - x + k$?

Q9. If α, β are the zeros of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeros are $\frac{1}{\alpha}$, $\frac{1}{\beta}$

Q10. If the sum and product of the zeros of the polynomial ax^2-6x+c is equal to 12 each, find the value of a and c each.

SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

SECTION – B

LEVEL 1

Q1. Check whether $x = -3$ is a zero of $x^3 + 11x^2 + 23x - 35$.

Q2. If 2 is a zero of $2x^2 + px + 5$,then find the value of p.

Q3. Find the zeroes of the polynomial $2x^2 - 9$ and verify the relationship between zeros and coeffients.

Q4. Find a quadratic polynomial the sum and product of whose zeros are 3 and -2/5 respectively.

LEVEL 2

Q5. If α and β are zeros of $3x^2 + 5x + 13$,then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

Q6. Find p and q if p and q are the zeros of the quadratic polynomial $x^2 + px + q$.

Q7. Prove that both zeroes of $x^2 + 99x + 127$ are negative.

Q8. Find the quadratic polynomial sum of whose zeros is 8 and their product is 12.Hence find the zeroes of the polynomial.

Q9. For what value of k ,-4 is a zero of $x^2 - x - (2k + 2)$?

Q10. Form a quadratic polynomial one of whose zeros is $2 + \sqrt{5}$ and the sum of zeros is 4.

LEVEL 3

Q11. Find the value of a in the polynomial $2a^2 + 2xa + 5a + 10$ if $(x + a)$ is one of its factors.

Q12. Show that $x^2 + 4x + 7$ has no zeros.

Q13. If the zeros of $x^2 - kx + 6$ are in the ratio $3: 2$,find k.

Q14. If the zeros of the polynomial $x^2 + px + q$ are double in value to the zeros of $2x^2 - 5x - 3$, find p and q.

Q15. The sum and product of the zeros of $4x^2 - 27x + 3k^2$ are equal, find the values of k.

Q16. If α and β are the zeros of the polynomial p(x)= $x^2 + 5x + q$ such that $\alpha - \beta = 1$. Find k.

Q17. If the sum of zeros of the quadratic polynomial $f(t) = kt^2$+2t+3k is equal to their product, find k.

Q18. If (x+1) is a factor of $x^2 - 3ax + 3a - 13$, find k.

Q19. If zeros of the polynomial $x^2 - 4x + 2p$ are a and $2/a$,then find the value of a.

Q20. If one of the zeros of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find k.

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

SECTION – C

LEVEL 1

Q1. Find the zeroes of the following polynomial by factorisation method and verify the relations between the zeroes and their coefficients

i) $7y^2 - \frac{11}{3}y - \frac{2}{3}$ ii) $\sqrt{3}x^2 + 10x + 7\sqrt{3}$ iii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

LEVEL 2

Q2. If one zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Q3. If p and q are the zeroes of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeroes are ${}^{1}/_{p}$ and ${}^{1}/_{q}$.

Q4. Find the zeros of the quadratic polynomial ($5u^2$ + $10u$) and verify the relation between the zeros and the coefficients.

Q5. Find zeroes of the Polynomial p(x) = $4x^2 + 5\sqrt{2}x - 3$ & verify relationship between the zeroes and the co-efficient of the polynomials.

Q6. Find the zeroes of the following quadratic polynomials $6x^2 - 3 - 7x$ and verify the relationship between the zeros and the coefficients.

LEVEL 3

Q7. If α and β are zeroes of the quadratic polynomial $x^2 - (k+6)x + 2(2k-1)$. Find the value of k if α + β = $\frac{1}{2}$ α β.

Q8. If m and n are zeroes of $ax^2 - 5x + c$, find the values of a and c if $m + n$ = mn = 10

Q9. Find the value of k in order that one zero of $3x^2 + (1+4k)x + k^2 + 5$ may be one third of the other.

Q10. The zeroes of $x^2 - kx + 6$ are in the ratio 3:2, find k.

Q11. Find the zeros of the quadratic polynomial ($5u^2 + 10u$) and verify the relation between the zeros and the coefficients.

Q12. Find zeroes of the Polynomial p(x) = $4x^2 + 5\sqrt{2}x - 3$ & verify relationship between the zeroes and the co-efficient of the polynomials.

Q13. Find the zeroes of the following quadratic polynomials $6x^2 - 3 - 7x$ and verify the relationship between the zeros and the coefficients.

Q14. If α, β are zero of quadratic polynomial $kx^2 + 4x + 4$, find the values of k such that $(\alpha + \beta)^2 - 2\alpha\beta = 24$

HOTS

Q15. If sum of the squares of the zeroes of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k.

Q16. If the sum of the zeroes of the polynomial $p(x) = (a+1)x^2 + (2a+3)x + (3a+4)$ is -1, then find the product of the zeroes.

Q17. If (x + a) is a factor of two polynomials $x^2 + px + q$ and $x^2 + mx + n$, then prove that $a = \frac{n-q}{m-P}$

Q18. Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer k > 1?

LONG ANSWER TYPE QUESTIONS (4 MARK QUESTIONS)

SECTION – D

HOTS

Q1. If α and β are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$, find the value of

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\,\alpha\beta$

Q2. If the squared difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Q3. If α and β are the roots of the equation $ax^2 + bx + c = 0$ and if $px^2 + qx + r = 0$ has roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$, then r is

Q4. If a and b are the zeroes of the quadratic polynomial $f(x) = x^2 - px + q$, prove that $\frac{a^2}{b^2} + \frac{b^2}{a^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2.$

Q5. If l and m are zeroes of the polynomial $p(x) = 2x^2 - 5x + 7$, find a polynomial whose zeroes are $2l + 3$ *and* $2m + 3$.

Q6. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c, a \neq 0, c \neq 0$.

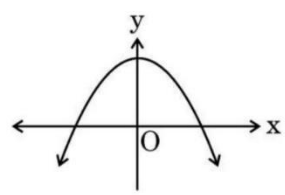
Q7. **If the polynomial** $16a^4 + 8a^2 - 15$ **have real zeroes, find them.**

Q8. If one zero of the polynomial $p(x) = 2x^2 - 4kx + 6x - 7$ is the negative of other find the zeros of $x^2 - kx - 1$.

CASE STUDY BASED QUESTIONS

CASE STUDY 1

1. Rainbow is an arch of colours that is visible in the sky after rain or when water droplets are present in the atmosphere. The colours of the rainbow are generally, red, orange, yellow, green, blue, indigo and violet. Each colour of the rainbow makes a parabola. We know that any quadratic polynomial **$p(x)=ax^2+bx+c (a\neq0)$** represents a parabola on the graph paper.



(CBSE 2022

Based on the above, answer the following questions :

i. The graph of a rainbow $y=f(x)$ is shown in the figure.Write the number of zeroes of the curve.

ii. If the graph of a rainbow does not intersect the x-axis but intersects y-axis at one point, then how many zeroes will it have?

iii) (a) If a rainbow is represented by the quadratic polynomial $p(x)=x^2+(a+1)x+b$, whose zeroes are 2 and -3 , find the value of a and b.

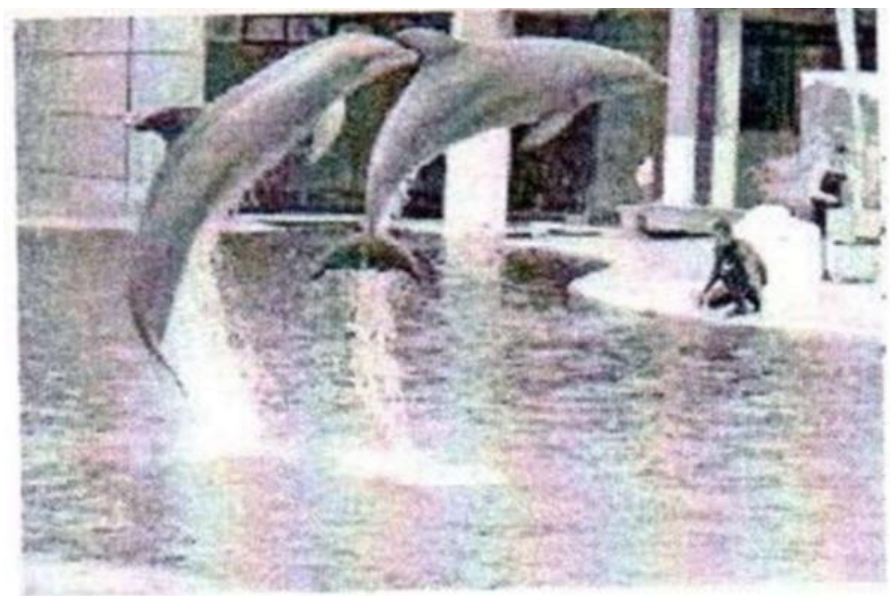
OR

(b) The polynomial $x^2-2x-(7p+3)$ represents a rainbow. If -4 is a zero of it, find the value of P

CASE BASED QUESTION - 2

In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h=20t-16t^2$ following questions :

(CBSE 2023 STANDARD)



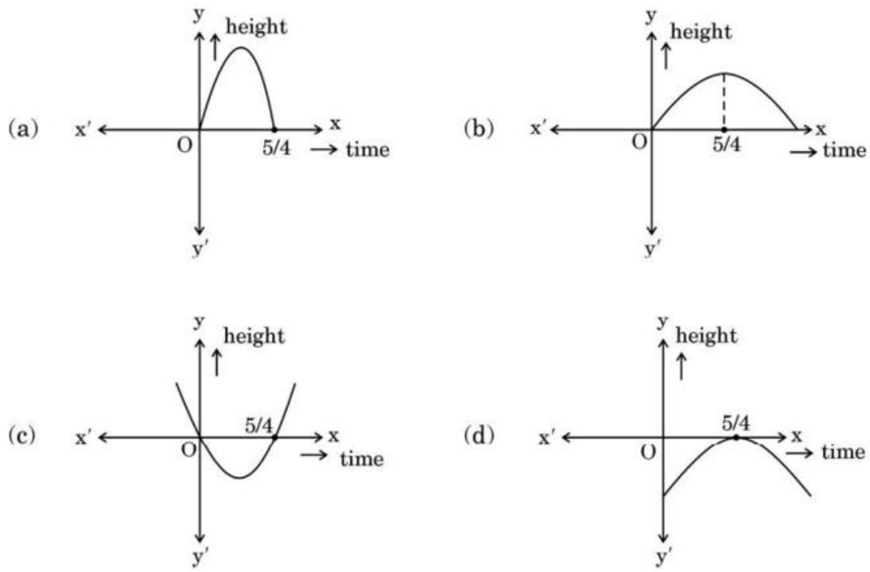
Based on the above answer the following questions

1. Find the Zeroes of the polynomial
2. What would the value of h at t = 3/2? Interpret the result

OR

How much distance has the dolphin covered before hitting the water level

Which of the following types of graph represents p(t)



MCQ (1 MARK)	
Q NO	ANSWER
1	(b) Since 2 is zero P(2)=0 P(2) = 2^2 + 3x2 +k=0 which gives k= -10
10	(b) Given $\alpha = 2$ $and\ \beta = \frac{1}{2}$ $\alpha + \beta = \frac{-5}{p}$ $and\ \alpha\beta = \frac{r}{p}$ $\alpha + \beta = 2 + \frac{1}{2} = \frac{5}{2} = \frac{-5}{p}$ Cross multiplying, we get $p = -2$ $\alpha\beta = \frac{r}{p} = 1$ Cross multiplying, we get $r = p = -2$
11.	(d) Since the graph touched the X-axis at three different points, the polynomial will have three zeros.
2	(a) Given $\alpha + \beta = 0$ $\alpha = 3$ $so\ \beta = -3$ $p(x) = k(x^2 - (\alpha + \beta)x + \alpha\beta)$ $p(x) = k(x^2 - 9)$
3	(a) Let the zeros be α , $\frac{1}{\alpha}$ So $\alpha\ X \frac{1}{\alpha} = 1 = \frac{4k}{k^2+4}$ cross multiplying we get $k^2 - 4k + 4 = 0 \Rightarrow (k-2)^2 = 0$ which gives k = 2
4	(b) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-1}{1} = -1$ $\{\alpha + \beta = -1\ and\ \alpha\beta = 1\}$
5	(d) $P(x) = k(x^2 - (-2+5)x + -2\ X\ 5) = k(x^2 = 3x - 10)$ Since k can take infinite number of values, there can be more than three polynomials.
6	(b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (\frac{-b}{a})^2 - 2X\frac{c}{a} = \frac{b^2 - 2ac}{a^2}$

	$\frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{b^2-2ac}{a^2} \div (\frac{c}{a})^2 = \frac{b^2-2ac}{a^2} X \frac{a^2}{c^2} = \frac{b^2-2ac}{c^2}$
7	(b) $P(x) = x^2 - p(x+1) - c = x^2 - px - (p+c)$ $\{a = 1 \quad b = -p, \quad c = -(p+c)\}$ $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$ $= \frac{c}{a} + \frac{-b}{a} + 1 \qquad = \frac{-(p+c)}{1} + \frac{-(-p)}{1} + 1 = 1 - c$
8	(c) $P(x) = x^2 - 6x + k \quad a = 1,\ b = -6,\ c = k$ $\alpha + \beta = \frac{-(-6)}{1} = 6 \ \dots\dots\dots(1) \qquad \alpha\beta = \frac{c}{a} = \frac{k}{1} = k$ Given $3\alpha + 2\beta = 20 \dots\dots\dots\dots(2)$ Multiplying equation (1) by 3 and subtracting from (2) we get $\beta = -2$. Substituting this in equation (1) get $\alpha = 8$ So $k = \alpha\beta = -2\,X 8 = -16$
9	(b) $P(x) = x^2 - 5x + 4 \qquad P(x) + 2 = x^2 - 5x + 4 + 2 = x^2 - 5x + 6 = (x-3)(x-2)$ So the zeros are 3, 2
OBJECTIVE QUESTIONS (1 MARK)	
Q NO	ANSWER
5	$f(x) = 9x^3 - 18x^2 - x - 2$ If -2 is a zero then $f(-2) = 0$ $f(-2) = 9X((-2)^3 + 18X(-2)^2 - (-2) - 2$ $= 9\,X(-8) + 18\,X\,(4) + 2 - 2$ $= -72 + 72 + 2 - 2 = 0$ Since $f(-2) = 0$ -2 is a zero of the given polynomial.
1	$x^2 - x - 6 = (x-3)(x+2)$ so the zeros are 3 and -2
2	Polynomial = $K(x^2 - (\alpha+\beta)x + \alpha\beta)$ $= K(x^2 - (2+\sqrt{3} + 2 - \sqrt{3})x + (2+\sqrt{3})(2-\sqrt{3}))$ $= K\left(x^2 - (4)x + 2^2 - (\sqrt{3})^2\right) = K(x^2 - 4x + (4-3))$ $= K(x^2 - 4x + 1)$
3	. $P(x) = K\,(x^2 - (\alpha+\beta)x + \alpha\beta) \quad = K(x^2 - 6x + 4)$
8	$\frac{-1}{4}$, $\frac{1}{4}$ $P(x) = k(x^2 - \left(\frac{-1}{4} + \frac{1}{4}\right)x + \frac{-1}{4}\,X\frac{1}{4})$ $= k\left(x^2 - 0x - \frac{1}{16}\right) = k(x^2 - \frac{1}{16})$ If $k = 16 \qquad P(x) = 16x^2 - 1$
4	Given $\alpha, \frac{1}{\alpha}$ are the zeros of the polynomial. Product of the zeros $= \frac{c}{a} = \frac{k-4}{4}$ $\alpha\,X\frac{1}{\alpha} = \frac{k-4}{4}$ $1 = \frac{k-4}{4}$ Cross multiplying we get k = 8
6	. $P(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ Sum= 5 and product = $4\sqrt{3}\,X\,-2\sqrt{3} = -8\,X3 = -24$ The numbers are -3 *and* $+8$ By splitting the middle term, we get

	$P(x) = 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$ $= 4x(\sqrt{3}\,x + 2) - \sqrt{3}(\sqrt{3}\,x + 2)$ $=(\sqrt{3}\,x + 2)(4x - \sqrt{3})$ The zeros are $\frac{-2}{\sqrt{3}}$ and $\frac{\sqrt{3}}{4}$
7	$P(x) = 2x^2 + x + k$ Given 3 is a zero so $P(3) = 0$ $P(3) = 2\,X\,(3)^2 + 3 + k = 0$ $2\,X\,9 + 3 + k = 0$ $21 + k = 0$ $\quad$ *which gives* $\quad k = -21$
9	Given $P(y) = 6y^2 - 7y + 2$ here $\alpha + \beta = \frac{7}{6}$ *and* $\alpha\beta = \frac{2}{6}$ The given zeros are $\frac{1}{\alpha}$ *and* $\frac{1}{\beta}$ sum of zeros = $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$ $\frac{\frac{7}{6}}{\frac{2}{6}} = \frac{7}{2}$ Product of zeros $= \frac{1}{\alpha}\,X\,\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{2}{6}} = \frac{6}{2}$ The new polynomial is $P(y) = k(y^2 - (sum)y + product)$ $P(y) = k(\,y^2 - \left(\frac{7}{2}\right)y + \frac{6}{2})$ When k = 2 $\quad P(y) = 2y^2 - 7y + 6$
10	$P(x) = ax^2 - 6x + c$ *given* $\alpha + \beta = 12$ $\frac{6}{a} = 12$ *which gives* $a = \frac{6}{12} = \frac{1}{2}$ $\alpha\,\beta = 12$ *which gives* $\frac{c}{a} = 12 = \frac{c}{\frac{1}{2}} = 12$ *which gives* $c = 12 \times \frac{1}{2} = 6$

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

Q NO	ANSWER
4	x= -3 is not a zero
5	$p = 1; q = -2$
1	$\pm\frac{3}{\sqrt{2}}$
2	$x^2 - 15x - 2$
3	$.-\frac{5}{13}$
6	$a = -\frac{13}{2}$
7	Applying quadratic formula we get x= -1.3 ,-97.7 .
8	$x^2 - 8x + 12$; zeros are 6,2
9	k=9
12	one zero is $2 + \sqrt{5}$ *sum is* 4 , *other root is* $2 - \sqrt{5}$; Quadratic polynomial is $x^2 - 4x - 1$
10	$a = -2$
11	we cannot find two numbers a and b with sum 4 and product 7.So polynomial has no zeros
13	± 5
14	$p = 5\,, q = -6$

15	$k = \pm\frac{3}{2}$
16	k=6
17	k=-2/3
18	a=2
19	a=1
20	K = 0

SHORT ANSWER TYPE QUESTIONS(3 MARKS)

Q NO	ANSWER	Q NO	ANSWER	Q NO	ANSWER
1	i) y = $\frac{14}{21}, -\frac{1}{7}$ ii) x = $-\sqrt{3}, -\frac{7}{\sqrt{3}}$ iii) x = $-\frac{2}{\sqrt{3}}, \frac{3}{4\sqrt{3}/2}$	6	x = $\frac{3}{2}, \frac{-1}{3}$	11	k = -1 , $\frac{2}{3}$
2	k = $\frac{2}{3}$	7	k = 7	12	k = 12
3	$\frac{1}{2}(2y^2 - 7y + 6)$	8	a = $\frac{1}{2}$ and c = 5	13	Product = -2
4	u = -2, 0	9	k = $\frac{79}{8}$	14	Correct proof
5	x = $\frac{1}{2\sqrt{2}}, -\frac{3}{\sqrt{2}}$	10	k = ± 5	15	cannot have equal zeros for any odd integer k > 1

LONG ANSWER TYPE QUESTIONS(4 MARKS)

Q NO	ANSWER
1	Sum of zeroes = $\alpha+\beta = \frac{-b}{a} = -(\frac{-6}{3}) = 2$ ....(i) Product of zeroes = $\alpha\beta = \frac{c}{a} = \frac{4}{3}$ -…………..(ii) Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\,(\frac{1}{\alpha} + \frac{1}{\beta}) + 3\,\alpha\beta = \frac{\alpha^2+\beta^2}{\alpha\beta} + 2\,(\frac{\alpha+\beta}{\alpha\beta}) + 3\,\alpha\beta$ $= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\,(\frac{\alpha+\beta}{\alpha\beta}$ $= \frac{(2)^2 - 2(\frac{4}{3})}{\frac{4}{3}} + 2\,(\frac{2}{\frac{4}{3}}) + 3(\frac{4}{3})$ $= 1 + 3 + 4 = 8$
2	f(x) = x^2+px +45 Sum of zeroes = $\alpha+\beta = \frac{-b}{a}$ = -p (i) Product of zeroes = $\alpha\beta = \frac{c}{a}$ = 45…(ii) Given $(\alpha - \beta)^2 = 144$ $(\alpha + \beta)^2 - 4\alpha\beta = 144$ $(-p)^2 - 4(45) = 144$, $P^2 = 144 + 180 = 3$, P = √324 = 18
3	Since α and β are the roots of the equation $ax^2 + bx + c = 0$, so,

<table>
<tr><td></td><td>

$\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

The equation with roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$ can be written as

$x^2 - \{\frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta}\}\, x + \{\frac{1-\alpha}{\alpha} * \frac{1-\beta}{\beta}\} = 0$ ……………….. 1

Now, sum of zeroes , $\{\frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta}\} = \frac{\alpha + \beta - 2\alpha\beta}{\alpha\beta}$

$= \frac{\alpha+\beta}{\alpha\beta} + \frac{-2\alpha\beta}{\alpha\beta} = \frac{\alpha+\beta}{\alpha\beta} - 2$, ………………………………....2

$= \frac{-b}{c} - 2 = \frac{-b-2c}{c}$, since $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

Product of zeroes

$\frac{1-\alpha}{\alpha} * \frac{1-\beta}{\beta} = \frac{1-(\alpha+\beta)+\alpha\beta}{\alpha\beta} = \frac{1-\frac{-b}{a}+\frac{c}{a}}{\frac{c}{a}} = \frac{a+b+c}{c}$ ………………….3

Putting 2 and 3 in 1

The required equation is $x^2 - \{\frac{-b-2c}{c}\}x + \frac{a+b+c}{c} = 0$

$cx^2 + (b + 2c)\, x + (a + b + c) = 0$ —- (i)

On comparing equation (i) with the equation given $px^2 + qx + r = 0$, $r = a + b + c$.

</td></tr>
<tr><td>4</td><td>

Sum of zeroes = a+ b = p

Product of zeroes = ab = q

$\frac{a^2}{b^2} + \frac{b^2}{a^2} = \frac{a^4 + b^4}{a^2b^2} = \frac{(a^2 + b^2)^2 - 2a^2b^2}{a^2b^2}$

$= \frac{[(a+b)^2 - 2ab]^2 - 2a^2b^2}{a^2b^2} = \frac{[p^2-2q]^2 - 2q^2}{q^2}$

$= \frac{p^4 - 4p^2q + 4q^2 - 2q^2}{q^2} = \frac{p^4-4p^2q+2q^2}{q^2}$

$= \frac{p^4}{q^2} - \frac{-4p^2q}{q^2} + \frac{2q^2}{q^2}$

$= \frac{p^4}{q^2} - \frac{-4p^2q}{q^2} + 2$

</td></tr>
<tr><td>5</td><td>

$l + m = \frac{5}{2}$, $lm = \frac{7}{2}$

a polynomial whose zeroes are $2l + 3$ and $2m + 3$ is

$x^2 - (2l + 3 + 2m + 3)x + (2l + 3)(2m + 3)$

$= x^2 - [2(l + m) + 6)]x + (4lm + 6(l + m) + 9)$

$= x^2 - 5x + 6x + 14 + 15 + 9$

$= x^2 + x + 38$

</td></tr>
<tr><td>6</td><td>

Let α and β be the zeroes of the polynomial $f(x) = ax^2 + bx + c$.

</td></tr>
</table>

	So, $\alpha + \beta = -b/a$, $\alpha\beta = c/a$ Now, the sum of zeroes = $(1/\alpha) + (1/\beta)$ = $(\alpha + \beta)/\alpha\beta$ = $(-b/a)/(c/a)$ = $-b/c$ Product of two zeroes = $(1/\alpha)(1/\beta)$ = $1/\alpha\beta$ = $1/(c/a)$ = a/c The required quadratic polynomial = $k[x^2 – (\text{sum of zeroes})x + (\text{product of zeroes})]$ $= k[x^2 – (-b/c)x + (a/c)] = k[x^2 + (b/c) + (a/c)]$
7	**The polynomial $16a^4 + 8a^2 -15$ = $(4a^2)^2 + 2(4a^2) -15$** **Put $4a^2 = x$,** **$x^2+ 2x -15 = 0$** $x^2+ 5x -3x -15 = 0$, $x(x+5) -3(x+5) = 0$, $(x+5)(x-3) = 0$ $x= -5$, $x=3$, If $x= -5$, $a = \sqrt{-5/2}$, If $x = 3$, $a = \frac{\sqrt{3}}{2}$
8	$p(x) = 2x^2 – 4kx + 6x – 7$ let the zeroes be a, -a , sum of zeroes = a + -a = 0 $2x^2 – 4kx + 6x – 7 = 2x^2 –x(4k – 6) – 7$ Sum of zeroes = $(4k-6)/ 2 = 2k-3$ But $2k-3 = 0$, $K = 3/2$ Now, $x^2 – kx – 1 = x^2 – \frac{3}{2}x – 1 = 2x^2 – 3x – 2$

CASE STUDY BASED QUESTIONS

9 CASE STUDY 1		10. CASE STUDY 2	
Q NO	ANSWER	Q NO	ANSWER
(i)	2	(i)	c) 0,5/4
(ii)	no zero	(ii)	C
(iii)a	a=0 , b= -6	(iii)	c) 25 cm
(iii)b	p = 3	(iv)	a)1.25 s

LINEAR EQUATIONS IN TWO VARIABLES

An equation which can be put in the form ***ax* + *by* + *c* = 0**, where *a*, *b* and *c* are real numbers, and ***a* and *b* are not both zero ($a^2 + b^2 \neq 0$)**, is called a linear equation in two variables *x* and *y*.

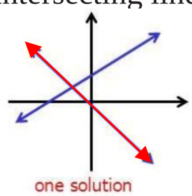
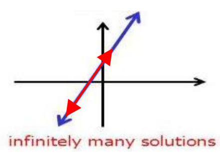
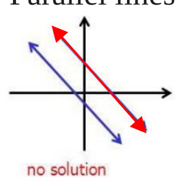
Each solution (x, y) of a linear equation in two variables, $ax + by + c = 0$, corresponds to a point on the line representing the equation, and vice versa.

The general form of a pair of linear equations is

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Interpretation of the pairs of equations

Ratio comparison	Graphical representation	Algebraic interpretation	Consistent/ Inconsistent
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines 	Exactly one solution (unique)	**consistent**
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coinciding lines 	Infinite solution	**dependent (consistent)**
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines 	no solution	**Inconsistent**

Algebraic Methods:

- **Substitution Method**

Following are the steps to solve the pair of linear equations by substitution method:

$a_1x + b_1y + c_1 = 0$ … (i) and

$a_2x + b_2y + c_2 = 0$ … (ii)

Step 1: We pick either of the equations and write one variable in terms of the other

Step 2: The expression obtained in Step1 should be substituted in the other equation to get a linear equation in one variable

Step 3: Solve this equation and get the value of one variable

- **Algebraic Methods:**

a) Substitution Method

Following are the steps to solve the pair of linear equations by substitution method:

$$a_1x + b_1y + c_1 = 0 \ldots (i) \text{ and}$$

$$a_2x + b_2y + c_2 = 0 \ldots (ii)$$

Step 1: We pick either of the equations and write one variable in terms of the other

Step 2: The expression obtained in Step1 should be substituted in the other equation to get a linear equation in one variable

Step 3: Solve this equation and get the value of one variable

Step 4: Substitute this value in the equation obtained in Step 1 to obtain the value of the other variable.

b) Elimination Method

Step 1: First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.

Step 2: Then add or subtract one equation from the other so that one variable gets eliminated.

❖ If you get an equation in one variable, go to Step 3.

❖ If in Step 2, we obtain a true statement involving no variable, then the original pair of equations has infinitely many solutions.

❖ If in Step 2, we obtain a false statement involving no variable, then the original pair of equations has no solution, i.e., it is inconsistent.

Step 3: Solve the equation in one variable (x or y) so obtained to get its value.

Step 4: Substitute this value of x (or y) in either of the original equations to get the value of the other variable.

MULTIPLE CHOICE QUESTIONS

SECTION A

LEVEL 1

Q1. The pair of equations 3x – 5y = 7 and – 6x + 10y = 7 have
a) a unique solution
b) infinitely many solutions
c) no solution
d) two solutions

Q2. The pair of equation x = – 4 and y = – 5 graphically represents lines which are
a) intersecting at (- 5, – 4)
b) intersecting at (- 4, – 5)
c) intersecting at (5, 4)
d) intersecting at (4, 5)

Q3. One equation of a pair of dependent linear equations is 2x + 5y = 3. The second equation will be
a) 2x + 5y = 6
b) 3x + 5y = 3
c) -10x – 25y + 15 = 0
d) 10x + 25y = 15

Q4. The value of k, for which equations 3x + 5y = 0 and kx + l0y = 0 has a non-zero solution is
a) 6 b) 0 c) 2 d) 5

Q5. Hrithu has only ₹1 and ₹2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹75, then the number of ₹1 and ₹2 coins are, respectively
a) 35 and 15
b) 15 and 35
c) 35 and 20
d) 25 and 25

Q6. How many solutions does the system of equations 3x-4y=5 and 12x-16y =20 have?
a) a unique solution
b) more than two solutions
c) no solution
d) two solutions

Q7. For what value of k will the equations 4x+6y=11 and 2x+ky=7 be inconsistent
a) 6 b) 3 c) 2 d) 5

Q8. Find c if the system of equations $cx + 3y + (3 - c) = 0$; $12x + cy - c = 0$ has infinitely many solutions?
a) 7 b) 3 c) 6 d) 5

Q9. For what value of m the system of linear equations has unique solution?

$$2x + 3y = 7$$
$$2mx + y = 28$$

a) $m \neq 1/6$
b) $m \neq 1/3$
c) $m \neq 1/2$
d) $m \neq 1/5$

Q10. Find the value of of k so that $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution
a) k ≠6 b) k ≠3 c) k ≠2 d) k ≠7

Q11. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is
a) $\frac{-5}{4}$ b) $\frac{2}{5}$ c) $\frac{15}{4}$ d) $\frac{3}{2}$

LEVEL 2

Q12. The pair of equations $ax + 2y = 9$ and $3x + by = 18$ represent parallel lines where a,b are integers if
a) $a = b$ b) $3a = 2b$ c) $2a = 3b$ d) $ab = 6$

Q13. The solution of the pair of linear equations $x + y = a + b$ and $ax - by = a^2 - b^2$ is

a) $x = b, y = a$
b) $x = -a, y = b$
c) $x = a, y = b$
d) $x = a, y = -b$

Q14. The value of k for which the pair of linear equations $x + y - 4 = 0$ and $2x + ky - 3 = 0$ have no solution is

a) 0
b) 2
c) 6
d) 8

Q15. If $am \neq bl$, then the system of equations $ax + by = c$ and $lx + my = n$

a) has a unique solution
b) has no solution
c) has infinitely many solutions
d) may or may not have a solution

Q16. Graphically, the pair of equations $7x - y = 5$, $28x + 4y = 11$, represents two lines which are

a) Intersecting at one point
b) Parallel
c) Coincident
d) Intersecting at two points

Q17. 3 chairs and 1 table cost ₹900 whereas 5 chairs and 3 tables cost ₹2100.If the cost of one chair is ₹x and the cost of one table is ₹y, then the situation can be represented algebraically as

a) $3x + y = 900, 3x + 5y = 2100$
b) $x + 3y = 900, 3x + 5y = 2100$
c) $3x + y = 900, 5x + 3y = 2100$
d) $x + 3y = 900, 5x + 3y = 2100$

LEVEL 3

Q18. If $x = a, y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are

a) $x = 3, y = -1$
b) $x = 1, y = 3$
c) $x = -3, y = 1$
d) $x = 3, y = 1$

Q19. The solution of the pair of linear equations $x = -5$ and $y = 6$ is

a) (-5,6)
b) (-5,0)
c) (0,6)
d) (0,0)

Q20. The value of a for which the system of equations $ax + 2y - 4 = 0$ and $x - y - 3 = 0$ will represent intersecting lines:

a) $a = -2$
b) $a \neq -2$
c) $a = 2$
d) $a \neq 2$

2 MARKS QUESTIONS

LEVEL 1

Q1. Solve for x and y: 141x + 93y = 189 ; 93x + 141y = 45

Q2. Find the value of k for which the pair of linear equations $kx + 3y = k - 2$ and $12x + ky = k$ has no solution.

Q3. Find the value of k so that the point (3, k) lies on the line represented by x-5y=5.

Q4. How many solutions does the pair of equations y = 0 and y = -5 have?

LEVEL 2

Q5. Without drawing the graph, find out whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident.

$18x - 7y = 24$; $\frac{9}{5}x - \frac{7}{10}y = \frac{9}{10}$

Q6. Anu's father is three times as old as Anu. After five years, his age will be two and half times as old as Anu. Represent this situation algebraically.

Q7. If sum of two positive numbers is 108 and the difference of these numbers is 8, then find the numbers.

LEVEL 3

Q8. Solve the following pair of linear equations by substitution method:

i. $3x - 7y - 4 = 0$

ii. $9x = 2y - 7$

Q9. Solve the pair of linear equations by elimination method:

i. $x - y + 1 = 0$

ii. $4x + 3y - 10 = 0$

Q10. Find the value of k for which the given system of equations has infinitely many solutions:

i. $(k - 3)\, x + 3\, y = k$

ii. $k\, x + k\, y = 12$

Q11. Write a pair of linear equations which has a unique solution $x = 2$ *and* $y = -1$. How many such pairs are possible?

Q12. If $ax + by = a^2 - b^2$ and $bx + ay = 0$, find the value of $(x + y)$.

Q13. Solve for x and y:

i $m\,x - n\,y = m^2 + n^2$;

ii $x - y = 2n$

Q14. Is the system of linear equations $2x + 3y - 9 = 0$ *and* $4x + 6y - 18 = 0$ consistent? Justify your answer.

Q15. For which value of a and b does the following pair of linear equations has infinite number of solutions?

i. $2x + 3y = 7$

ii. $a(x + y) - b\,(x - y) = 3a + b - 2$

Q16. There are 20 vehicles – cars and motorcycles in a parking area. If there are 56 wheels together, how many cars and motorcycles are there.

Q17. If $x - 4$ is a factor of $x^3 + ax^2 + 2bx - 24$ and $a - b = 8$, find the value of a and b.

Q18. Are the following pair of linear equations consistent? Justify your answer.

$2ax + by = a;\ 4ax + 2by - 2a = 0\ ;\ a\,,b \neq 0$

Q19. If $2x + y = 23$ and $4x - y = 19$, find the values of $5x - 3y$.

Q20. Find the solutions of the pair of linear equations $5x + 10y - 50 = 0$ and $x + 8y = 10$.

Hence find the value of m if $y = mx + 5$.

SECTION C

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

LEVEL 1

Q1. Solve by elimination:

a. $x - y + 1 = 0$ and $4x + 3y - 10 = 0$

b. $3x - 4y = 15$ and $2x - 2y = 8$

Q2. Determine the values of a and b for which the following system of linear equations have infinite solutions

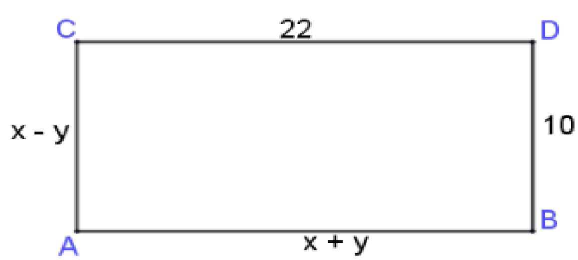
$2x - (a - 4)y = 2b + 1\ ;$

$4x - (a - 1)y = 5b - 1$

Q3. Solve for x and y:

$\frac{x}{a} + \frac{y}{b} = 2$ and $ax - by = a^2 - b^2$

Q4. In the given figure ABCD is a rectangle. Find the value of x and y



LEVEL 2

Q5. For each of the following system of equations determine the values of k for which the given system has no solution

$3x - 4y + 7 = 0$

$kx + 3y - 5 = 0$

Q6. For what value of k, will the following system of equations have infinitely many solutions

$2x + 3y = 4$

$(k+2)\,x\ +6y\ =\ 3k+2$

Q7. Find the two-digit numbers whose sum is 75 and difference is 15.

LEVEL 3

Q8. The age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father

Q9. On reversing the digit of a two-digit number, the number obtained is 9 less than three times the original number. If the difference of these two numbers is 45, find the original number

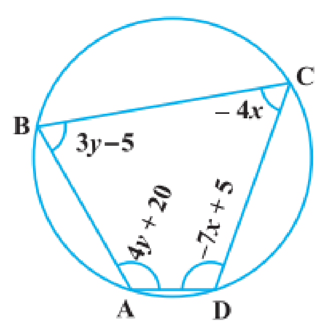
Q10. A and B each have a certain number of oranges. A says to B, "if you give me 10 of your oranges, I will have twice the number of oranges left with you." B replies," if you give me 10 of your oranges, I will have the same number of oranges as left with you. Find the number of oranges with A and B separately.

SECTION D

LONG ANSWER TYPE QUESTIONS (3 MARKS)

Q1. Draw the graph of $2x\ +\ y\ =\ 6\ and\ 2x - y\ +\ 2\ =\ 0$. Shade the region bounded by these lines and x-axis. Also find the area of the shaded region.

Q2. ABCD is a cyclic quadrilateral. Find the angles of the cyclic quadrilateral.



Q3. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Q4. The sum of the numerator and the denominator of a fraction is 3 less than twice the denominator. If the numerator and the denominator are decreased by one, the numerator becomes half the denominator. Determine the fraction.

Q5. A number consists of two digits. When the number is divided by the sum of its digits, the quotient is 7. If 27 is subtracted from the number, the digits interchange their places. Find the number.

Q6. A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first-class ticket from the station A to B costs ₹2530. Also one reserved first class ticket and one reserved first class half ticket from A to B costs ₹3810.Find the full first class fare from station A to B and also the reservation charges for a ticket.

Q7.The annual income of A and B are in the ratio 3:4 and their annual expenditure are in the ratio 5:7 .If each saves Rs 15000 annually, find their annual incomes?

Q8. Solve the following:

$$2(a\,x - by) + (a + 4b) = 0$$

$$2(bx + ay) + (b - 4a) = 0$$

Q9. A and B each have certain number of mangoes. A says to B , 'If you give me 30 of your mangoes ,I will have twice as many as left with you. B replies 'If you give me 10 , I will have thrice as many as left with you .' How many mangoes does each have?

Q10. In a competitive examination, one mark is awarded for each correct answer while 21 mark is deducted for every wrong answer. Jayanthi answered 120 questions and got 90 Marks. How many questions did she answer correctly.

ASSERTION REASONING QUESTIONS

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
c) Assertion (A) is true but reason (R) is false.
d) Assertion (A) is false but reason (R) is true.

Q1. **Assertion:** The graph of the linear equations 3x+7y=11 and 5x-2y=4 gives a pair of intersecting lines.
Reason: The graph of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ gives a pair of intersecting lines if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Q2. **Assertion:** If one equation of a pair of dependent linear equations is

-3x+5y-2=0, then -6x+10y-4=0 can be the second equation.

Reason: The condition for a pair of linear equations to be dependent is

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Q3. **Assertion:** The value of k for which the system of linear equations 3x-4y=7 and 6x-8y=k have infinite number of solutions is 14.
Reason: The graph of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ gives a pair of intersecting lines if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Q4. **Assertion:**The pairs of equations x+2y-5 = 0 and -4x-8y+20=0 have infinitely many solution.
Reason: if a1/a2 = b1/b2 = c1/c2 then the pair of equations has infinitely many solutions.

Q5. **Assertion:** If a pair of linear equations is consistent, then the lines are intersecting or coincident
Reason: Because the two lines definitely have a solution.

Q6. **Assertion**: The pairs of equations 9x + 3y + 12 = 0 and 18x + 6y + 26 = 0 have no solution.

Reason: a1/a2 = b1/b2 ≠ c1/c2 So, the pairs of equations are parallel and the lines never intersect each other at any point, therefore there is no possible solution.

Q7. **Assertion:** If the lines 3x+2ky – 2 = 0 and 2x+5y+1 = 0 are parallel, then the value of k is 15/4

Reason: The condition for parallel lines is a1/a2 = b1/b2 ≠ c1/c2

Q8. **Assertion**: The value of k for which the system of equations 3x+ky=0 and 2x-y=0 has a unique solution is k ≠ -3/2
Reason: The graph of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ gives a pair of parallel lines if $a_1/a_2 = b_1/b_2 = c_1/c_2$.

Q9. **Assertion:** The number of common solutions for the system of linear equations 5x+4y+6=0 and 10x+8y=12 is zero.
Reason: The graph of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ gives a pair of intersecting lines if $a_1/a_2 \neq b_1/b_2$

Q10. **Assertion:** A pair of linear equations has no solution (s) if it is represented by intersecting lines graphically.
Reason: If the pair of lines are intersecting, then the pair has unique solution and is called consistent pair of equations.

ANSWERS: Section E

1a 2a 3b 4a 5a 6a 7a 8c 9d 10 d

CASE STUDY BASED QUESTIONS

CASE STUDY -1

Special offers are short-term pricing strategies that businesses, especially shops will adopt to encourage customers to buy from them. During winter season, a shopkeeper sells a jacket at 8% profit and a sweater at 10 % discount thereby getting a sum of ₹1008. If she had sold the jacket at 10 % profit and the sweater at 8 % discount, she would have got ₹1028. Denoting the cost price of one jacket by ₹ x and the list price of one sweater by ₹ y, answer the following situations.

I. Represent the first situation algebraically.
   a) 12x+10y=11200
   b) 10x+12y=11200
   c) 12x-10y=11200
   d) 10x-12y=1120

II. Represent the second situation algebraically
   a) 46x+55y=51400
   b) 55x+46y=51400
   c) 55x-46y=51400
   d) 46x-55y=51400

III. The system of linear equations representing both the situations will have.
   a) Infinite number of solutions
   b) Unique solution
   c) No Solutions
   d) Exactly two solutions

IV. The graph of the system of linear equations representing both the situations will be
   a) Parallel lines
   b) Coincident lines
   c) Intersecting lines
   d) None of these

CASE STUDY- 2:

The tradition of pottery making in India is very old. In fact it is older than Indus Valley Civilisation The shaping and baking of clay articles has continued through the ages. The picture of a potter is shown below (CBSE -2023)

A potter makes a certain number of pottery articles on a day. It was observed on a particular day the cost of production of each article (in₹) was 1 more than twice the number of articles produced on that day. The total cost of production on that day was ₹210

I. Take the number of articles produced on that day as , form a quadratic equation in .

II. Find the number of articles produced on that day and the cost of each article

CASE STUDY -3:

An alumni association is an association of former students. These associations often organize social events, publish newsletters or magazines and raise funds for the organisation.The alumni meet of two batches of a college- batch A & batch B were held on the same day in the same hotel in two separate halls “Rose” and “Jasmine”. The rents were the same for both the halls. The expense for each hall is equal to the fixed rent of each hall and proportional to the number of persons attending each meet. 50 persons attended the meet in “Rose” hall, and the organisers had to pay ₹ 10000 towards the hotel charges. 25 guests attended the meet in “Jasmine” hall and the organisers had to pay ₹ 7500 towards the hotel charges. Denote the fixed rent by ₹ x and proportional expense per person by ₹ y.

I. Represent algebraically the situation in hall "Rose".

a) $50x + y = 10000$
b) $50x - y = 10000$
c) $x + 50y = 10000$
d) $x - 50y = 10000$

II. Represent algebraically the situation in hall "Jasmine"

a) $x + 25y = 7500$
b) $x - 25y = 7500$
c) $25x + y = 7500$
d) $25x - y = 7500$

III. What is the fixed rent of the halls?

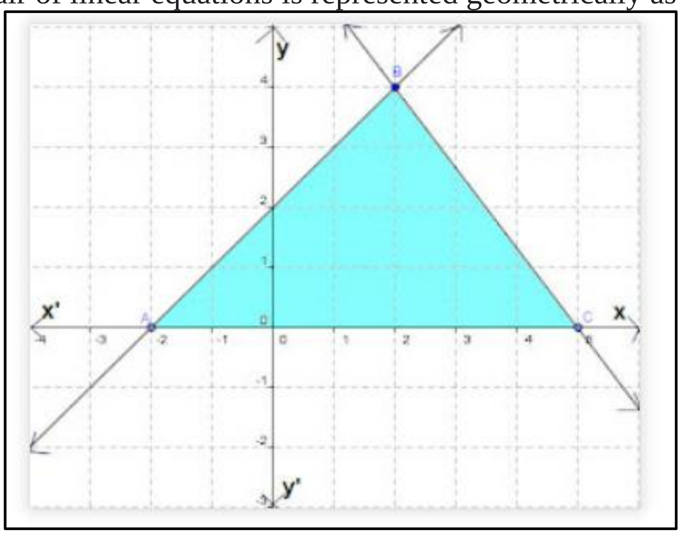
a) ₹2500
b) ₹3300
c) ₹ 4000
d) ₹5000

IV. Find the amount the hotel charged per person.

a) ₹ 150
b) ₹ 190
c) ₹130
d) ₹ 100

CASE STUDY -4:

A pair of linear equations is represented geometrically as shown below.



.

a) What can you say about the pair of linear equations?
   a) Consistent
   b) Inconsistent
   c) Dependent

b) From the graph, find the coordinates of the point, where the line AB intersects the X-axis
   a) (5,0)
   b) (-2,0)
   c) (0,2)
   d) (0,0)

c) From the graph, find the solution of the pair of linear equations

a) (4,2)

b) (2,4)

c) (-2,0)

d) (5,0)

d) What is the area of the shaded region?

a) 11 sq. units

b) 12 sq. units

c) 13 sq. units

d) 14 sq.units

ANSWERS

Question		n		on		on	n
1	c	6	b	11	c	16	a
2	b	7	b	12	d	17	c
3	c	8	c	13	c	18	d
4	a	9	b	14	b	19	a
5	d	10	a	15	a	20	b

SECTION B ANSWERS 2 - MARKS QUESTIONS	
Q1	Add two given equations $x + y = 1 \quad (1)$ $x - y = 3 \quad (2)$ $(1) + (2) \rightarrow \quad 2x = 4$ $x = 2$ $Sub\ x = 2\ in\ (1)\ y = -1$
Q2	$\frac{k}{12} = \frac{3}{k}$ $k^2 = 36$ $k = \pm 6$
Q3	$x - 5y = 5$ $at\ (3, k), \quad 3 - 5k = 5$ $-5k = 2$ $k = -2/5$
Q4	y=0 and y=-5 represent parallel lines. So, no. of solution is zero.
Q5	$\frac{18}{\frac{9}{5}} = \frac{10}{1}$ $\frac{-7}{\frac{-7}{10}} = 10$ $\frac{24}{\frac{9}{10}} = \frac{8x10}{3} = \frac{80}{3}$ $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$
Q6	$Let\ Anu's\ age = x$ $Father's\ age = y$ $x = 3y.......(1)$ $y + 5 = (2\frac{1}{2})(x + 5)$

	$2y + 10 = 5x + 25$ $5x - 2y = 15$…………(2)
Q7	$x + y = 108$ $x - y = 8$ $2x = 116$ $x = 58$ $y = 50$
Q8.	x = $\frac{7y+4}{3}$ $9 \times \frac{7y+4}{3} = 2y - 7$ $3(7y + 4) = 2y - 7$ 21y+12=2y-7 y=-1 x=(-7+4)/3 =-1
Q9.	$(1) \times 4 \rightarrow$ 4x – 4y = -4 4x + 3y=10……(2) $(1) - (2) \rightarrow \quad -7y = -14$ $y = 2$ $x = 1$
Q10.	$\frac{k-3}{k} = \frac{3}{k}$, k≠0 k = 6
Q11.	$+ y = 1$ Infinitely many solutions
Q12	$ax + by = a^2 - b^2$ ……… (1) $bx + ay = 0$ ……………… (2) adding (1) and (2) $a(x + y) + b(x + y) = a^2 - b^2$ $(x + y)(a + b) = (a + b)(a - b)$

	$x + y = a - b$
Q13.	$mx - ny = m^2 + n^2$ $mx - my = 2\ nm$ (1) – (2) $(m - n)\ y = m^2 + n^2 - 2\ nm = (m - n)^2$ $y = m - n$ $x - (m - n) = 2n$ $x = m + n$
Q14.	$\frac{2}{4} = \frac{1}{2}$, $\frac{-9}{-18} = \frac{1}{2}$, $\frac{3}{6} = \frac{1}{2}$ It has infinite number of solutions. It is consistent.
Q15.	$2x + 3y = 7$ $x(a - b) + y\ (a + b) = 3a + b - 2$ Since it has infinitely many solutions, $\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$ After equating $a = 5b$ (1) $2a - 5b = 6$ (2) Solve (1) and (2) $a = 5$ and $b = 1$
Q16.	Let no. of cars = x and no. of motor cycles = y According to the given condition $x + y = 20$ (i) $4x + 2y = 56$ (ii) Solve (i) and (ii) $x = 8$ and $y = 12$
Q17.	Since $x - 4$ is a factor of $x^3 + ax^2 + 2bx - 24$ $4^3 + a \times 4^2 + 2b \times 4 - 24 = 0$

	a + 2b + 10 =0 (i) a -b = 8 (ii) Solve (i) and (ii) a=2 , b=-6
Q18.	$\frac{a1}{a2} = \frac{2a}{4a} = \frac{1}{2}$ $\frac{b1}{b2} = \frac{b}{2b} = \frac{1}{2}$ $\frac{c1}{c2} = \frac{a}{2a} = \frac{1}{2}$ It has infinitely many solution, it is consistent
Q19.	$Solve\ the\ given\ equations$ $x = 7, \quad y = 9$ $So, \quad 5x - 3y = 8$
Q20.	$Solve\ the\ given\ equations$ $x = 10, \quad y = 0$ $Sub\ x = 10\ and\ y = 0\ in\ y = mx + 5$ $0 = m \times 10 + 5$ $m = -1/2$

SECTION -C ANSWERS -3 MARK QUESTIONS	
Q1	a)$4x + 3y = 10$ ………………..(1) $x - y = -1$ ………………….(2) Multiplying (2) by 4 and subtracting from (1), we get 7y= 14 y = 2 Putting y = 2 in (1), we get 4x + 3 x 2= 10 x = 1 Hence x = 1, y=2

	b) $3x - 4y = 15$ ………………..(1) $2x - 2y = 8$ …………………(2) Multiplying (2) by 2 and subtracting from (1), we get -x = -1 x = 1 Putting x = 1 in (1), we get 3 (1) - 4y = 15 y = -3 Hence x = 1, y=-3
Q2	A pair of linear equation has infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Therefore $\frac{2}{4} = -\frac{a-4}{-(a-1)} = \frac{2b+1}{5b-1}$ Solving, a = 7 and b = 3
Q3	$\frac{x}{a} + \frac{y}{b} = 2 \Longrightarrow bx + ay = 2ab \dots\dots\dots\dots.(i)$ $ax - by = a^2 - b^2$ …………….(ii) $(i) \times a \Longrightarrow abx + a^2y = 2a^2b \dots\dots\dots\dots\dots(iii)$ (ii) $\times b \Longrightarrow abx - b^2y = a^2b - b^3$……….(iv) Solving y = b and x = a
Q4	From the given figure; x - y = 10 ………..(i) x + y = 22 …………(ii) Solving we get , x = 16 and y = 6
Q5	Here $a_1 = 3$, $b_1 = -4$, $c_1 = 7$ $a_2 = k$, $b_2 = 3$, $c_2 = -5$ For no solution, we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

	$We\ have\ \frac{b_1}{b_2} = -\frac{4}{3}\ and\ \frac{c_1}{c_2} = -\frac{7}{5}$ $Clearly, \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. $So\ the\ given\ system\ will\ have\ no\ solution.$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{3}{k} = -\frac{4}{3}$ $\Rightarrow k = -\frac{9}{4}$
Q6	A pair of linear equation has infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Therefore $\frac{2}{k+2} = \frac{3}{6} = \frac{4}{3k+2}$ Solving. k=2 $5x - 2y = 15$…………(2)
Q7	Let the numbers be x and y. x +y = 75 …………(1) x – y = 15…………(2) adding (1) and (2) 2x= 90, x=45. Putting x=45 in (1) , x= 30. Hence the numbers are x = 30 and y= 45
Q8.	Let the present ages of children be **x** years and **y** years respectively. Present age of father is twice the sum of ages of his 2 children = 2(x+y) ………..(i) Then by question, (x+20) + (y+20) = 2(x + y) + 20 x + y + 40 = 2x + 2y + 20 x + y = 20 Putting (x + y) in (i), 2(x + y) = 2 × 20=40
Q9.	Let the digit on unit place be x and tens digit be y

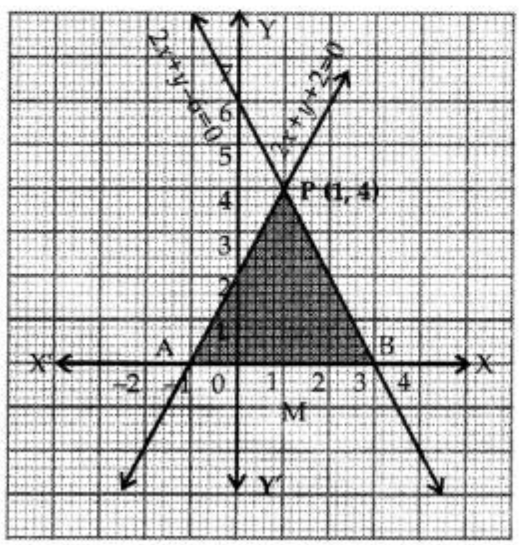
	Then the number = 10y + x Number formed by reversing the digits = 10x + y Then, 10x + y = 3(10y + x) - 9 7x - 29y = -9 ………………(i) Also, x - y = 5 x = y + 5 ……………(ii) (ii) in (i) 9(y + 5) - 29y = -9 y = 44/22 = 2 x = 2+5 =7 the number = 10(2) + 7 = 27
Q10.	Suppose A has x. oranges and B has y oranges. Then x +10 = 2(y-10) $\Rightarrow x - 2y + 30 = 0$ $y + 10 = x - 10 \Rightarrow x - y - 20 = 0$ Solving , we get y=50 and x=70 Hence A has 70 oranges and B has 50 oranges
SECTION -D ANSWER – 5 MARKS QUESTIONS	
Q1	1) The given system of equation is 2x + y – 6 = 0 ………. (i) 2x – y + 2 = 0 ……… (ii) Let us write three solutions for each equation of the system in a table. (i) ⇒ y = 6 – 2x Table of solutions for 2x + y – 6 = 0 <table><tr><td>x</td><td>0</td><td>2</td><td>3</td></tr><tr><td>y</td><td>6</td><td>2</td><td>0</td></tr><tr><td>(x,y)</td><td>(0,6)</td><td>(2,2)</td><td>(3,0)</td></tr></table>

Similarly (ii) ⇒ $y = 2x + 2$

Table of solutions for $2x – y + 2 = 0$

x	0	-1	2
y	2	0	6
(x,y)	$(0,2)$	$(-1,0)$	$(2,6)$

In graph, area bounded by the lines and x−axis is ΔPAB which is shaded.



Area of shaded region
= Area of ΔPAB
$= 1/2 \times AB \times PM$
$= 1/2 \times 4 \times 4 = 8$
sq. units.

Q2.

1) Angle A=70°

Angle B=120°

Angle C=60°

Angle D=110°

Q3.

1) Let the age of father be x years

Let the age of one of his children be y years and age of other children be z years

From given information,

	$x=2\times(y+z)$ ......(1) After 20 years, $(x+20)=(y+20)+(z+20)$ ....(2) $2y+2z-y-z=40-20$ $y+z=20$ ......(3) Substituting value of (3) in (1), we get $x=2\times(20)$ $\Rightarrow x=40$ Age of father is 40 years.
Q4.	1) Let the numerator be x and the denominator be y $Fraction\ is\ x/y$ According to the first condition $x+y=2y-3$ $x+y-2y=-3$ $x-y=-3----(1)$ According to the second condition $x-1=12y-1$ $2x-1=y-1$ $2x-2=y-1$ $2x-y=1\ -----(2)$ Subtracting eqn (2) from eqn (1) we get $-x=-4$ $x=4$ Substituting x=4 in equation 2, we get $2(4)-y=1$ $8-y=1$ $y=7$ Fraction is 4/7
Q5.	1) Let the digit in ones place be y and the digit in tens place be y.

	Two digit number = 10x+y Given 10x+yx+y=7 $\Rightarrow$ 10x + y = 7 (x + y) $\therefore$ 10x+y-7x-7y=0 3x-6y=0 x-2y=0--------(1) **According to the second condition.** 10x + y - 27 = 10y + x 10x+y-10y-x=27 9x-9y=27 x-y=3-------(2) Equation (1)-(2) x-2y-x-y=0-3 x-2y-x+y=-3 -y=-3 y=3 Substituting y=3 in equation (2), we get x-3=3 x=6 Two-digit number =10x+y =10x6+3 =60+3=63 Substitute y value in eqn(1) we get, x = 2× 3 $\Rightarrow$ x = 6 Hence the required number is 63**.**
Q6.	Let the cost of full fare be ₹ x and the cost of half first class fare be ₹ $\frac{x}{2}$, respectively and reservation charges

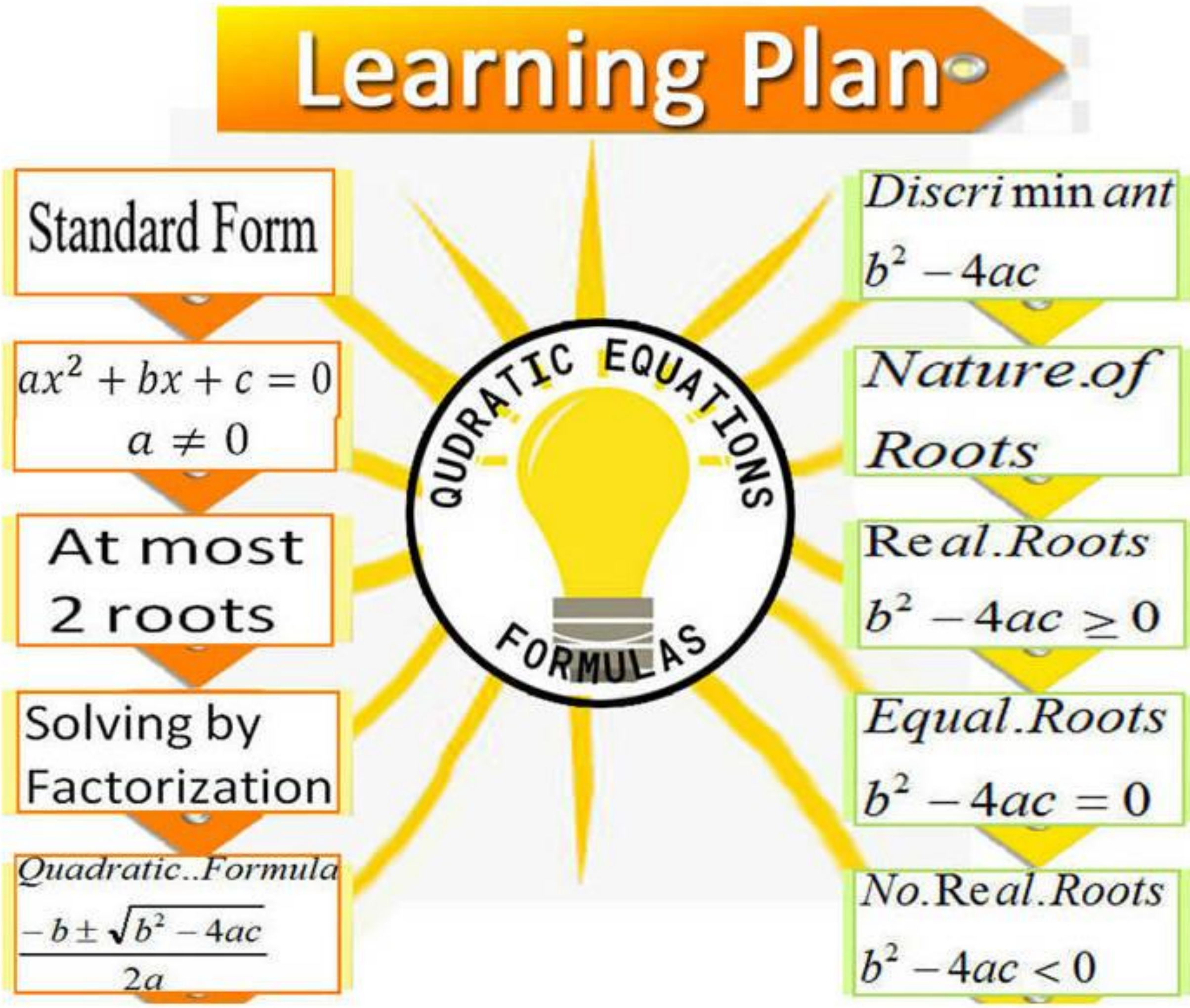
	be ₹ y per ticket. Case I The cost of one reserved first class ticket from the stations A to B =₹ 2530 x + y = 2530 ... (i) Case II The cost of one reserved first class ticket and one reserved first class half ticket from stations A to B = ₹ 3810 $\Rightarrow x + y + \frac{x}{2} + y = 3810$ $\Rightarrow x + \frac{x}{2} + y + y = 3810$ $\Rightarrow \frac{3x}{2} + 2y = 3810$ Multiplying throughout by 2, we get $\Rightarrow 3x + 4y = 7620 \ldots (ii)$ Now, multiplying Eq. (i) by 4 and then subtracting from Eq. (ii), we get $3x + 4y - 4x - 4y = 7620 - 10120$ $-x = -2500$ $\Rightarrow x = 2500$ On putting the value of x in Eq. (i), we get 2500+y=2530 $\Rightarrow y = 30$ Hence, full first-class fare from stations A to B is ₹ 2500 and the reservation for a ticket is ₹ 30.
Q7.	Let the annual incomes of A and B be 3x and 4x And the annual expenditures of A and B be 5y and 7y Since each of them saves Rs5000. 3x−5y=5000 ----- (1)

	$4x-7y=5000$ ---- (2) Multiplying equation (1) with 4 we get, $12x-20y=20000$ ----- equation (3) Multiplying equation (2) with 3 we get, $12x-21y=15000$ ----- equation (4) Subtracting equation (4) from (3), we get $y=5000$ Substituting $y=5000$ in the equation (1), we get $3x-5(5000)=5000=x=10000$ Hence,, annual income of A $=3x=$ Rs 30000 and of B $=4x=$ Rs 40000 It has infinitely many solution, it is consistent
Q8.	$x = -1/2$ $Y = 2$
Q9.	Suppose A has x mangoes and B has y mangoes, According to the given conditions, $x+30=2(y-30)$ $x+30=2y-60$ $x-2y+30+60=0$ $x-2y+90=0$ ......(1) $y+10=3(x-10)$ $y+0=3x-30$ $y-3x+10+30=0$ $y-3x+40=0$ ......(2) Multiplying eq. 1 by (3), $x-2\times 62+90=0$ 3x + 6y + 270 = 0 …… (3) and $x-124+90=0$ Now adding eq.2 and eq.3 $x-34=0$ 5y = 310 $y = \frac{310}{5}$ $\boxed{y=62}$

	$x-2\times62+90=0$ $x-124+90=0$ $x-34=0$ $\boxed{x=34}$ Hence A has 34 mangoes and B has 62 mangoes.
Q10.	Let x be the number of correct answers and y be the number of wrong answers. Since Jayanti answered 120 questions therefore, x+y=120 that is $x=120-y$..........(1) Also, it is given that one mark is awarded for each correct answer while 21 mark is deducted for every wrong answer and Jayanti got 90 marks, therefore, $x-0.5y=90$..........(2) Substituting the value of equation 1 in equation 2: $120-y-0.5y=90$ $-1.5y=-30$ y=20 Therefore, $x=120-20=100$
	Hence, she answered 100 questions correctly.

	ANSWERS
	CASE STUDY 1
1	b) x + 10y = 75, x + 15y = 110
2	c) Rs.355
3	a) x + 8y =91, x + 14y = 145

4	b) Rs.289
5	(c)
	CASE STUDY 2
1	(a) $2x + y = 19$, $x + y = 13$
2	(c) 54m
3	(b) area of bedroom = 30 sq.m, area of kitchen = 35 sq.m
4	(a)75 sq.m
5	(d)Rs.1750



LEARNING PLAN

- **TOPIC 1**: Standard form of a quadratic equation is $ax^2 + bx + c = 0\ where\ a \neq 0$, where a, b, c are real numbers. It has at most two roots generally called as $\alpha\ and\ \beta$
- **TOPIC 2:** A Quadratic equation can be solved by **Factorisation method**
- **Quadratic formula.** Quadratic formula is**,** $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ provided $b^2 - 4ac \geq 0$
- **TOPIC 3:**: $b^2 - 4ac$ **i**s called DISCRIMINANT.
- **TOPIC 4** : A quadratic equation has
  - ➢ two distinct real roots if $b^2 - 4ac > 0$
  - ➢ two equal real roots if $b^2 - 4ac = o$
  - ➢ no real roots if $b^2 - 4ac < 0$

VERY SHORT ANSWER TYPE & MULTIPLE CHOICE QUESTIONS (1MARK)

SECTION A

Q1. What is the positive root of $\sqrt{3x^2+6} = 9$
(a) 3
(b) 5
(c) 0
(d) none of these

Q2. For what value(s) of α quadratic equation $3\alpha x^2 - 6x + 1 = 0$ *has no real roots*?
(a) α > 3
(b) α < 3
(c) α =3
(d) none

Q3. Find the nature of the roots of the Quadratic equation $2x^2 - 4x + 3 = 0$?
(a) real roots
(b) no real roots
(c) equal roots
(d) none

Q4. Find the positive values of k for which the Quadratic equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$, both will have the real roots?
(a) k = 4
(b) k =16
(c) k > 16
(d) k< 4

Q5. If the sum of the roots of the quadratic equation $3x^2 + (2k+1)x - (k+5) = 0$ is equal to the product of roots, then the value of k is
(a) 2
(b) 3
(c) 4
(d) 5

Q6. If the equation $x^2 - bx + 1$ =0 does not possess real roots, then
(a) $-3 < b \leq +3$
(b) $-2 < b \leq +2$
(c) b > 2
(d) $b < -2$

Q7. Find the roots of the quadratic equation $x - \frac{1}{x}$ is
(a) $\frac{3}{2}, \frac{-3}{2}$
(b) $\frac{3+\sqrt{13}}{2}$, $\frac{3-\sqrt{13}}{2}$
(c) $\frac{2}{3}$, $\frac{-2}{3}$
(d) none

Q8. If $(x+4)(x-4)$ =9, then the values of x are
(a) ± 5
(b) $\pm \frac{1}{5}$
(c) 5, 5
(d) $\frac{1}{5}, \frac{1}{5}$

Q9. How many real roots does the equation $(x+1)^2 - x^2 = 0$ have?
(a) 1
(b) 2
(c) 3
(d) 4

Q10. The product of two successive integral multiples of 5 is 300. Then the numbers are

(a) 25, 30
(b) 10, 15
(c) 30, 35
(d) 15, 20

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

SECTION B

LEVEL 1

Q1. For what value of p for equation $2x^2 + 3x + p = 0$ will have real roots?

Q2. Find the sum of the roots of the quadratic equation$3x^2 - 9x + 5 = 0$?

Q3. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4}$ =0, then what is the value of k?

Q4. If the one root of the equation $4x^2 - 2x + p - 4$ =0 be the reciprocal of other, then what is the value of p?

Q5. What is the value of k for which the quadratic equation$2x^2 - kx + k$ =0 has equal roots?

Q6. Find the roots of the quadratic equation $x^2 - 3x$ =0

Q7. If $p^2x^2 - q^2 = 0$, then find the value of x?

LEVEL 2

Q8. Find the value of m for which the quadratic equation $(m - 1)x^2 + 2(m - 1)x + 1 = 0$ has two real and equal roots

Q9. Solve the following quadratic equation for x: $\sqrt{3}\,x^2 + 10x + 7\sqrt{3} = 0$

Q10. The product of Rahana's age (in years) 5 years ago and his age 7 years from now, is one more than twice his present age. Find their present age?

Q11. Find the roots of the equation $x^2 + x - p(p + 1) = 0$

Q12. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation

Q13. $4x^2 - 2px + k = 0$ has an equal root, find the value of k?

Q14. Find the roots of the quadratic equation $4x^2 - 4px + (p^2 - q^2)$=0

LEVEL 3

Q15. One year ago, father's age was 8times as old as his son and now his age is equal to the square of his son's age. Find the son's age?

Q16. The sum of a number and its reciprocal is $\frac{5}{2}$ .Find the numbers?

Q17. The product of two consecutive natural numbers is 72. Find the numbers?

Q18. What is the discriminant of the quadratic equation $7\sqrt{3}x + 10x - \sqrt{3} = 0$?

Q19. If a and b are the roots of the equation $x^2 + ax + b = 0$ then what is the value of $a + b$?

Q20. If one root of the equation $2x^2 + kx + 4 = 0$ is 2, then find its other root?

Q21. What is the discriminant of the quadratic equation: $(x+5)^2 = 2(5x-3)$

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

SECTION C

LEVEL 1(3MARKS)

Q1. The sum of the squares of two consecutive natural numbers is 421, find the numbers

Q2. Solve the quadratic equation

Q3. Solve the following equation: -

Q4. Write all the values of p for which the quadratic equation, Find the roots of the equation so obtained

Q5. Find the nature of the roots of the quadratic equation

LEVEL2 (3MARKS)

Q6. If α *and* β *are the roots of the equation* $2\,x^2 - 6x + a$ *and* $2\alpha + 5\beta = 12$ find the roots of the equation

Q7. The sum of ages of a son and his father is 35 years and the product of their ages is 150 years. Find their ages

Q8. If -5 is the root of the quadratic equation and the quadratic equation has equal roots, then find the value of k

Q9. Find the positive value of k for which the equation

Q10. If $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$. *Prove that* $x/a = y/b$

HOT QUESTIONS (3MARK)

Q1. Solve for x. $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$

Q2. Three consecutive positive integers are such that the sum of the square of the first and the product of the other two is 46, find the integers.

Q3. If the roots of the quadratic equation (x – a) (x – b) + (x – b) (x – c) + (x – c) (x – a) = 0 are equal, then show that a = b = c.

Q4. In a rectangular park of dimensions 50 m × 40 m, a rectangular pond is constructed so that the area of grass strip of uniform width surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.

Q5. P and Q are centres of circles of radii 9 cm and 2 cm respectively. PQ = 17 cm. R is the centre of the circle of radius x cm which touches given circles externally. Given that angle PRQ is 90°. Write an equation in x and solve it.

LONG ANSWER TYPE QUESTIONS (4 MARKS)

SECTION D

Q1. Seven years ago, Rahul's age was five times the square of Reena's age. Three years hence, Reena's age will be two fifth of Rahul's age. Find their present ages.

Q2. The diagonal of a rectangular field is 16metres more than the shorter side. If the longer side is14 metres more than the shorter side, then find the length of the sides of the field.

Q3. One fourth of a herd of camel was seen in the forest. Twice the square root of the herd had gone to the mountains and the remaining 15 camels were seen on the bank of the river. Find the total number of camels.

Q4. A train travels 180 km at a uniform speed. If the speed had been 9km/hr more,it would have taken 1 hour less. Find the speed of the train.

Q5. Rs 9000 were divided equally among certain number of persons. Had there been 20 more persons, each would have got Rs160 less. Find the original number of persons.

Q6. Two taps running together can fill a tank in $3\frac{1}{13}$hours.If one tap takes 3hours more than the other to fill the tank, then how much time will each tap take to fill the tank.

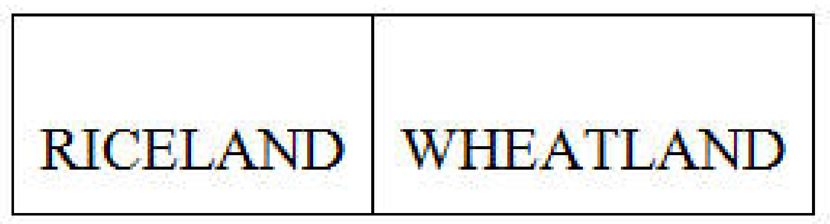
Q7. Solve the following quadratic equations

$$9x^2 - 9(a + b) x + [2a^2+5ab + 2b^2] = 0$$

Q8. Solve for x: $\frac{x-3}{x-4} + \frac{x-5}{x-6} = \frac{10}{3}$,$x \neq 4,6$

Q9. A motor boat whose speed is 24 km/hr in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Q10. Madhav has a field with total area 1260 square metre. He uses it to grow wheat and rice. The land used to grow wheat is rectangular in shape while the rice land is in the shape of a square as shown in the following figure. The length of wheat land is 3m more than twice the length of Rice land. find the area of wheat land.



	LEVEL I
1	Assertion(A): The poynomial P(x)=$x^2 + 3x + 3$ has two real roots Reason:(R) :A quadratic polynomial can have at most two real roots

	a) Both assertion and Reason are true and Reason R is the correct explanation of Assertion b) Assertion is true, but Reason is false c)Assertion is false, but Reason is true d) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion
2	The equation $8x^2 + 3kx + 2 = 0$ has equal roots. then the value of k is =+8/3 and -8/3 Reason: The equation $ax^2 + bx + c = 0$ has equal roots a) Both assertion and Reason are true and Reason R is the correct explanation of Assertion b) Assertion is true, but Reason is false c)Assertion is false, but Reason is true d) Both Assertion and Reason are true, but Reason is not the correct explanation Assertion
	ANSWER KEY
1	**C (explanation)** b^2 -4ac is -3. it has no real roots)
2	**D (explanation)** b^2 -4ac =0, k =+8/3 and -8/3 and reason is also correct

CASE STUDY BASED QUESTIONS

CASE STUDY 1

John and Jayant are very close friends. They decided to go to Ranikhet with their families in separate cars. John's car travels at a speed of x km/hr while Jayant's car travels 5km/hr faster than Johan's car. Johan took 4 hours more than Jayant to complete the journey of 400 km.

1. Find the distance covered by Jayant's car in two hours
2. Form a quadratic equation describing the speed of Johan's ca
3. Find the speed of Johan's car in km/hr
4. Find the speed of Jayant's car in km/hr
5. What is the time taken by Jayant to travel 400 km?

CASE STUDY 2

An Auditorium was booked for School Annual Day Celebrations and the seats are arranged in a particular manner. The number of rows is equal to the number of seats in each row. When the number of rows was doubled and the number of seats in each row was reduced by 10, the total number of seats increased by 300

Based on the above information answer the following questions

1. If x is taken as number of row in original arrangement form a quadratic equation describe the situation?
2. Find the number of rows are there in the original arrangement?
3. How many seats are there in the auditorium in original arrangement?
4. How many seats are there in the auditorium after re-arrangement?

CASE STUDY 3

The speed of a motor boat is 20 km/hr. For covering the distance of 15 km the boat took 1 hour more for upstream than downstream.

1. If the speed of the stream be x km/hr. then find the speed of the motorboat in upstream.
2. If the speed of stream is 10 km/hr, then what is the speed of the motor boat in downstream.
3. Form a quadratic equation for the speed of current.
4. Find the speed of current.
5. How much time the motor boat takes to cover 15 km upstream

**************** ************************* *******************

ANSWER KEY

VERY SHORT ANSWER TYPE QUESTIONS (1MARK)

QN NO	ANS	QN NO	ANS
1	b	6	b
2	a	7	b
3	b	8	a
4	b	9	a
5	c	10	d

ONE MARK QUESTIONS (1)

1.	-5,-2	6.	6 years ,12 years
2.	All values of a greater than $\frac{3}{10}$	7.	Other zero is $-\frac{3}{2}$
3.	$\pm\frac{4}{3}$	8.	51/4
4.	$\frac{-9}{4}$	9.	1,2
5.	$5b^2 - 4ac = -31$	10.	13,15

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

QN no	ANS	QN no	ANS	QN no	ANS	QN no	ANS
1	$p \leq \frac{9}{8}$	6	$x = 0$ $x = 3$	11	p, -(p+1)	16	8,9
2	Sum is 3	7	$\pm\frac{q}{p}$	12	$k = 1$	17	184
3	2	8	$m \neq 1$, $m = 2$	13	$\frac{p \pm q}{2}$	18	$a + b = -1$

4	P=8	9	$x = -\frac{7}{\sqrt{3}}$,-$x = \sqrt{3}$	14	7years and 49years	19	Other root is 1
5	K= 0 and 8	10	6 years	15	$x = 2$ *and* $x = \frac{1}{2}$	20	-124

3 MARKS QUESTIONS

1	One number is 14 Other number is 15
2	X=0.7
3	$\boldsymbol{x = \frac{-1 \pm \sqrt{3}}{3}}$
4	p = ±8, $x^2 \pm 8x + 16 = 0$, ∴ $(x \pm 4)^2 = 0$ $x \pm 4 = 0$ ∴ Roots are x = –4 and x = 4 ,
5	We have: $2x^2 + 4x - 8 = 0$ given equation has two distinct real roots and they are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, x = $\frac{-2 \pm \sqrt{(2)^2-4x1x(-4)}}{2x1}$ ∴ x =$\frac{-2 \pm \sqrt{20}}{2}$, x = $-1 \pm \sqrt{5}$
1	α + β = -((-6)/2) =3 2α+5β = 12 2(3-β) +5β = 12 β= 2 ie ,α = 1 α.β = a/2 a = 4
2	−5 is a root of the quadratic eqn. $2x^2 + px - 15 = 0$ ⇒ $2(-5)^2 + p(-5) - 15 = 0$ ⇒ $2(25) - 5p - 15 = 0$.p=7 The quadratic equation $px^2 + px + k = 0$ has equal roots. ⇒ $b^2 - 4ac = 0$ $p^2 - 4(p)(k) = 0$ ⇒ $72 - 4(7)(k) = 0$ $k = 49/28 = 7/4$
3	Let the age of father be x years and age of son be 35 – x years. $x(35 - x) = 150$ ⇒ $x^2 - 35x + 150 = 0$ 1 Hence, the age of father = 30 years and the age of son = 5 years

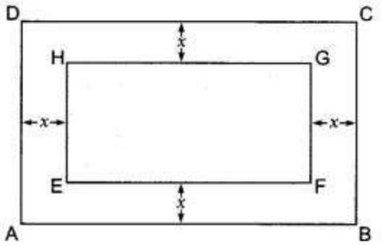
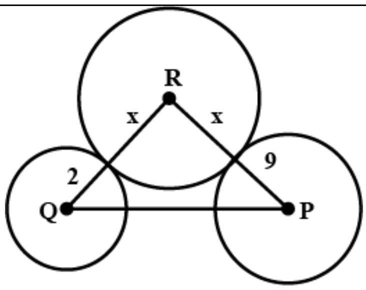
4	Here for the equation $x^2+kx+64 = 0$, $D \geq 0$ $k^2 - 4\times64 \geq 0$ $k^2 \geq 256$ $k \geq 16$ ....(1) Also the equation $x^2-8x+k = 0$ we have , $D \geq 0$ $64-4k \geq 0$ $4k \leq 64$ $k \leq 16$ ....(2) The value of satisfying both the eqns is $k = 16$
5	Number of persons=25 Given, $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$ $\Rightarrow x^2a^2 + x^2b^2 + y^2a^2 + y^2b^2 = a^2x^2 + b^2y^2 + 2abxy$ $\Rightarrow x^2b^2 + y^2a^2 - 2abxy = 0$ $\Rightarrow (xb - ya)^2 = 0$, $xb = ya$, $x/a= y/b$

SHORT ANSWER TYPE QUESTIONS (3 MARKS) & HOT QUESTIONS

1.	We have: $2x^2 + 4x - 8 = 0$ Dividing by 2, we get $x^2 + 2x - 4 = 0$ ....(i) Comparing (i) with $ax^2 + bx + c = 0$, $a = 1$, $b = 2$, $c = -4$ $\therefore b^2 - 4ac = (2)^2 - 4(1)(-4)$ $= 4 + 16 = 20 > 0$ Since $b^2 - 4ac > 0$, the given equation has two distinct real roots and they are given by $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ $x = \frac{-2\pm\sqrt{(2)^2-4x1x(-4)}}{2x1}$ $\therefore x = \frac{-2\pm\sqrt{20}}{2}$ $\Rightarrow x = \frac{-2\pm2\sqrt{5}}{2}$ $\Rightarrow x = -1 \pm \sqrt{5}$ Thus, the required roots $x = -1 + \sqrt{5}$ and $x = -1 - \sqrt{5}$.
2.	$x=\frac{q^2}{p^2}$, $x= -1$
3.	$\alpha + \beta = -((-6)/2) = 3$ $2\alpha+5\beta = 12$ $2(3-\beta) + 5\beta = 12$ $\beta= 2$ ie ,$\alpha = 1$ $\alpha.\beta = a/2$ $a = 4$

4.	-5 is a root of the quadratic eqn. $2x^2 + px - 15 = 0$ $\Rightarrow 2(-5)^2 + p(-5) - 15 = 0$ $\Rightarrow 2(25) - 5p - 15 = 0$ $\Rightarrow 50 - 5p - 15 = 0$ $\Rightarrow 35 - 5p = 0$ $\Rightarrow 5p = 35 \Rightarrow p = 7$ The quadratic equation $px^2 + px + k = 0$ has equal roots. $\Rightarrow b^2 - 4ac = 0$ $\Rightarrow p^2 - 4(p)(k) = 0$ $\Rightarrow 72 - 4(7)(k) = 0$ $\Rightarrow 49 - 28k = 0$ $\Rightarrow 28k = 49$ $\Rightarrow k = 49/28 = 7/4$
5.	Here for the equation $x^2+kx+64 = 0$, $D \geq 0$ $k^2 - 4\times64 \geq 0$ $k^2 \geq 256$ $k \geq 16$ …………. 1 Also the equation $x^2-8x+k = 0$ we have , $D \geq 0$ $64-4k \geq 0$ $4k \leq 64$ $k \leq 16$ ………. 2 The value of satisfying both the eqns is $k = 16$
6.	We have, $\frac{(5x+1)+3(x+1)}{(x+1)(5x+1)} = \frac{5}{x+4}$ $17x^2-6x-11 = 0$ $x = \frac{-11}{17}$,1
7.	Let the age of father be x years and age of son be $35 - x$ years. $x(35 - x) = 150$ $\Rightarrow x^2 - 35x + 150 = 0$ 1 $\Rightarrow (x - 30)(x - 5) = 0$ $\Rightarrow x = 30$ or $x = 5$ (rejected) Hence, the age of father = 30 years and the age of son = 5 years
8.	Let the two consecutive natural numbers be x and $x + 1$ According to the question, $x^2 + (x + 1)^2 = 421$ $\Rightarrow x^2 + x^2 + 2x + 1 = 421$ $\Rightarrow x^2 + x - 210 = 0$ $\Rightarrow (x + 15)(x - 14) = 0$ $\Rightarrow x + 15 = 0$ or $x - 14 = 0$ $\Rightarrow x = -15$ or $x = 14$ Rejecting negative value, first number = 14 and second consecutive number = 15
9.	Let the usual speed be x km/hr. Then , $\frac{300}{x} - \frac{300}{x+5} = 2$ $x^2 + 5x - 750 = 0$ $x = -30$ or $x = 25$ Usual speed = 25 km/hr

10.	Let the speed of the steam be x km/hr. Then ,$\frac{12}{11+x}$ +$\frac{12}{11-x}$ =$2\frac{3}{4}$ x= ±5 Speed of the stream is 5km/hr.
11.	Let the usual speed be x km/hr. Then, $\frac{1500}{x}$ - $\frac{1500}{x+100}$ = $\frac{1}{2}$ x^2 + 100x- 300000 = 0 x = -600 or x = 500 Usual speed of the plane = 500 km/hr
12.	Let the no of days taken by B to finish the work = x days No of days taken by A = (x-6) ie, $\frac{1}{x-6}$ + $\frac{1}{x}$ = $\frac{1}{4}$ x = 12, x = 2 (not possible) No of days taken by B = 12
13.	Given, $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$ $\Rightarrow x^2a^2 + x^2b^2 + y^2a^2 + y^2b^2 = a^2x^2 + b^2y^2 + 2abxy$ $\Rightarrow x^2b^2 + y^2a^2 – 2abxy = 0$ $\Rightarrow (xb – ya)^2 = 0$ ⇒xb = ya ⇒x/a= y/b
14.	(a) $x = -5\sqrt{3}$, $2\sqrt{3}$ (b) x = a – 2 , x = – (a + 3) (c) x = 0 , 7 (d) $x = \frac{1}{b^2}$, $x = \frac{-1}{a^2}$
15.	Let time taken by pipe A be x minutes. Then time taken by pipe B = x + 5 minutes. In one minute, pipe A will fill 1/x part and in one minute, pipe B will fill 1/(x+5) part Hence, pipes A + B will fill in one minute = $\frac{1}{x} + \frac{1}{x+5}$ part Now according to the question, $\frac{1}{x} + \frac{1}{x+5} = \frac{9}{100}$ i.e ; $9x^2 – 155x – 500 = 0$ ⇒(x – 20)(9x + 25) = 0 ⇒ x = 20 or x = -25/9 rejecting negative value,x = 20 minutesand x + 5 = 25 minutes Hence, pipe A will fill the tank in 20 minutes andpipe B will fill it in 25 minutes.
ANSWERS TO HOT QUESTIONS	
1.	Let y = $x^{\frac{1}{3}}$ y^2+y-2 = 0 y = -2, y = 1 ie, x = -8, x = 1
2.	Let the consecutive positive integers be x, x+1 and x+2. x^2+(x+1)(x+2)= 46 x = 4 or x = $\frac{-11}{2}$ (rejected) Integers are 4,5,6

3.	Given $(x-a)(x-b)+(x-b)(x-c)+(x-6)(x-a)=0$ $\Rightarrow x^2-ax-bx+ab+x^2-bx-cx+bc+x^2-cx-ax+ac=0$ $\Rightarrow 3x^2-2(a+b+c)x+ab+bc+ca=0$ Now, for equal roots, D = 0 $\Rightarrow 4(a+b+c)^2-12(ab+bc+ca)=0$ $\Rightarrow 4a^2+4b^2+4c^2+8ab+8bc+8ca-12ab-12bc-12ca=0$ $\Rightarrow 2[2a^2+2b^2+2c^2-2ab-2bc-2ca]=0$ $\Rightarrow 2[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ca)]=0$ $\Rightarrow [(a-b)^2+(b-c)^2+(c-a)^2]=0$ $\Rightarrow a-b=0, b-c=0, c-a=0$ $\Rightarrow a=b, b=c, c=a$ $\Rightarrow a=b=c$ (Hence Proved)
4.	 Let ABCD be rectangular lawn and EFGH be rectangular pond. Let x mbe the width of grass area, which is same around the pond. Given, Length of lawn = 50 m Width of lawn = 40 m Length of pond = $(50-2x)$ m Breadth of pond = $(40-2x)$ m Also given, Area of grass surrounding the pond = 1184 m^2 $\Rightarrow$ Area of rectangular lawn – Area of pond = 1184 m^2 $\Rightarrow 50\times 40-\{(50-2x)\times(40-2x)\}=1184$ $\Rightarrow 2000-(2000-80x-100x+4x^2)=1184$ $\Rightarrow 2000-2000+180x-4x^2=1184$ $\Rightarrow 4x^2-180x+1184=0$ $\Rightarrow x^2-45x+296=0$ $\Rightarrow x=37$ or $x=8$ x = 37 is not possible (as length of pond will becomes $50-2\times37=-24$ which is not possible) Hence, x = 8 is acceptable. $\therefore$ Length of pond = $50-2\times 8=34$ m Breadth of pond= $40-2\times 8=24$ m
5.	 In right DPQR, by Pythagoras theorem $PQ^2=PR^2+RQ^2$

	$\Rightarrow 17^2 = (x + 9)^2 + (x + 2)^2$ $\Rightarrow 2x^2 + 22x – 204 = 0$ $\Rightarrow x^2 + 11x – 102 = 0$ $\Rightarrow x^2 + 17x – 6x – 102 = 0$ $(x – 6)(x + 17) = 0$ Þ x = 6 or x – 17 (x can't be negative) ½ Thus, x = 6 cm

LONG ANSWER TYPE QUESTIONS (4 MARKS)

1	Rahul's age=27 years Reena's age=9years	6	Larger tap=5 hours Smaller tap=8 hours
2	10m and 24m	7	x=$\frac{2a+b}{3}$ or x=$\frac{a+2b}{3}$
3	Total number of camels=36	8	$7,\frac{9}{2}$
4	Speed of train=36km/hr	9	Speed of stream=8km/hr
5	Number of persons=25	10	Area=860m^2

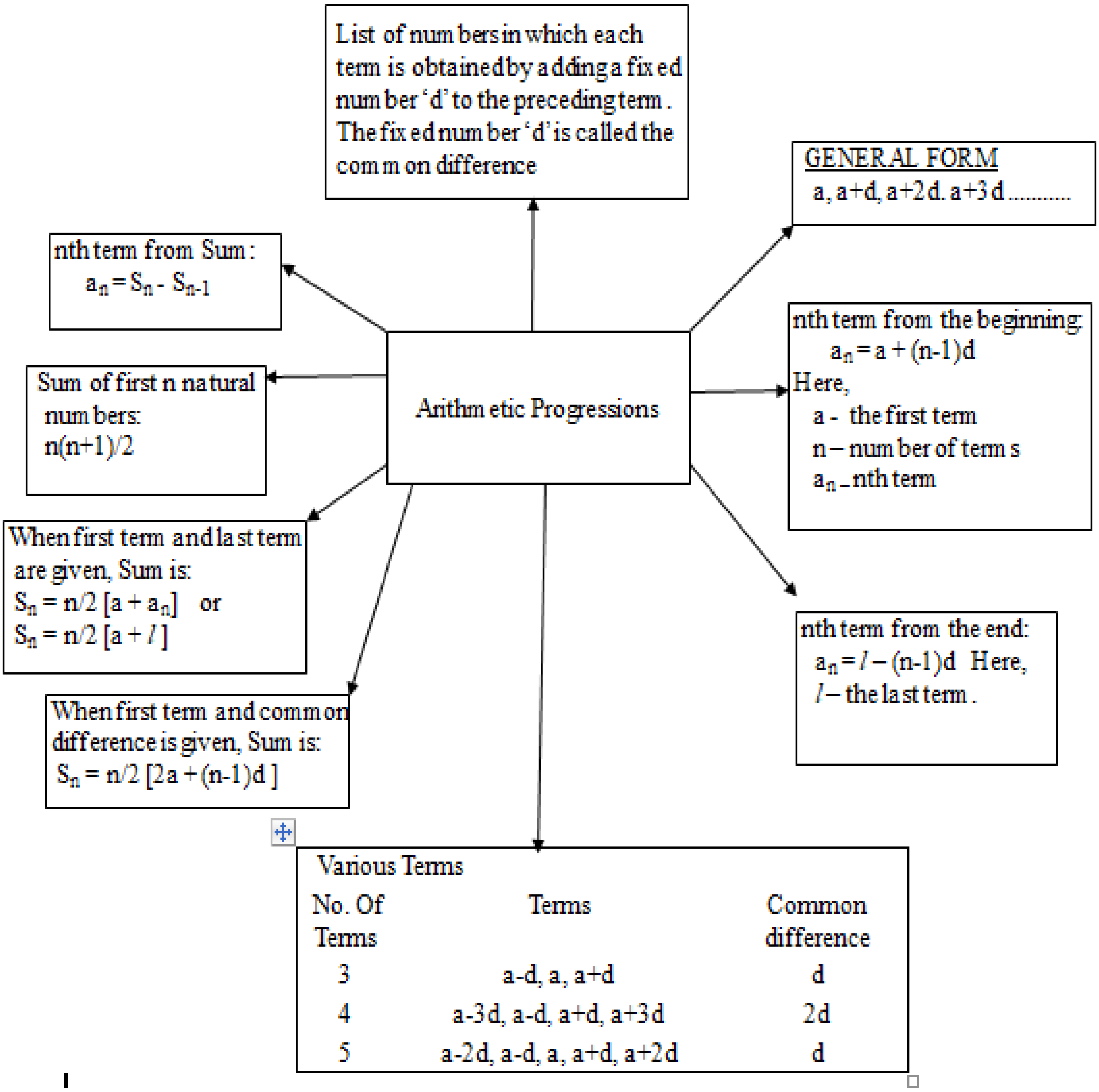
CASE STUDY BASED QUESTIONS

CASE STUDY 1	CASE STUDY 2	CASE STUDY 3
1. 2(x+5)km	1.b	1.(20-X)km/hr
2.c	2.30	2.(20+x) km/hr
3. 20km/hr	3.900	3.c
4.25km/hr	4.1200	4.10
5.16 hours		5.$1\frac{1}{2}$hours

CHAPTER 5

ARITHMETIC PROGRESSIONS

CONCEPT MAP



MULTIPLE CHOICE QUESTIONS (1 MARK)

LEVEL 1

Q1. The common difference of an AP whose n^{th} term is 3n+7

a) 2 b) 3 c) 4 d) 5

Q2. The sum of first n natural numbers is

a) $\frac{n}{2}$

b) $\frac{n+1}{2}$

c) $\frac{n(n+1)}{2}$

d) $\frac{n(n+1)(n+2)}{2}$

Q3. The next term of the AP: √8, √18 , √32….is

a) 5√2 b) 5√3 c) 3√3 d) 3√5

Q4. The list of numbers -10, -6, -2, 2, ,….is

a) an AP with d=-16
b) an AP with d=4
c) an AP with d=-4
d) not an AP

Q5. The 27^{th} positive odd number is:

a) 50 b) 51 c) 52 d) 53

Q6. The common difference of an AP is 5, then the value of $a_{18} – a_{13}$ is:

a) 5 b) 20 c) 25 d) 30

Q7. A man receives Rs. 60 for the first week and Rs. 3 more each week than the preceding week. How much does he earn by the 20th week ?

a) Rs. 1760
b) Rs. 1770
c) Rs. 1780
d) Rs. 1790

Q8. If the first term of an AP is p and the common difference is q, its 10^{th} term is

a) p+9q
b) p+q
c) p+10q
d) 9p+q

Q9. If an AP has a_1=1, a_n= 20 and S_n=399, then the value of n is

a) 20 b) 32 c) 38 d) 40

Q10. Two APs have the same common difference. The first term of one of these is –1 and that of the other is – 8. Then the difference between their 4th terms is

a) –1
b) – 8
c) 7
d) –9

LEVEL 2

Q11. The 7^{th} term from the end of the AP 7,11, 15,…….,107

a) 79 b) 83 c) 81 d) 87

Q12. If the common difference of an AP is 7 then a_{25}-a_{21} is equal to

a) 14 b) 20 c) 28 d) 35

Q13. If a_1, a_2, a_3, are in AP such that a_{20}- a_{12} = -32, then the common difference of the APis

a) 4 b) -4 c) -3 d) 3

Q14. The sum of first n odd natural number is

a) 2n
b) 2n+1
c) n^2
d) $n^2 – 1$

Q15. The sum of first n even natural numbers

a) 2n
b) n^2
c) $n^2 + n$
d) n^2 -1

LEVEL 3

Q16. If the sum of first n natural number is 225, then the value of n is

a) 15 b) 25 c) 35 d) 45

Q17. The nth term of an AP ,if the sum of whose n term is S_n

a) $S_n + S_{n-1}$
b) S_n-S_{n-1}
c) S_n+ S_{n+1}
d) S_n-S_{n+1}

Q18. If 18, a, b, -3 are in AP, then a+b=

(a) 19 (b)7 (c)11 (d)15

Q19. The value of x for which $2x, x + 10, and\ 3x + 2$ are the three consecutive terms of an AP

(a) -6 (b)18 (c)6 (d)-18

Q20. If the ratio of 18th term to 11th term of an AP is 3:2 then the ratio of the 21st term to 5th term is

(a)3:2 (b)3:1 (c)1:3 (d)2:3

VERY SHORT ANSWER QUESTIONS

LEVEL 1

Q1. For what value of k: 2k, k + 10 and 3k + 2 are in AP?

Q2. The first, second and last terms of an AP are respectively 4, 7 and 31. How many terms are there in the given AP?

Q3. Write first four terms of the AP,when first term is 1.25 and common difference is -0.25.

Q4. Find the common difference of an AP in which $a_{18} – a_{14} = 32$.

Q5. If the nth term of an AP is 2n+1, then find the sum of its first three terms.

LEVEL 2

Q6. Find the common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$

Q7. Find the 9th term from the end (towards the first term) of the AP 5, 9, 13, …, 185.

Q8. Find the sum of all natural numbers from 1 to 100.

Q9. In an AP,if the common difference is -4 and the seventh term is 4,then find the first term.

Q10. Find the missing terms in the given AP 2, ---, 26, ---

SECTION B

SHORT ANSWER QUESTIONS (2 mark questions)

LEVEL 1

Q1. How many terms of the AP 27, 24, 21, … should be taken so that their sum is zero.

Q2. Three numbers are in AP and their sum is 24. Find the middle term.

Q3. Check whether -150 is a term of the AP: 11,8, 5, 2, …

Q4. Find the middle term of the AP -11, -7, -3, …, 45.

Q5. How many two-digit numbers are divisible by 3?

Q6. Find the sum: 34+ 32+30+……+10

LEVEL 2

Q7. Which term of the AP 3, 15, 27, 39, … is 132 more than its 54 th term?

Q8. Find the number of terms of an AP 5, 9, 13, …, 185.

Q9. Find the sum of all odd numbers between 10 and 200.

Q10. If the sum of first n terms of an AP is n^2 find the 5^{th} term.

Q11. Which term of the AP. 20, 17, 14, ...; is the first negative term?

Q12. If the sum of first m terms of an AP is $am^2 + bm$, find the common difference.

Q13. Find the sum of first 8 multiples of 3.

LEVEL 3

Q14. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5

Q15. The fourth term of an AP is 11 and the eleventh term is 25. Determine the first term and common difference.

Q16. If an AP has 8 as the first term, -5 as the common difference and its first 3 terms are 8,A,B ,then find A+B

Q17. In an AP a=15, d=-3, a_n =0, then find the value of n.

Q18. The sum of first n terms of an AP is given by $S_n= 2n^2+n$. Then find its nth term

Q19. The 4^{th} term of an AP is zero. Prove that 25^{th} term is three times its 11^{th} term.

Q20. Find the nth term of the AP $\frac{1}{m}$, $\frac{1+m}{m}$, $\frac{1+2m}{m}$, …….?

SHORT ANSWER TYPE QUESTIONS

SECTION C (3 Mark Questions)

LEVEL 1

Q1. If sum of the 3rd and the 8th terms of an AP is 7 and the sum of the 7th and the 14th terms is –3, find the 10th term.

Q2. Find the sum of all 3-digit natural numbers which are multiples of 11.

Q3. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.

Q4. The p^{th} term of an AP is $\frac{1}{7}(2p-1)$. Find the sum of its first n terms.

Q5. How many terms of the AP: 9,17, 25…. must be taken to get a sum of 636?

LEVEL 2

Q6. If m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$. Show that $(mn)^{th}$ term of this AP is 1.

Q7. The sum of the first 9 terms of an AP is 171 and the sum of of its first 24 terms is 996.Find the first term and the common difference.

Q8. If the sum of first m terms of an A.P. is the same as the sum of its first n terms, then show that the sum of its first (m + n) terms is zero.

Q9. For what value of n, are the nth terms of two APs: 63, 65, 67, . . . and 3, 10, 17, . . . equal?

Q10. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20^{th} term.

LEVEL 3

Q11. In an AP, ratio of 4^{th} term and 9^{th} term is 1:3,find the ratio of 12^{th} and 5^{th} term.

Q12. The 14^{th} term of an A.P. is twice its 8^{th} term. If the 6^{th} term is -8, then find the sum of its first 20 terms.

Q13. Find the sum of n terms of the series: $(4-\frac{1}{n})+\left(4-\frac{2}{n}\right)+(4-\frac{3}{n})+\ldots$

Q14. If the 10^{th} term of an A.P. is 52 and the 17^{th} term is 20 more than the 13^{th} term, find A.P.

Q15. The 5th term of an AP is 20 and the sum of its 7th and 11th terms is 64. Find the common difference of the AP

CASE STUDY BASED QUESTIONS (4 marks questions)

Q1. **CASE STUDY QUESTION 1:**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

Based on the above information, answer the following questions:

i. Find the production during the first year.
ii. In which year, the production is 29,200.

Q2. **CASE STUDY QUESTION 2:**

Your friend Veer wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

Q1. What is the minimum number of days he needs to practice till his goal is achieved?

Q2. If nth term of an AP is given by an = 2n + 3 then find the common difference of the AP.

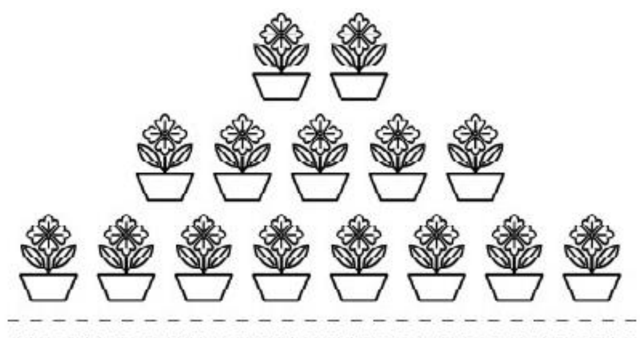
CASE STUDY QUESTION 3:

Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month , answer the following:

i. Find the amount paid by him in 30th instalment.
ii. Find the total amount paid by him after 30 instalments.

CASE STUDY QUESTION 4:

Aahana being a plant lover decides to convert her balcony into beautiful garden full of plants. She bought few plants with pots for her balcony. She placed the pots in such a way that the number of pots in the first row is 2, second row is 5, third row is 8 and so on (CBSE 2023



Based on the above information, answer the following questions:

i. Find the number of pots p = 29laced in the 10^{th} row.

ii. Find the difference in the number of pots placed in 5^{th} row and 2^{nd} row.

iii. If Aahana wants to place 100 pots in total, then the find total no of rows formed in the arrangement.

iv. If Aahana has sufficient space for 12 rows, then how many total number of plants are placed by her with the same arrangement?

ANSWER KEY

MULTIPLE CHOICE QUESTIONS (1mark questions)

Qn no	Answer	Qn no	Answer

1	3	6	25
2	$\frac{n(n+!)}{2}$	7	Rs. 1770
3	$5\sqrt{2}$	8	p+9q
4	an AP with d=4	9	38
5	53	10	7

1	k+10= $\frac{2k+3k+2}{2}$= k+10=$\frac{5k+2}{2}$ Also by cross multiplying we get 2(k+10)=5k+2 2k+20=5k+2, 18= 3k, k=6
2	a_1=4 , a_2 =7 , an=31 ,d= a_2- a_1=7-4=3 31= 4+(n-1)3,(n-1)3=27,n-1=9,n=10
3	a, a+d, a+2d, a+3d= 1.25, 1 ,0.75, 0.50
4	$a_{18} – a_{14}$ = 32.,a+17d-(a+13d)=32, 4d=32,d=32÷4=8
5	a_n = 2n+1, a_1 = 2x1+1=3 , a_3= 2x3+1=7, $S_3=\frac{3}{2}(3+7)$= 15
6	d= a_2-a_1=$\frac{1-p}{p}$ - $\frac{1}{p}$ = $\frac{1-p-1}{p}$=$\frac{-p}{p}$= -1
7	nth term from the end = l- (n-1)d, where l is the last term. 9^{th} term from the end =185-(9-1)4, 185-32=153
8	$\frac{n(n+!)}{2}$ = $\frac{100(100+!)}{2}$ = 5050
9	a_7= 4,d=-4, a+6d=4, a+6x-4=4, a-24=4,a=28.
10	a_2= $\frac{a1+a3}{2}$, a_2= $\frac{2+26}{2}$ =14, d= a_2 - a_1=12, a_4= a_3+d = 26+12=38

SECTION B (2 marks questions)

1	The first term (a) = 27, The sum of first n terms (Sn) = 0 Common difference of the A.P. (d) = a2 – a1 = 24 – 27 = -3. On substituting the values in Sn, we get 0 = $\frac{n}{2}[2(27) + (n − 1)(− 3)]$, 0 = (n)[54 + (n – 1)(-3)] , 0 = (n)[54 – 3n + 3] ⟹ 0 = n [57 – 3n] Further we have, n = 0 Or, 57 – 3n = 0 ⟹ 3n = 57 ⟹ n = $\frac{57}{3}$= 19. The number of terms cannot be zero. Hence n=19
2	Let the three numbers of the AP be a-d, a, a+d. So a-d +a+a+d= 24 ⟹ 3a=24,a=$\frac{24}{3}$=8. Hence the middle term =8.
3	11, 8, 5, 2, …-150, a = 11, d = 8- 11 = -3, an = -150, 11 + (n – 1) (-3) = -150, 11 – 3n + 3 = -150 , -3n + 14 = -150 , -3n = -150 – 14, -3n = -164, 3n = 164 ∴ n = $\frac{164}{3}$, Here value of 'n' is not a positive integer. Hence -150 is not a term of the given AP

4	Given AP is -11,-7,-3,......,45 Here a = -11, d = -7- -11=-7+11=4 and last term l = 45 45 = (-11) + (n - 1)4, 56 = (n - 1)4 , n - 1 = 14, Therefore, n = 15 That is there are 15 terms. Hence 8th term is the middle most term of the given AP a_8 = a + 7d = (-11) + 7(4) = 17. Thus the middle term is 17
5	The Required A.P = 12, 15 ,18 .... 99 ,First term (a) = 12, Last term = 99, Common difference = 3, nth term = a+(n-1)d,. Putting the values in the formula:=> 99 = 12+(n-1)3, 99-12 = 3(n-1), 87 = 3(n-1), $\frac{87}{3}$= n-1, 29 = n-1, 29+1 = n, n = 30
6	Given, 34 + 32 + 30 + … + 10, first term, a = 34, d = $a_2 - a_1$ = 32−34 = −2, Let 10 be the n^{th} term of this A.P., a_n= a +(n−1)d, 10 = 34+(n−1)(−2), −24 = (n −1)(−2), 12 = n −1, n = 13, $S_n = \frac{n}{2}(a + l)$, l = 10, $S_n = \frac{n}{2}$ (34 + 10) = $\frac{13}{2}$ x44= 286
7	a_1 = 3, a_2= 15, d = 15 - 3 = 12, 54^{th} term of the AP is a_{54} = a + (54 - 1)d = 3 + 53 ×12 = 639, Let n^{th} term of AP be 132 more than 54^{th} term ,We get, 132 + 639 = 771, a_n = 771, 771 = 3 + (n - 1)12, 768 = (n - 1)12, (n - 1) = 64, n = 65, Therefore, the 65^{th} term will be 132 more than the 54^{th} term
8	a_1=5, d=9-5=4, a_n=185, 185 = 5 + (n - 1)4, 185 = 5 + 4n -4,185 = 1 + 4n, 185 - 1 = 4n, 184 = 4n, n= 184/4 = **46**
9	Odd numbers between 10 and 200 are 11,13,15....199. a_1 = 11, Last term l = 199, d = 2, an = a + (n-1) d, 199 = 11 + (n - 1) 2,199 - 11 = (n - 1) 2,188 = (n - 1) 2, 94 = n – 1,95 = n Sum of n terms = $\frac{n}{2}$(a + l), =$\frac{95}{2}$ (11 + 199)= 9975
10	Given $S_n = n^2$,we know , $a_n = S_n - S_{(n-1)}$, $a_5 = S_5 - S_{(5-1)} = S_5 - S_4 = 5^2 - 4^2$= 25-16=9
11	a_n<0 , 20 + (n-1)-3<0 ,20-3n+3<0,23-3n<0, 23<3n ,$\frac{23}{3}$ <n, 7.6<n. Next natural number greater than 7.6 is 8.Hence 8^{th} term is the first negative number.
12	$S_m = am^2 + bm$, S_1= a+b= a_1, S_2= 4a+2b= $a_1 + a_2$ $a_2 = S_2 - S_1$= 4a+2b- (a+b) = 3a +b , d= $a_2 - a_1$= 3a+b-(a+b)= 2a
13	First 8 multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 These numbers are in A.P. where a = 3, d = 3 and n = 8 , a_n=24, $S_n = \frac{n}{2}(a_1 + a_n)$, $S_8 = \frac{8}{2}$(3+ 24)= 4x27=108

14	Since, the number is divisible by both 2 and 5, means it must be divisible by ***10***.AP = 110, 120, 130,..., 990, a = 110, d = 10, nth term of the AP = 990 a+(n-1)d=990, 110+(n-1)10=990, (n-1)10=990-110,(n-1)= 880/10, n-1=88, n=88+1, n=89
15	a + 3d = 11….(1) a + 10d = 25…..(2)Subtracting equation (1) from equation (2) a + 10d – (a + 3d) = 25 – 11, 7d = 14, d = 2, Putting value of d=2 in the equation 2, a + 10x2 = 25, a + 20 = 25, a = 25 – 20, a = 5
16	The first term of the AP =8,Common difference d = -5 Given that A is the second term, So, A = 8+(-5) = 8-5 = 3 Given that B is the third term So, B= 3+(-5) = 3-5 = -2 So (A+B) = 3+(-2) = 3-2 = 1
17	First term (a) =15,Common difference (d) = -3,Last term(an) = 0, 0 = 15 + (n - 1)-3 -15 = -3n + 3, -15 - 3 = -3n, -18 = -3n, n=6
18	The sum of the first n terms of an A.P. is given by $S_n= 2n^2 +n$, At n=1 , $S_1= 2\mathrm{x}1^2+1= 3$, At n=2 , $S_2= 2\mathrm{x}2^2+2=10$, Since $a_1=S_1$,$S_2=a_1+a_2$, So , $a_1=3$,a1+a2=10,⇒3+a_2=10 so $a_2=7$,d= 7-3=4,a_n= 3+(n-1)4=4n-1
19	a + 3d = 0 or a = -3d…(1) ,a_{25} = a + 24d = -3d + 24d = 21d …(2) a_{11} = a + 10d = -3d + 10d = 7d ….(3) From (2) and (3), we have 21d= 3x7d , a_{25} = 3 x a_{11} Hence proved.
20	$d=\frac{1+m}{m} - \frac{1}{m}= 1$, $a_n = \frac{1}{m} +(n-1)1= \frac{1}{m} +n- 1 = \frac{1+m(n-1)}{m}$

SHORT ANSWER TYPE QUESTIONS

SECTION C (3 MARKS QUESTIONS)

1	Let the first term, common difference of an AP are a and d, respectively. According to the question, $a_3 + a_8 = 7$ and $a_7 + a_{14} = -3$ ⇒ a + (3−1)d + a + (8−1)d=7 [∵ an=a + (n−1)d] And a + (7 - 1)d + a + (14 - 1)d = - 3 a + 2d + a + 7d = 7 And a + 6d + a + 13d =-3 2a + 9d = 7 . . . . (i) And 2a + 19d = - 3 . . . . (ii) On subtracting eq. (i) from eq.(ii), we get; 10d = −10 ⇒ d = −1 2a + 9 (-1) = 7 ⇒ 2a - 9 = 7

	$\Rightarrow 2a = 16 \Rightarrow a = 8$ $\therefore\quad a_{10} = a + (10-1)d$ $= 8 + 9(-1)$ $= 8 - 9 = -1$
2	First three-digit number which is a multiple of 11 is 110 Last three-digit number which is a multiple of 11 is 990 the sequence of three-digit numbers which are multiples of 11 are 110, 121, 132, …, 990. Clearly, it is an A.P. $\therefore a=110 \quad a_n = 990 \quad d = 11$ $a_n = a + (n-1)\,d, \quad 990=110+(n-1)11$ $\frac{880}{11} = n - 1$ $80 = n - 1,\ n = 81$ $\therefore$ sum of all terms of A.P is given by $S_n = \frac{n}{2}\ [a_1 + a_n]$ $= \frac{81}{2}[110+990]$ $= \frac{81}{2} \times 1100$ $= 81 \times 550$ $= 44550$ Hence, the required sum is 44550.
3	$S_n = 3n^2 + 5n$ $S_1 = 3 \times 1^2 + 5 \times 1 = 8 = a_1$ $S_2 = 3 \times 2^2 + 5 \times 2 = 22 = a_1 + a_2$ $a_2 = 22 - 8 = 14 \Longrightarrow$ a+d , d= 14-8=6 $a_k = 164 \Longrightarrow 8 + (k-1)6 = 164$, k= 27
4	$a_p = \frac{(2p-1}{7}$, $a_1 = \frac{(2x1-1)}{7} = \frac{1}{7}$, $a_2 = \frac{(2x2-1)}{7} = \frac{3}{7}$, $d = \frac{3}{7} - \frac{1}{7} = \frac{2}{7}$, a = First Term = $\frac{1}{7}$ nth term = a + (n-1)d $= \frac{1}{7} + (n-1)\frac{2}{7}$ $= \frac{1+2n-2}{7} = \frac{2n-1}{7}$ Sum of n terms $= \frac{n}{2}$ (First term + nth term) $= \frac{n}{2}\left(\frac{1}{7} + \frac{2n-1}{7}\right) = \frac{n}{2}\ x \frac{1+2n-1}{7} = \frac{n}{2}\ x \frac{2n}{7} = \frac{n^2}{7}$

5	Given that first term, $a = 9$, Common difference, $d = 17 - 9 = 8$,Sum up to nth terms, $S_n = 636$ where $S_n = \frac{n}{2}[2a + (n - 1) d]$, $636 = \frac{n}{2}[2 \times 9 + (n - 1) 8]$ $636 = \frac{n}{2}[18 + 8n - 8]$, $636 = \frac{n}{2}[10 + 8n]$, $636 = n[5 + 4n]$, $636 = 5n + 4n^2$, $4n^2 + 5n - 636 = 0$, $4n^2 + 53n - 48n - 636 = 0$, $n (4n + 53) - 12 (4n + 53) = 0$ $(4n + 53)(n - 12) = 0$ Either $4n + 53 = 0$ or $n - 12 = 0$ $n = - 53/4$ or $n =12$ n cannot be -53/4 because the number of terms can neither be negative nor fractional, therefore, $n = 12$
6	Given that, mth term=$\frac{1}{n}$ and nth term=.$\frac{1}{m}$ then ,let a and d be the first term and the common difference of the A.P. so $a+(m-1)d=.\frac{1}{n}$.........(1) and $a+(n-1)d=\frac{1}{m}$..........(2). subtracting equation (2) from(1) we get, $md-d-nd+d=\frac{1}{n}-\frac{1}{m}$ $\Rightarrow d(m-n)= \frac{m-n}{nm}$ $\Rightarrow d= \frac{1}{nm}$ again if we put this value in equation (1) or (2) we get, $a+(m-1) \frac{1}{nm} =\frac{1}{n}$, $a=\frac{1}{n} - .\frac{1}{nm} (m-1)= \frac{1}{n} - \frac{m}{nm} -(- \frac{1}{nm})= \frac{1}{nm}$ then, the mnth term of the AP $a+(mn-1)d=\frac{1}{nm} +(mn-1) \frac{1}{nm} = \frac{mn}{mn} =1$ hence proved.
7	$S_9=171$, $S_{24}= 996$, $\frac{9}{2}[2a + (9- 1) d]=171$….(.1), $\frac{24}{2}[2a + (24- 1) d]= 996$……(.2) $2a+8d= \frac{171x2}{9} =$ $2a+ 8d= 38$…..(3) $2a+23d= \frac{996x2}{24}$ $= 2a+23d=83$……..(4) Solving (3) and (4) $23d-8d= 83-38$, $15d= 45$, $d=3$. Put $d=3$ in equation (3) $2a+ 8x3= 38$,$2a= 38-24= 14$, $a= 7$.
8	Let a be the first term and d be the common difference of the given AP. Then, $S_m = S_n$ $\Rightarrow \frac{m}{2}[2a + (m-1) d] = \frac{n}{2}[2a + (n-1) d]$ $\Rightarrow 2ma + d (m^2 - m) = 2an + d (n^2 - n)$ $\Rightarrow 2ma - 2na+d(m^2-m) - d(n^2-n) = 0$ $\Rightarrow 2a(m-n) + d(m^2-m) - d(n^2-n) = 0$ $\Rightarrow 2a(m-n) + d(m^2-n^2) - d(m-n) = 0$ $2a(m-n) + d(m+n)(m-n)- d(m-n) = 0$ $\Rightarrow 2a(m-n) +d(m-n)(m+n-1) = 0$ $(m-n) 2a+(m+n-1)d=0$ $[\because m -n \neq 0]$ $2a+(m+n-1)d=0$ ……(i) $\therefore S_{m+n} = \frac{m+n}{2}[2a + (m+n-1) d]$

	$S_{m+n} = \frac{m+n}{2} \times 0 = 0$ [from Eq. (i)] Hence proved.
9	Let a, d , and A , D be the first term and common different of the 2 A.P.s respectively. Here, $a = 63, \ d = 2$ $A = 3, D = 7$ Given, $a_n = A_n$ $\Rightarrow a + (n - 1)\, d = A + (n - 1)\, D$ $\Rightarrow 63 + (n - 1)\ 2 = 3 + (n - 1)\ 7$ $\Rightarrow 63 + 2n – 2 = 3 + 7n – 7$ $\Rightarrow 61 + 2n = 7n – 4$ $\Rightarrow 5n = 65$ $\Rightarrow n = 13$ ∴ When n is 13, the nth terms are equal i.e., $a_{13} = A_{13}$
10	Here, $S_{14} = 1050$, n = 14, a = 10. We know that $Sn = \frac{n}{2}[2a + (n-1)\, d]$ Substituting the values we have, $\Rightarrow 1050 = \frac{14}{2}[20 + 13d] \Rightarrow 1050 = 140 + 91\, d$ $\Rightarrow 910 = 91d$ $\Rightarrow d = 10$, Therefore, $a_{20} = 10 + (20 – 1) \times 10 = 200$ i.e. 20^{th} term is 200.
11	$\frac{a4}{a9} = \frac{1}{3}$ $\frac{a+3d}{a+8d} = \frac{1}{3}$, $(a+3d)3 = (a+8d)$ $3a+9d = a+8d$ $2a+d = 0$ $d = -2a$…..(1) $\frac{a12}{a5} = \frac{a+11d}{a+4d} = \frac{a+11x-2a}{a+4x-2a} = \frac{a-22a}{a-8a} = \frac{-21a}{-7a} = \frac{3}{1} = 3:1$
12	Let the first term is a and common difference is d Here, $a_{14} = 2\, a_8$ Or, $a + 13\, d = 2(a+7d)$ $a + 13d = 2a + 14d$ $-a - d = 0, \ a = - d$ ………………(1) again $a_6 = - 8$ or $a + 5d = - 8$ ……………………..(2) solving eq. (1) and (2) we get $a = 2$, $d = -2$ $S_{20} = 10\, (4 + (-38))$ $= 10\, (4 – 38)$ $= -340$

13	Let sum of first n terms be Sn. $\therefore$ Sn = $(4-\frac{1}{n})+\left(4-\frac{2}{n}\right)+(4-\frac{3}{n})+$ ……………up to n terms = (4 + 4 + 4 + 4 + 4 + ... up to n terms) + (-1/n - 2/n - 3/n - ....up to n terms) = 4 (1+1+1+1.......... up to n terms) - $\frac{1}{n}$ (1 + 2 + 3 +4 ....up to n terms) = $4\,n-\frac{1}{n}\times\frac{n(n+1)}{2}$ = $4n-\frac{n+1}{2}$ = .$\frac{8n-(n+1)}{2}$......(taking L.C.M) = $\frac{7n-1}{2}$ Therefore, the sum of n terms is $\frac{7n-1}{2}$.
14	Given, a_{10} = 52 ⇒ a+9d=52 ....(1) also, $a_{17} = 20 + a_{13}$ ⇒a + 16d = 20 + a + 12d ⇒16d − 12d = 20 ⇒4d = 20 ⇒d=5 putting the value of d in eq. (1), we get, ⇒a+9(5)=52 ⇒a+45=52 ⇒a=7 hence, the required AP is 7,12,17,.........
15	5th term is 20., a+4d = 20.---- (1) 7th term + 11th term is 64, a+6d +a + 10d = 64. , 2a + 16d = 64., a+8d = 32.---(2) solving equations (1) and(2). 4d = 12, d= 3

CASE STUDY QUESTIONS & LONG ANSWER QUESTIONS

SECTION D (4 MARKS QUESTIONS)

1	i) a_9 = a +8d =22600...(1), a_6 = a +5d = 16000....(2) ,by solving the two equations we get d= 2200 Put the value of d in second equation, a+ 5x 2200 = 16000 , a+ 11000 = 16000, a = Rs 5000 ii) an = 29200, a+ (n-1)d = 29200, 5000 + (n-1)2200 = 29200 2200n -2200= 29200 -5000 2200n = 24200 + 2200 n = 26400/2200= 12
2	i) AP is 51,49,47...31 a = 51 d = -2 an = a + (n-1)d, 31 = 51 + (n-1)-2 , 31-51 = -2n +2 -20 -2 = -2n, -22 = -2n

	n = 11 , 11 days ii) $a_n = 2n + 3$, $a_1 = 2x1 + 3 = 5$, $a_2 = 2x2 + 3 = 7$, d= a2 -a1 = 7-5 = 2
3	i) AP is 1000, 1100, 1200... a=1000 d = 100 $a_{30} = a + 29d = 1000 + 29x\ 100 = 3900$ ii) $S_{30} = n/2\ (a + a_{30})$ = 30/2 (1000 + 3900) = 15 x 4900 = 73500

LONG ANSWER QUESTIONS

4	$a_8 = \frac{1}{2}a_2$, $a+7d = \frac{1}{2}(a+d)$, $2a + 14d = a+d$, $a= -13d$ $a_{11} = \frac{1}{3}(a_4) + 1$, $a+10d = \frac{1}{3}(a + 3d) + 1$ Put a= -13d, $-13d + 10d = \frac{1}{3}(-13d+3d)+1$ $-3d = \frac{1}{3}(-10d)+1 = \frac{-9d+10d}{3} = 1$ d/3 =1, d =3, a= -13x3 = -39 $a_{15} = a + 14d = -39+14x3 = 3$
5	Assume that the first term is a and the common difference is d.Here the total terms are 37, which is an odd number.so, the middle term is:$\frac{37+1}{2}$= 19. Here the three middle most numbers are 18^{th},19^{th}&20^{th} terms . $a_{18}+a_{19}+a_{20}=225$ a+17d +a+18d+a+19d =225 3a+ 54d =225, a+18d =75 ...(1) Given $a_{35} + a_{36} + a_{37} = 429$, a+34d + a+ 35d + a+ 36d =429 3a +105d = 429, a+35d = 143....(2) Solving (1) and (2) a+ 35d -a -18d = 143-75 17d = 68, d= 4 Put d =4 in (1) a+18x4 = 75, a= 75 - 72 = 3 . AP is a,a+d,a+2d…. AP is 3,7,11,15....
6	Let the a be first term and d be common difference of an AP Given $S_5 + S_7 = 167$ $\frac{5}{2}(2a+ (5-1)d) + \frac{7}{2}(2a+(7-1)d)= 167$ 5[2a + 4d] + 7[2a + 6d] = 167x2 , 10a + 20d + 14a + 42d = 334 24a + 62d = 334, 12a + 31d = 167 12a = 167 - 31d …..(1) Also S_{10} = 235 , $\frac{10}{2}(2a+ (10-1)d)= 235$ 5[2a + 9d] = 235 2a + 9d = 47…….(2), Multiplying equation (2) by 6 we get

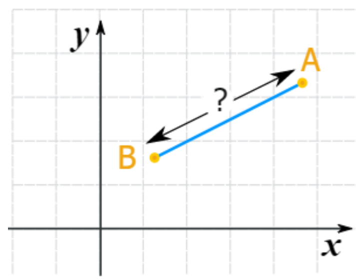
	12a + 54d = 282 167 - 31d + 54d = 282 [using eqn (1)] 23d = 115, d = 5 Put d=5 in equation (1) 12a = 167 – 31x5, 12a = 167 - 155 = 12 a = 1 , So, the AP is, a, a + d, a + 2d, a + 3d, … , AP : 1, 6, 11, 16….
7	Given,The sum of first n terms of an AP is equal to the sum of first 2n terms of another AP.We have to find the value of n. Given, the first AP series has First term, a = 8, d = 20, Also, the second AP series has , First term, a = -30, d = 8.Then the sum of first n terms of AP is given by $S_n = n/2[2a + (n-1)d]$ For the first series, $S_n = \frac{n}{2}[2(8) + (n - 1)(20)] = \frac{n}{2}[16 + 20n - 20] = \frac{n}{2}[20n - 4]$ $S_n = n[10n - 2]$ For the second series, $S_{2n} = \frac{2n}{2}[2(-30) + (2n - 1)(8)] = n[-60 + 16n - 8]$ $S_{2n} = n[16n - 68]$, Given, $S_n = S_{2n}$ $n[10n - 2] = n[16n - 68]$, $10n - 2 = 16n – 68$ $10n - 16n = -68 + 2$, $-6n = -66$, $n = 66/6$ $n = 11$, Therefore, the value of n is 11.
8	Let the three term in A.P is (a-d), a and (a+ d) According to question, a - d + a + a + d = 21, 3a = 21, a = 7 Also $(a-d)^2+a^2+(a+d)^2= 155$ $a^2-2ad+d^2+a^2+a^2+2ad+d^2= 155$, $3a^2+2d^2= 155$ $3x7^2+2d^2= 155$, $2d^2= 155-147$, $2d^2=8$, $d^2= 4$, $d=\pm2$ So the AP is when d=2 , AP: 7-2,7,7+2,… = 5,7,9… Or when d=-2 AP: 7-(-)2, 7, 7+ -2,….. = 9,7,5,…
9	$a_3 + a_7 = 6$ ----- (1) $a_3 \times a_7 = 8$ ----- (2) (a + 2d) + (a + 6d) = 6, 2a + 8d = 6, a + 4d = 3, a = 3 - 4d ----- (3) (a + 2d) × (a + 6d) = 8…..(4)

	Substituting the value of a from equation (3) $(3 - 4d + 2d) \times (3 - 4d + 6d) = 8$ $(3 - 2d) \times (3 + 2d) = 8$ $(3)^2 - (2d)^2 = 8$ [Since $(a + b)(a - b) = a^2 - b^2$] $9 - 4d^2 = 8$ $4d^2 = 1$, $d^2 = ¼$, $d = ½, -½$ Case 1: When $d = ½$ $a = 3 - 4d, = 3 - 4 \times ½ = 3 - 2= 1$ $S_{16} = \frac{16}{2}[2 \times 1 + (16 - 1) \times ½]$ $= 8 \times[2+ \frac{15}{2}]= 76$ Case 2: When $d = - ½$ $a = 3 - 4d, = 3 - 4 \times (- ½) = 3 + 2 = 5$ $S_{16} = \frac{16}{2}[2 \times 5 + (16 - 1) \times (-½)]$ $= 8 [10 -\frac{15}{2}] = 8 \times \frac{5}{2}= 20$
10	AP 8,10,12... a=8, d=2, n=60 $a_{60} = a+59d$ $=8+59x2 = 8+118 = 126$ Now the last term is 126 By reversing AP ,new AP: 126....12, 10, 8 Sum of last 10 terms= Sum of first 10 terms of new AP a= 126 d = 8-10 = -2 $S_{10} = \frac{n}{2}(2a +(n-1)d)$ $=\frac{10}{2}$ (2x 126 +(10-1) x -2)= 5(252 -18) = 5x234 = 1170
	Case study 4: i) a_{10} ii) $a_5 – a_2 =9$ iii) n = 8 iv) $S_{12}=222$

UNIT 3
COORDINATE GEOMETRY

IMPORTANT FORMULAS & CONCEPTS

DISTANCE FORMULA



Let $A(x_1,y_1)$ and $B(x_2,y_2)$ be two points in the Cartesian plane.

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$or \ \ AB = \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$$

Problems based on distance formula

- To show that a given figure is a
- **Parallelogram** – prove that the opposite sides are equal
- **Rectangle** – prove that the opposite sides are equal and the diagonals are equal.
- **Parallelogram but not rectangle** – prove that the opposite sides are equal and the diagonals are not equal.
- **Rhombus** – prove that the four sides are equal
- **Square** – prove that the four sides are equal and the diagonals are equal.
- **Rhombus but not square** – prove that the four sides are equal and the diagonals are not equal.

- **Isosceles triangle** – prove any two sides are equal.
- **Equilateral triangle** – prove that all three sides are equal.
- **Right triangle** – prove that sides of triangle satisfy Pythagoras theorem.

DISTANCE OF A POINT P(X,Y) FROM ORIGIN.

Since coordinate of origin is (0,0), Then by applying distance formula,

distance from P(x,y) is OP $=\sqrt{x^2+y^2}$

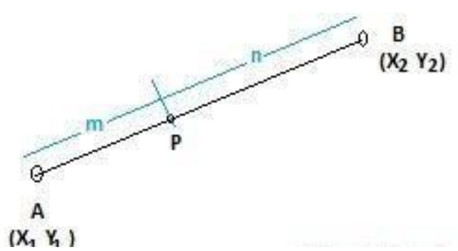
COLLINEAR POINTS:

A given number of points are said to be collinear if they lie on the same line. To prove that three points A ,B and C are collinear (using distance formula), we need to prove that sum of any two of the distances AB, BC and AC is equal to the third distance.

SECTION FORMULA

The coordinates of the point P (x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2), internally, in the ratio m:n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

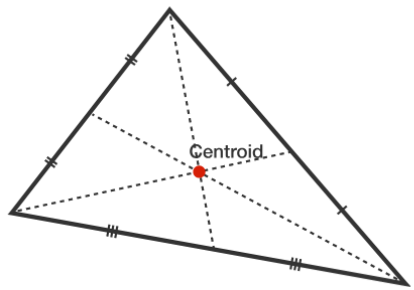


MID POINT FORMULA

If point P(x,y) divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2), internally, in the ratio 1:1 (*i.e.* P is the mid point of AB) Then coordinates of point P are given by, $\mathrm{P}(x,y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

CENTROID OF TRIANGLE

The centroid of a triangle is the center of the triangle. It is referred to as the point of concurrency of medians of a triangle.



The coordinates of the vertices of a triangle are A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3), then centroid C (x, y) of given triangle ABC can be find out using,

$$C\ (x, y)\ =\ (\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3})$$

MULTIPLE CHOICE QUESTIONS

SECTION - A

Q1. The distance of the point P (-6, 8) from the origin is:

(a) 14 (c) 8

(b) 6 (d) 10

Q2. If (a, b) is the mid-point of the line segment joining the points A (10, -6) and B (k, 4) and $a - 2b = 18$, the value of k is:

(a) 40 (c) 4

(b) 22 (d) 36

Q3. The distance between the points ($a \cos\theta + b \sin\theta$, 0) and (0, $a \sin\theta - b \cos\theta$), is :

(a) $\sqrt{a^2 - b^2}$ (c) $a^2 - b^2$

(b) $a^2 + b^2$ (d) $\sqrt{a^2 + b^2}$

Q4. If the point P (k, 0) divides the line segment joining the points A (2, -2) and B (-7, 4) in the ratio 1:2, then the value of k is :

(a) 1 (c) -1

(b) 2 (d) -2

Q5. If the point P (6, 2) divides the line segment joining A (6, 5) and B (4, y) in the ratio 3 : 1, then the value of y is :

(a) 4 (c) 1

(b) 2 (d) 3

Q6. Distance between two points (3, 2) and (6, 6) is:

(a) 5 (c) 2

(b) 3 (d) 8

Q7. The line segment joining the points P (-3, 2) and Q (5, 7) is divided by the *y*- axis in the ratio:

(a) 3 : 1 (c) 3 : 4

(b) 3 : 2 (d) 3 : 5

Q8. The point P on *x*- axis is equidistant from the points A (-1, 0) and B (5, 0) is:

(a) (2, 2) (c) (2, 0)

(b) (0, 2) (d) (3, 2)

Q9. The mid-point of the line segment joining the points A (-2, 8) and B (-6,-4) is:

(a) (-4, -6) (c) (2, 6)

(b) (-4, 2) (d) (6, -2)

Q10. Point A (-1, *y*) and B (5, 7) lie on a circle with centre O (2, -3*y*). The values of *y* are:

(a) 1, -7 (c) -2, -7

(b) -2, 7 (d) -1, 7

Q11. Find the perpendicular distance of A (5, 12) from the *y* -axis.

Q12. Find the value of *y* for which the distance between the points (2, - 3) and (10,*y*) is 10 units.

Q13. To locate a point Q on line segment AB such that $BQ = \frac{5}{7} \times AB$. What is the ratio of line segment in which AB is divided?

Q14. Find the distance of the point (- 4, -7) from the *y* -axis.

Q15. If (2, *p*) is the midpoint of the line segment joining the points *A* (6, -5) and *B* (-2, 11), find the value of *p*.

Q16. If the centre and radius of circle is (3, 4) and 7 units respectively, then what is the position of the point *AB* with respect to circle.

Q17. If the distance between the points (4, *k*) and (1, 0) is 5, then what will be the possible values of *k*?

Q18. *ABCD* is a rectangle whose three vertices are *B* (4, 0), *C* (4, 3) and *D* (0, 3). Find the length of one of its diagonals

Q19. *A* (5,1), *B* (1,5) and *C* (-3, -1) are the vertices of *ΔABC*. Find the length of median *AD.*

Q20. Find the perimeter of a triangle with vertices (0,4), (0,0) and (3,0).

SHORT ANSWER TYPE QUESTION (2 MARKS)
SECTION – B

Q1. Find the point on the *x*-axis which is equidistant from the points (2, -5) and (-2, 9)

Q2. Find the distance of the point P (2, 3) from the x-axis.

Q3. Find the ratio in which the point (-3, *k*) divides the line-segment joining the points (-5, 4) and (-2, 3). Also find the value of *k.*

Q4. If A (5,2), B (2, -2) and C (-2, t) are the vertices of a right-angled triangle with $\angle B$ = 90°, then find the value of t.

Q5. In what ratio does the point *P* (2, -5) divide the line segment joining A (-3, 5) and B (4, -9).

Q6. If the point *P* (*x*, *y*) is equidistant from the points *A (a* + *b*, *b* - *a*) and *B* (*a* - *b*, *a* + *b*), then prove that $bx = ay$.

Q7. If the mid-point of the line segment joining A $\left(\frac{x}{2}, \frac{y+1}{2}\right)$ and B $(x + 1,\ y - 3)$ is C (5, -2), find *x*, *y*.

Q8. Find a point on y-axis which is equidistant from A (6,5) and B (-4, 3).

Q9. If *A* and *B* are (-2, -2) and (2, -4), respectively, find the coordinates of *P* such that *AB* = 5*AB* and *P* lies on the line segment *AB.*

Q10. Find the third vertex of a *Δ*, if two of its vertices are at (1, 2) and (3, 5) and the centroid at the origin.

Q11. In a seating arrangement of desks in a classroom, three students are seated at A (3, 1), B (6,4) and C (8, 6) respectively. Are they seated in line?

Q12. Name the type of triangle formed by the points A (–5, 6), B (–4, –2) and C (7, 5).

Q13. Find a relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5).

Q14. Find the mid-point of side BC of *ΔABC*, with A(1, -4) and the mid-points of the sides through A being (2, -1) and (0, -1)

Q15. The coordinates of the points P and Q are respectively (4, -3) and (-1, 7). Find the abscissa of a point R on the line segment PQ such that PRPQ = 35.

Q16. Write the coordinates of a point on x-axis which is equidistant from the points (-3, 4) and (2, 5).

Q17. Find the ratio in which the line segment joining the points P (3, -6) and Q (5,3) is divided by the x -axis.

Q18. Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

Q19. Find the area of a rhombus if its vertices (3, 0), (4, 5), (- 1, 4) and (-2, – 1) are taken in order.

Q20. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1,4).

<u>SHORT ANSWER TYPE QUESTION (3 MARKS)</u>

SECTION – C

Q1. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Q2. Find the values of y for which the distance between the points P (2, -3) and Q (10, y) is 10 units.

Q3. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order.

Q4. If A (-2,1), B (a, 0), C (4, b) and D (1, 2) are the vertices of a parallelogram *ABCD*, find the values of a and b. Hence find the lengths of its sides.

Q5. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order find x and y.

Q6. Find the point on the x-axis which is equidistant from (2, -5) and (- 2, 9).

Q7. Find the centre of the circle passing through A (6, -6), B (3, -7) and C (3, -3).

Q8. Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

Q9. Find the coordinates of the points which divide the line segment joining A (- 2, 2) and B (2, 8) into four equal parts.

Q10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (- 3, 4).

Q11. If two adjacent vertices of a parallelogram are (3,2) and (-1,0) and the diagonals intersect at (2, -5), then find the coordinates of the other two vertices.

Q12. Find the type of quadrilateral formed by the points (- 1, – 2), (1, 0), (- 1, 2), (- 3, 0) and justify your answer.

Q13. Find the ratio in which the line segment joining A (1, – 5) and B (- 4, 5) is divided by the *x*-axis. Also find the coordinates of the point of division.

Q14. Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segments joining A (2, -2) and B (3,7).

Q15. If Q (0, 1) is equidistant from P (5, – 3) and R (x, 6), find the values of x. Also find the distance QR and PR.

LONG ANSWER TYPE QUESTIONS (4 MARKS)

SECTION – D

Q1. The vertices of quadrilateral *ABCD* are A (5, -1), B (8,3), C (4, 0) and D (1, -4). Prove that *ABCD* is a rhombus.

Q2. Find the centre and radius of the circumcircle (i.e., circumcentre and circum-radius) of the triangle whose vertices are (-2, 3), (2, -1) and (4, 0).

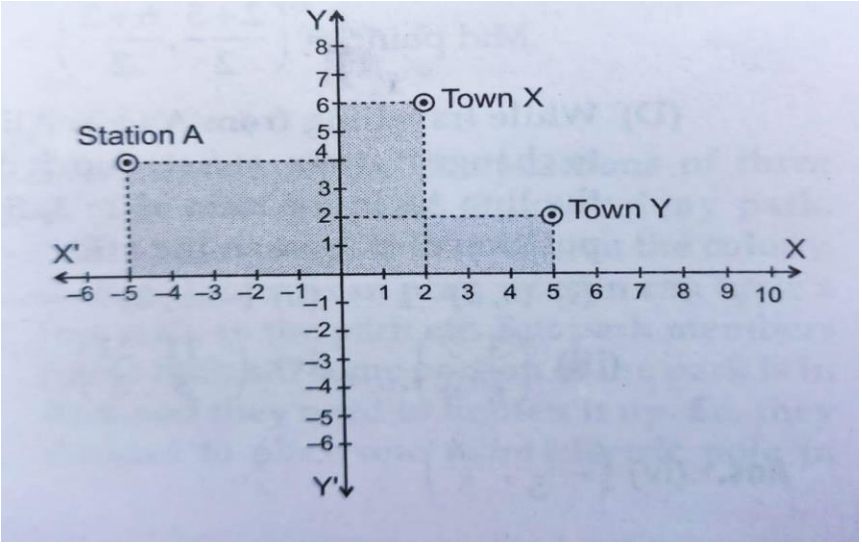
Q3. Find the coordinates of the points of trisection (*i.e., Points dividing in three equal parts*) of the line segment joining the points A (2, -2) and B (-7, 4).

Q4. An equilateral triangle has one vertex at (3, 4) and another at (-2, 3). Find the co-ordinates of the third vertex.

Q5. The three vertices of a parallelogram *ABCD* are *A* (3, -4), *B* (-1, -3) and *C* (-6, 2). Find the coordinates of vertex *D* and find the area of *ABCD*.

Q6. The base *QR* of an equilateral triangle *PQR* lies on *x*-axis. The co-ordinates of point *Q* are (-4, 0) and the origin is the mid-point of the base. Find the co-ordinates of the point *P* and *R*.

Q7. Two friends Dalvin and Alice works in the same office in Toronto. In the Christmas vacation, they both decided to go to their home towns represented by *Town X* and *Town Y*. *Town X* and *Town Y* are connected by trains from the same station *C* in Toronto. The situation of *Town X*, *Town Y* and *station A* is shown on the coordinate axis.



Based on the given situation, answer the following questions:

i. What is the distance that Dalvin have to travel to reach his hometown *X* ?

(a) $\sqrt{51}$ units (c) $\sqrt{35}$ units

(b) $\sqrt{53}$ units (d) $\sqrt{47}$ units

(e)

ii. What is the distance that Alice has to travel to reach her hometown *Y*?

(a) $2\sqrt{26}$ units (c) $2\sqrt{10}$ units

(b) $\sqrt{107}$ units (d) $\sqrt{51}$ units

iii. Now, both of them plan to meet at a place between Town X and Town Y, such that it is a mid-point between both. Calculate the coordinates of the mid-point of X and Y.

(a) (1, 3) (c) (2.5,3)

(b) (2, - 4) (d) (3.5, 4)

iv. While travelling from *A* to *Y*, Alice had to change the train, at a station, it divides the line *AY* in the ratio of 2: 3, find the position of station on the grid.

(a) $\left(0,\ \frac{7}{9}\right)$

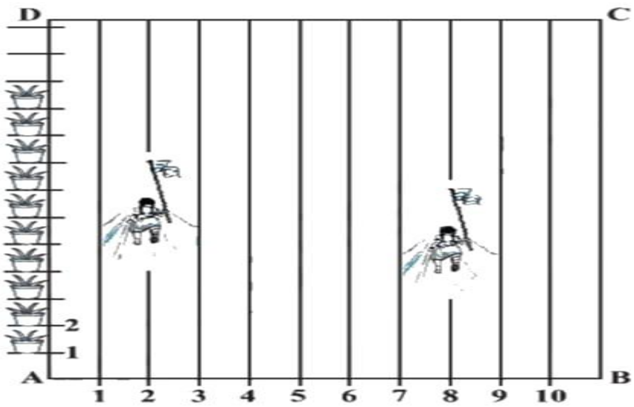
(b) $\left(-\frac{11}{5},\ \frac{24}{5}\right)$

(c) $\left(\frac{11}{8},\ \frac{17}{3}\right)$

(d) (12, 7)

Q8. To conduct Sport Day activities, in your rectangular shaped school ground *ABCD*, lines have been drawn with chalk powder at a distance of 1m each. 80 flower pots have been placed at a distance of 1 m from each other along *AD*, as shown in figure Hannah runs $\frac{1}{4}^{th}$ the distance AD in the 2^{nd} line and posts a blue flag. Preeta runs $\frac{1}{5}^{th}$ the distance AD on the 8^{th} line and posts a green flag.

i. What is the distance between both the flags?
ii. If Uthara has to post an orange flag exactly halfway between the line segment joining the two flags, where should she post her flag?
iii. Which mathematical concept is used in the above problem?
iv. What value is depicted in this problem?



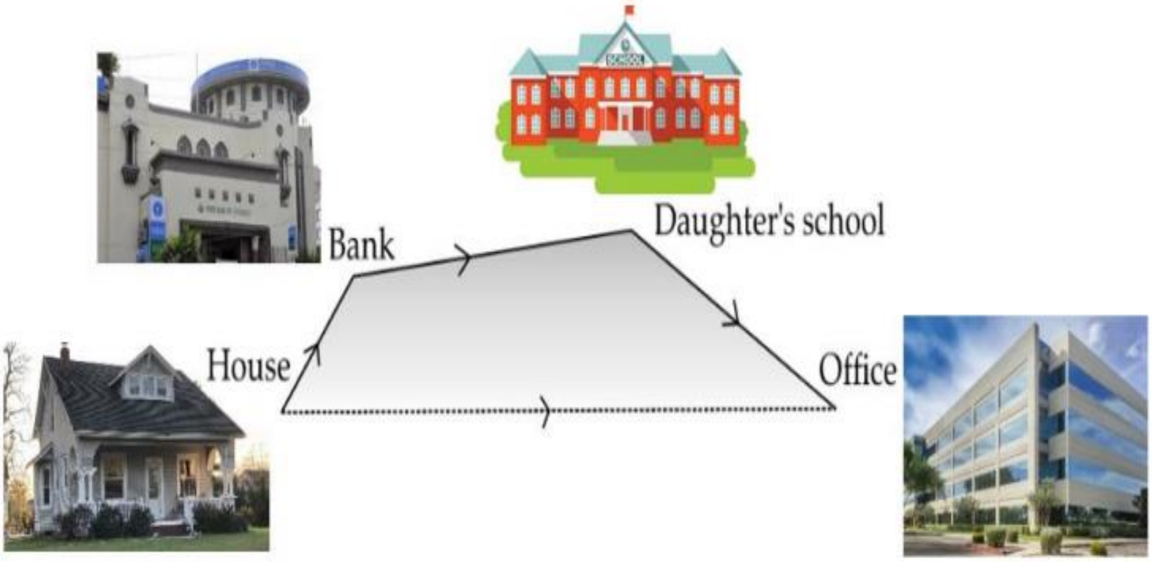
Q9. Find the ratio in which the point *P* (*x*, 2) divides the line segment joining the points *A* (12, 5) and *B* (4, -3). Also find *x*.

Rajeev went out from his house to reach the office. But he had to get some work done before going to the office. So, he first of all went to the bank first, from there he went to his son's school, and then reaches to office. The position of home, school, bank and office on coordinate axis is shown in the following figure: (Assume that all distances covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km.

Q10.

Now, answer the following questions:

(i) If Rajeev goes directly from bank to his office, how much distance he would travel?



(a) $2\sqrt{97}$ units (b) $9\sqrt{17}$ units (c) $7\sqrt{7}$ units (d) $7\sqrt{45}$ units

(ii) How much distance he will travel, if goes directly from home to the office?

(a) $15\sqrt{7}$ units (b) 10 units (c) $14\sqrt{3}$ units (d) $11\sqrt{5}$ units

(iii) If at the mid-point of the bank and school, there is a park, what are the coordinates of the park?

(a) (13, 14) (b) (9,11) (c) (-5, 10) (d) (10, 12)

(iv) Find the distance of the point (-6, 8) from the origin.

(a) 8 units (b) 10 units (c) 11 units (d) 9 units

ANSWERS OF SECTION - A			
1. (d) 10	6. (a) 5	11. 5 units	16. Inside the circle
2. (b) 22	7. (d) 3 : 5	12. y = -9 or y = 3	17. $k = \pm 4$
3. (d) $\sqrt{a^2+b^2}$	8. (c) (2, 0)	13. 2 : 5	18. BD = 5
4. (c) -1	9. (b) (-4, 2)	14. 4 units	19. **$\sqrt{37}$ units**
5. (c) 1	10. (d) -1, 7	15. p = 3	20. 12

ANSWERS OF SECTION - B			
1. (-7, 0)	6. $bx = ay.$	11. Yes	16. $\left(\frac{2}{5}, 0\right)$
2. 3	7. $y = -1$	12. Scalene Triangle	17. 2 : 1
3. Ratio is 2 : 1 & $k = \frac{2}{3}$	8. (0, 9)	13. $x - y = 2$	18. Yes
4. t = 1	9. $\left(\frac{-2}{7}, \frac{-20}{7}\right)$	14. BC = (1, 2)	19. 24 square units
5. $k = \frac{5}{2}$ or $k = 5 : 2$	10. (-4, - 7)	15. Abscissa of R = 1	20. A (3, -10)

ANSWERS OF SECTION - C		
1. **Non- collinear**	6. **(-7,0)**	11. (1,-12) and (5,-10)
2. $\boldsymbol{y = 3}$ **or** $\boldsymbol{y = -9}$	7. (4,-5)	12. **Square**
3. **24 square units**	8. **(2, -5/3) and (0, -7/3)**	13. **k =1 and (-3/2, 0)**
4. $a = 1,\ b = 1$ AB = CD = $\sqrt{\mathbf{10}}$ **units** BC = AD = $\sqrt{\mathbf{10}}$ **units**	9. $\left(\mathbf{-1, \frac{7}{2}}\right)$, **(0, 5)**, $\left(\mathbf{1, \frac{13}{2}}\right)$	14. 2 : 9
5. $\boldsymbol{x = 6}$ **and** $\boldsymbol{y = 3}$	10. $\boldsymbol{3x + y - 5 = 0}$	15. $\boldsymbol{x = 4}$ **or** $\boldsymbol{x = -4}$ **and** **QR=** $\sqrt{\mathbf{41}}$ **, PR=** $\sqrt{\mathbf{82}}$

ANSWERS OF SECTION – D

1. The sides of the quadrilateral $AB = BC = CD = AD = 5$ units & the diagonals $AC = \sqrt{2}$ units and $BD = 7\sqrt{2}$ units
   As the length of all the sides are equal and the length of the diagonals are not equal.
   $\Rightarrow$ *ABCD* is a rhombus
2. Circumcentre of the ΔABC is $\left(\frac{3}{2}, \frac{5}{2}\right)$ and Circumradius of ΔABC is $\frac{5\sqrt{2}}{2}$
3. The coordinates of the points of trisection of the line segment joining A and B are (-1, 0) and (-4, 2)

4. Third vertex has the coordinates $\left(\frac{1+\sqrt{3}}{2}, \frac{7-5\sqrt{3}}{2}\right)$ or $\left(\frac{1-\sqrt{3}}{2}, \frac{7+5\sqrt{3}}{2}\right)$

5. 15 square units

6. Coordinates of P are $(0, 4\sqrt{3})$ or $(0, -4\sqrt{3})$

7. (i) (b) $\sqrt{53}$ units
   (ii) (a) $2\sqrt{26}$ units
   (iii) (d) (3.5, 4)
   (iv) (b) $\left(-\frac{11}{5}, \frac{24}{5}\right)$

8. (i) $\sqrt{61}$m
   (ii) $\left(5, \frac{45}{2}\right)$
   (iii) Co-ordinate Geometry
   (iv) Team Spirit

9. Ratio is 3 :5 and $x = 9$

10. (i) (a) $2\sqrt{97}$ units
   (ii) (d) $11\sqrt{5}$ units
   (iii) (b) (9,11)
   (iv) (b) 10 units

HOTS QUESTIONS

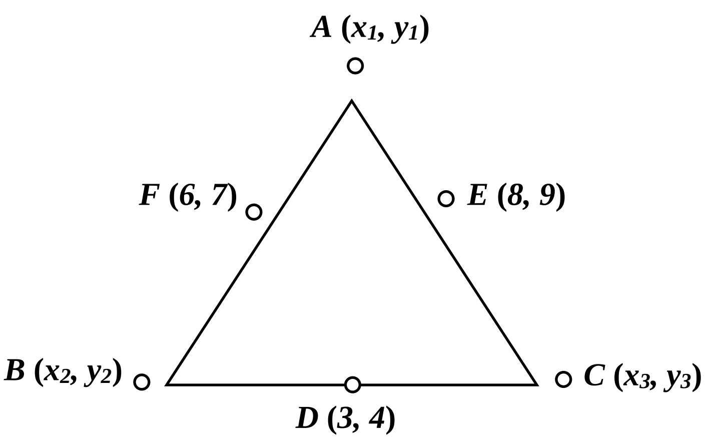
1. (1, -1), (0, 4) and (- 5, 3) are vertices of a triangle. Check whether it is a scalene triangle, isosceles triangle or an equilateral triangle. Also, find the length of its median joining the vertex (1, -1) the mid-point of the opposite side.

Ans: ΔABC is isosceles.

$\therefore$ Length of the median AD is $\frac{\sqrt{130}}{2}$ units.

2. The midpoints D, E, F of the sides of a triangle ABC are (3, 4), (8, 9) and (6, 7). Find the coordinates of the vertices of the triangle.

Ans:



Hence, the vertices of the ΔABC are A (11, 12), B (1, 2) and C (5, 6)

ANSWERS OF SECTION - A

1. (d) 10	6. (a) 5	11. 5 units	16. Inside the circle
2. (b) 22	7. (d) 3 : 5	12. y = -9 or y = 3	17. $k = \pm 4$
3. (d) $\sqrt{a^2 + b^2}$	8. (c) (2, 0)	13. 2 : 5	18. BD = 5
4. (c) -1	9. (b) (-4, 2)	14. 4 units	19. $\sqrt{37}$ **units**
5. (c) 1	10. (d) -1, 7	15. p = 3	20. 12

ANSWERS OF SECTION - B

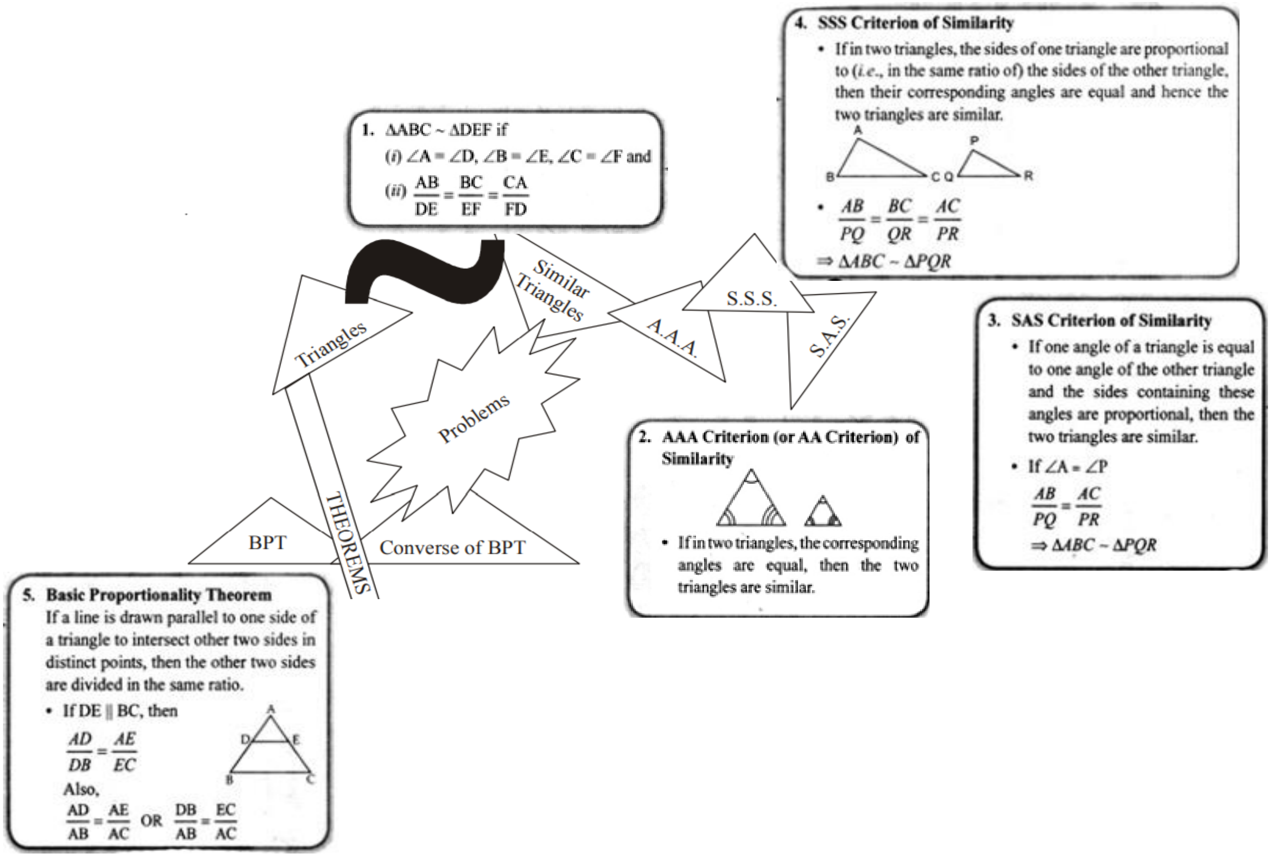
1. (-7, 0)	6. $bx = ay$.	11. Yes	16. $\left(\frac{2}{5}, 0\right)$
2. 3	7. y = -1	12. Scalene Triangle	17. 2 : 1
3. Ratio is 2 : 1 & $k = \frac{2}{3}$	8. (0, 9)	13. $x - y = 2$	18. Yes
4. t = 1	9. $\left(\frac{-2}{7}, \frac{-20}{7}\right)$	14. BC = (1, 2)	19. 24 square units
5. $k = \frac{5}{2}$ or k = 5 : 2	10. (-4, - 7)	15. Abscissa of R = 1	20. A (3, -10)

ANSWERS OF SECTION - C

1. **Non- collinear**	6. **(-7,0)**	11. (1,-12) and (5,-10)
2. **y = 3 or y = -9**	7. (4,-5)	12. **Square**
3. **24 square units**	8. **(2, -5/3) and (0, -7/3)**	13. **k =1 and (-3/2, 0)**
4. a = 1, b = 1 AB = CD = $\sqrt{10}$ **units** BC = AD = $\sqrt{10}$ **units**	9. $\left(-1, \frac{7}{2}\right)$, **(0, 5)**, $\left(1, \frac{13}{2}\right)$	14. 2 : 9
5. **x = 6 and y = 3**	10. **3x + y - 5 = 0**	15. **x = 4 or x = -4 and** **QR= $\sqrt{41}$, PR= $\sqrt{82}$**

UNIT 4 - GEOMETRY

LESSON 6 : TRIANGLES



LEARNING PLAN

- **TOPIC 1**: similar triangles, Definition, examples, Basic proportionality theorem
- **TOPIC 2:** Criterion of Similarity (AAA, SSS, SAS) Results based on it

TOPIC 1

- Two figures having same shapes (size may or may not same) are called similar figures
- Pair of all regular polygons are similar figures
- All circles are similar figures
- Film 35mm is enlarged into 70mm, and then they are called similar figures.

Similar triangles: If two triangles are said to be similar if

(a) Their corresponding angles are equal

(b) Ratio of their corresponding sides are equal/proportional

Basic proportionality Theorem/ Thales Theorem: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

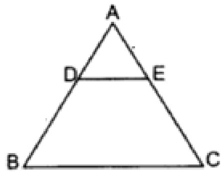
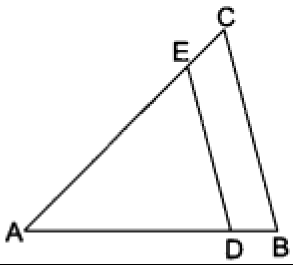
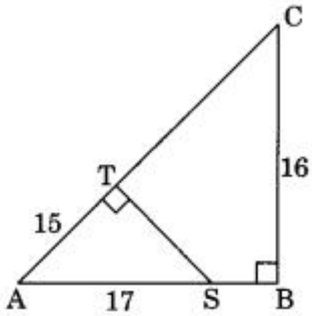
TOPIC 2

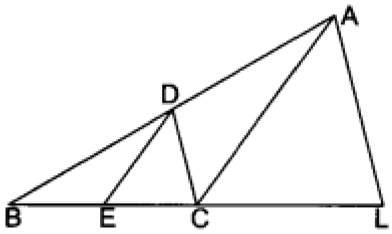
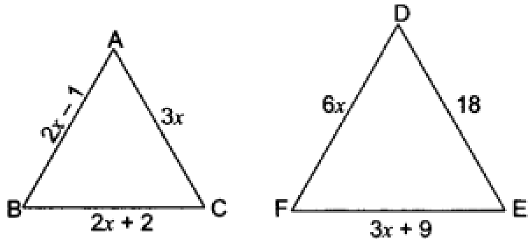
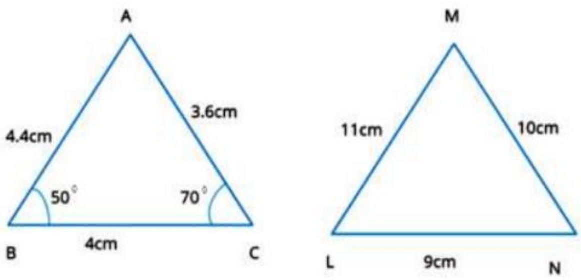
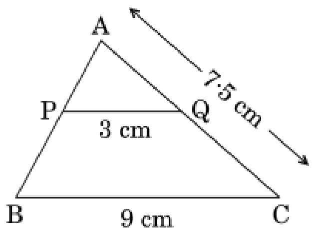
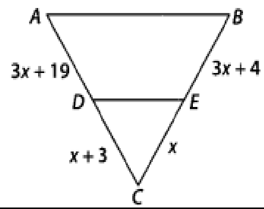
Criterion of similarity (AAA, SSS, SAS) and Results related on it.

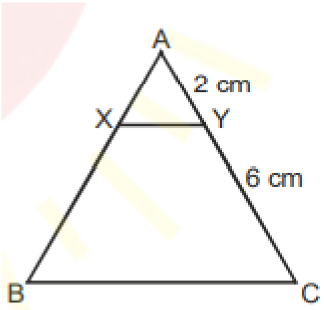
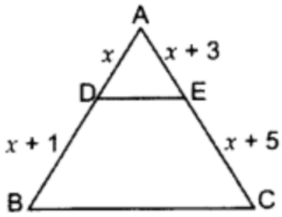
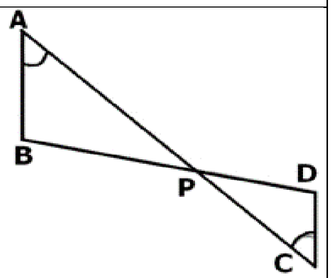
Revision notes

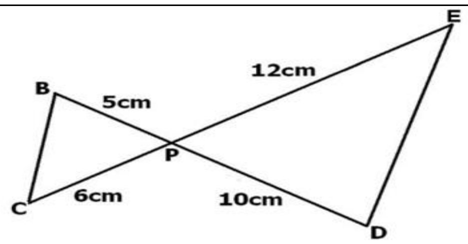
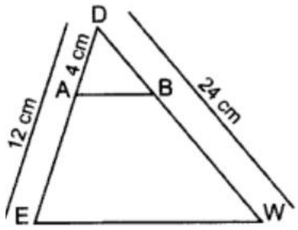
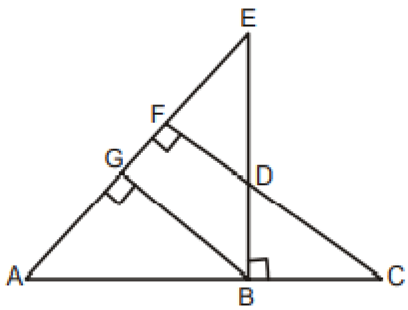
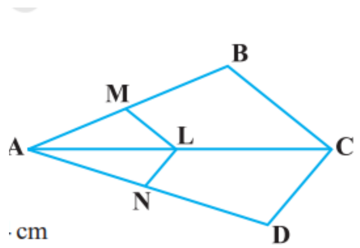
- In two triangles, if the corresponding angles are equal, then the corresponding sides are in the same ratio, then the triangles are similar (AAA similarity criterion)
- If the corresponding sides of any two triangles are proportional, then the corresponding angles are equal and the two triangles are similar (SSS similarity criterion)
- If one angle of a triangle is equal to one angle of the other triangle and the corresponding sides including are proportional. Then the triangle are similar (SAS criterion
- of the other triangle and the corresponding sides including are proportional. Then the triangle are similar (SAS criterion
- The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

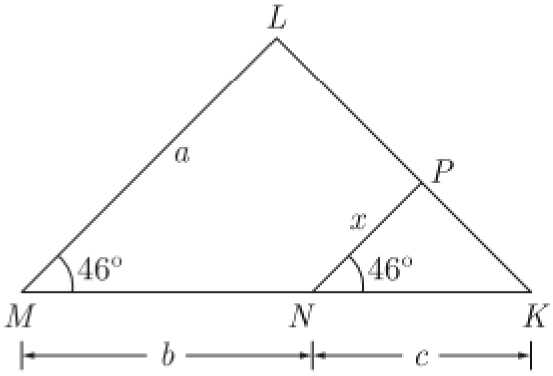
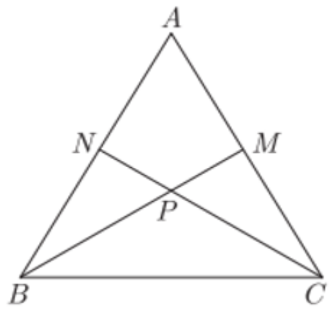
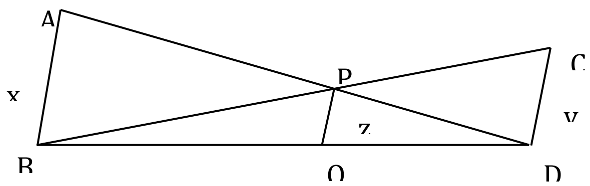
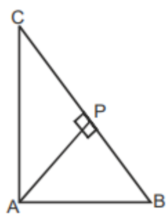
	SECTION A (1 MARK)
	LEVEL -1
1	Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?
2	A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm, and PB = 4 cm. Is AB ∥ QR? Give reason.
3	In triangles PQR and TSM, ∠P = 55°, ∠Q = 25°, ∠M = 100°, and ∠S = 25°. Is ΔQPR ~ ΔTSM? Why?
4	If in ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar , when ∠B=∠E (b) ∠A=∠D (c) ∠B=∠D (d) ∠A=∠F
5	All the congruent figures are similar but the converse is not true. True or false?
6	In ΔABC, ∠B=90^0, BD perpendicular to AC. If AC= 9cm, AD= 3 cm, then BD is equal to

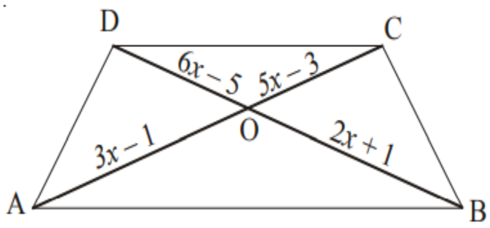
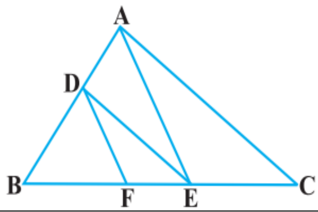
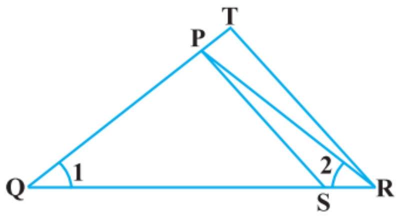
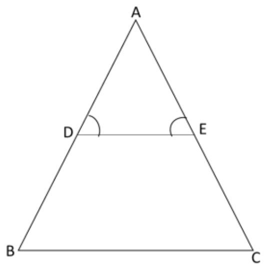
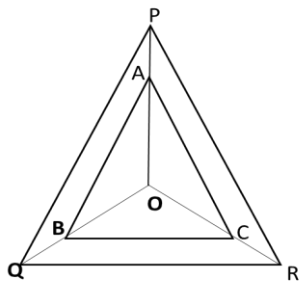
<table>
<tr><td></td><td>$(a) 2\sqrt{2}cm$ $(b) 3\sqrt{2}cm$ $(c) 2\sqrt{3}cm$ $(d) 3\sqrt{3}cm$</td></tr>
<tr><td>7</td><td>If ABC and DEF are similar triangles such that ∠A = 47° and ∠E = 63°, then the measures of ∠ C = 70°. Is it true? Give reason</td></tr>
<tr><td></td><td>LEVEL -2</td></tr>
<tr><td>8</td><td>If triangle ABC is similar to triangle DEF such that 2AB = DE and BC = 8 cm. Then find the length of EF.</td></tr>
<tr><td>9</td><td>In an isosceles ΔABC, if AC = BC and $AB^2 = 2AC^2$, then find ∠C.</td></tr>
<tr><td>10</td><td>The length of the diagonals of a rhombus are 16 cm and 12 cm. Find the length of side of the rhombus.</td></tr>
<tr><td>11</td><td>In ΔABC and ΔDEF, < B = <E, <F = <C and AB= 3DE. Then the two triangles are
(a) Congruent but not similar (b) Similar but not congruent
(c) neither congruent nor similar (d) none of the above</td></tr>
<tr><td>12</td><td>A man goes 24 m towards West and then 10 m towards North. How far is he from the starting point?</td></tr>
<tr><td>13</td><td>In figure, if DE || BC, AD=3 cm, BD= 4 cm and BC= 14 cm, then DE equals
(a) 7 cm (b) 6 cm (c) 4 cm (d) 3 cm

</td></tr>
<tr><td>14</td><td>ΔABC ~ ΔDEF such that AB = 9.1 cm and DE = 6.5 cm. If the perimeter of ΔDEF is 25 cm, what is the perimeter of ΔABC?</td></tr>
<tr><td>15</td><td>In Fig., DE || BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.

</td></tr>
<tr><td></td><td>LEVEL -3</td></tr>
<tr><td>16</td><td>In the given figure, ∠T and ∠B are right angles. If the length of AT, BC and AS (in centimeters) are 15, 16, and 17 respectively, then the length of TC (in centimeters) is:
(a) 18
(b) 16
(c) 19
(d) 12

</td></tr>
</table>

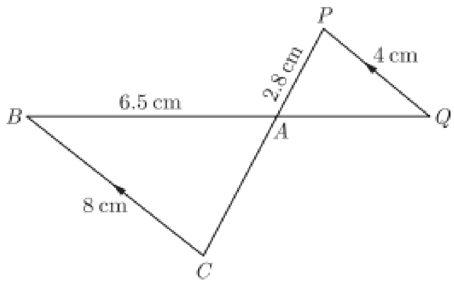
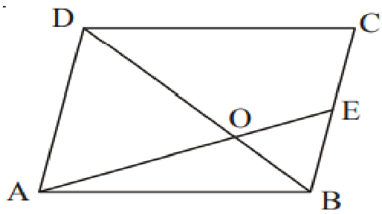
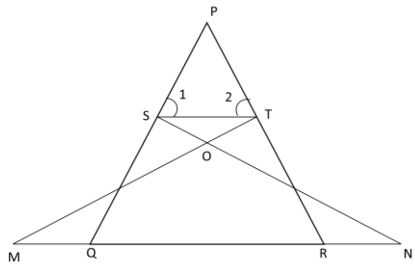
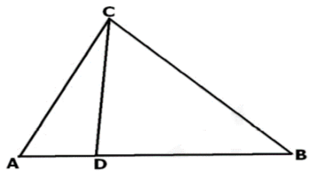
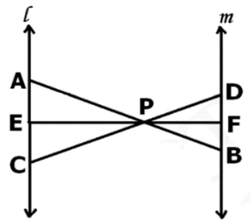
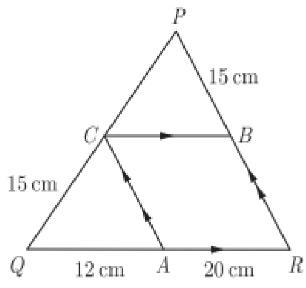
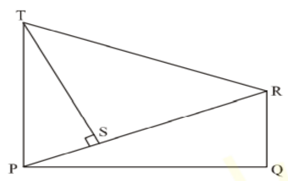
17	XY is drawn parallel to the base BC of a ΔABC cutting AB at X and AC at Y. If AB = 4 BX and YC =2cm, then AY is (a) 2cm (b) 4cm (c) 6 cm (d) 8cm
18	In the given Fig, CD ∥ LA and DE ∥ AC. Find the length of CL, if BE = 4 cm and EC = 2 cm. 
19	In Fig, if ΔABC ~ ΔDEF and their sides are of lengths (in cm) as marked along with them, then find the lengths of the sides of each triangle. 
20	If in two triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FE} = \frac{CA}{FD}$ then (a) ΔFDE ~ ΔCAB (b) ΔFDE ~ ΔABC (c) ΔCBA ~ ΔFDE (d) ΔBCA ~ ΔFDE
	SECTION B (1 MARK)
	LEVEL -1
1	Find ∠M. 
2	In Figure, PQ ∥ BC, PQ = 3 cm, BC = 9cm and AC = 7·5 cm. Find the length of AQ. 
3	Find the value of x for which DE ∥ AB is given  figure
4	X and Y are points on the sides AB and AC respectively of a triangle ABC such that

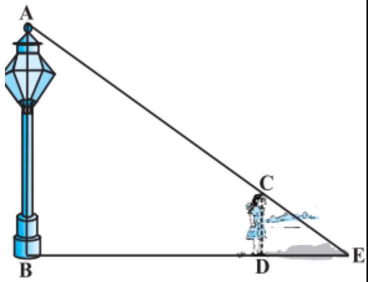
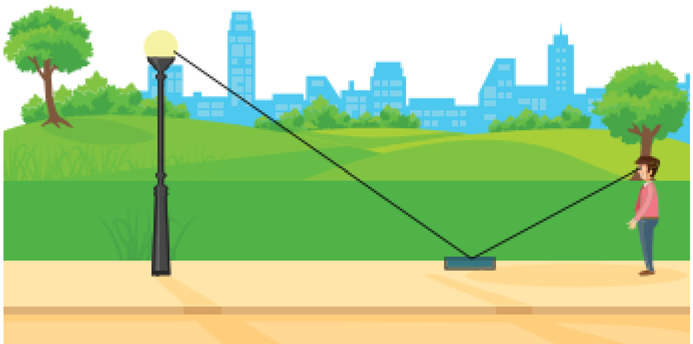
	= AX/AB=1/4 , AY = 2 cm and YC = 6 cm. Find whether XY || BC or not. 
5	In ΔABC, DE || BC, find the value of x. 
6	If the corresponding Medians of two similar triangles are in the ratio 5 : 7. Then find the ratio of their sides.
7	Diagonals AC and BD of a trapezium ABCD with AB ∥ DC intersect each other at the point O. Using a similarity criterion for two triangles, show that OA/OC= OB/OD.
	LEVEL -2
8	A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3m, what is the length of the shadow of the pole?
9	A 15 metres high tower casts a shadow 24 metres long at a certain time and at the same time, a telephone pole casts a shadow 16 metres long. Find the height of the telephone pole.
10	In the below figure, if ∠A = ∠C, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD. 

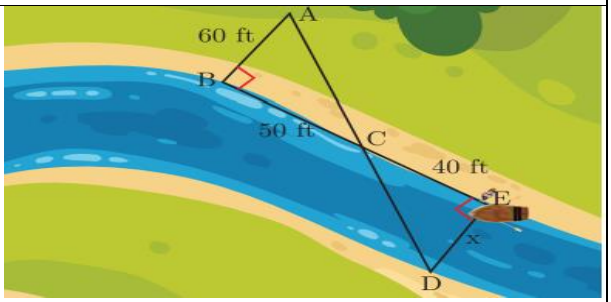
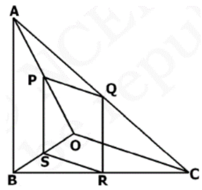
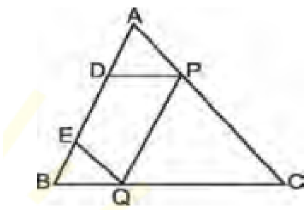
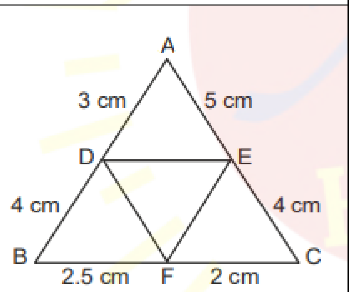
11	In the below Figure, BD and CE intersect each other at the point P. Is ΔPBC ~ ΔPDE? Why? 
12	In ΔDEW, AB || EW. If AD = 4 cm, DE = 12 cm and DW = 24 cm, then find the value of DB. 
13	f the perimeters of two similar triangles ABC and DEF are 50 cm and 70 cm respectively and one side of ΔABC = 20 cm, then find the corresponding side of ΔDEF.
	LEVEL -3
14	In given figure, EB perpendicular to AC, BG perpendicular to AE and CF perpendicular toAE. Prove that : (i)Δ ABG ~ ΔDCB (ii) BC/BD = BE/ BA 
15	In the figure given, if LM || CB and LN || CD, prove that $\frac{AM}{AN} = \frac{AB}{AD}$. 

16	In the figure above, find x 
17	In the figure, AM: MC =3:4, BP:PM =3:2 and BN = 12 cm. Then find AN. 
18	In the adjoining figure, AB PQ CD, AB = x units, CD= y units and PQ= z units. Then prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$ 
19	CM and RN are respectively the medians of ΔABC and ΔPQR. If ΔABC ~ ΔPQR, then Prove that (a) ΔAMC ~ ΔPNR (b) $\frac{CM}{RN}=\frac{AB}{PQ}$
20	In triangle ABC, if AP perpendicular to BC and $AC^2 = BC^2 – AB^2$, then prove that $PA^2 = PB \times CP$. 
	SECTION C (3 MARK)
	Level 1
1.	In ΔABC, DE‖ BC such that AD = 7x - 4 cm, AE = 5x - 2 cm , DB = 3x + 4 cm and EC = 3x cm . Then find the value of x.

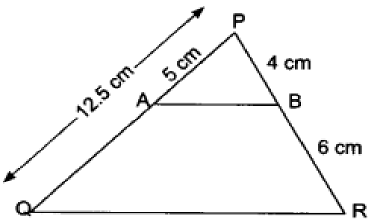
2.	In the given figure, AB ‖ DC and diagonals AC and BD intersect at O. If OA = 3x – 1 and OB = 2x + 1 , OC = 5x – 3 and OD = 6x – 5 , find the value of x. 
3.	State and prove Basic Proportionality theorem.
4.	In the Figure , DE || AC and DF || AE. Prove that $\frac{BF}{BE}=\frac{FE}{EC}$ 
5.	In Figure below, $\frac{QR}{QT}=\frac{QS}{PR}$ and ∠ 1 = ∠ 2. Show that Δ PQS ~ Δ TQR. 
	Level 2
6.	If ΔABC ~ ΔDEF, AB = 4 cm, DE = 6 cm, EF = 9 cm and FD = 12 cm, find the perimeter of ΔABC.
7.	In figure, if ∠D = ∠E and AD /AE = DB / EC, Prove that ΔBAC is an isosceles triangle 
8.	In figure, A, B, C are points on OP, OQ and OR respectively such that AB ‖ PQ and AC ‖PR. Show that BC ‖QR. 
	Level 3

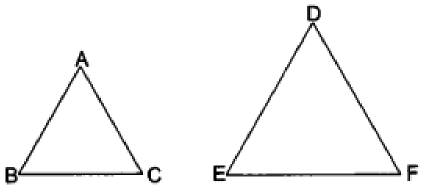
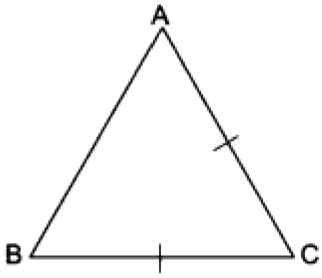
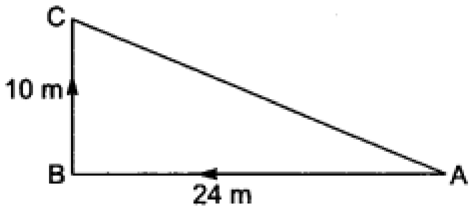
<table>
<tr><td>9.</td><td>In the given figure, BC || PQ and BC = 8 cm, PQ = 4 cm, BA = 6 5. cm AP = 2 8. cm. Find the length of CA.

</td></tr>
<tr><td>10</td><td>In the given figure, ABCD is a parallelogram. AE divides the line segment BD in the ratio 1 : 2. If BE = 1.5 cm find BC.

</td></tr>
<tr><td>11</td><td>In figure, ∠1 = ∠2 and ΔNSQ ≅ΔMTR, then prove that ΔPTS ~ ΔPRQ.

</td></tr>
<tr><td>12</td><td>In the figure, if ∠ACB= ∠ CDA, AC=8 cm and AD = 3 cm, find BD.

</td></tr>
<tr><td>13</td><td>In the figure, $l \parallel m$ and line segments AB, CD and EF are concurrent at point P. Prove that AE/BF=AC/BD=CE/FD.

</td></tr>
<tr><td>14</td><td>In the given figure below, CB || QR and CA || PR Also AQ = 12 cm, AR = 20 cm, PB = CQ = 15 cm. Calculate PC and BR.

</td></tr>
<tr><td>15</td><td>In the given figure, RQ and TP are perpendicular to PQ, also TS perpendicular to PR . Prove that ST.RQ = PS.PQ.

</td></tr>
</table>

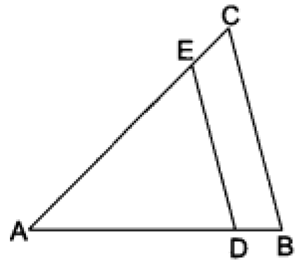
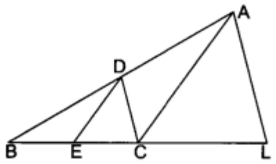
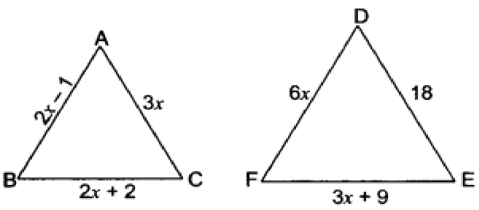
	Section –D (4 mark questions)
	Level-1
1.	Aakesh wanted to determine the height of a tree on the corner of his block. He knew that a certain fence by the tree was 4 feet tall. At 3 PM, he measured the shadow of the fence to be 2.5 feet tall. Then he measured the tree's shadow to be 11.3 feet. What is the height of the tree? 
2.	A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds. 
3.	Ramesh places a mirror on level ground to determine the height of a pole (with traffic light fired on it). He stands at a certain distance so that he can see the top of the pole reflected from the mirror. Ramesh's eye level is 1.5 m above the ground. The distance of Ramesh and the pole from the mirror are 1.8 m and 6 m respectively.  Q1. Which criterion of similarity is applicable to similar triangles? (a) SSA (b) ASA (c) SSS (d) AA Q2. What is the height of the pole? (a) 6 metres (b) 8 metres (c) 5 metres (d) 4 metres Q3. Now Ramesh move behind such that distance between pole and Ramesh is 13 meters. He place mirror between him and pole to see the reflection of light in right position. What is the distance between mirror and Ramesh ? (a) 7 metres (b) 3 metres (c) 5 metres (d) 4 metre Q4. What is the distance between mirror and pole? (a) 9 metres (b) 8 metres (c) 12 metres (d) 10 metres

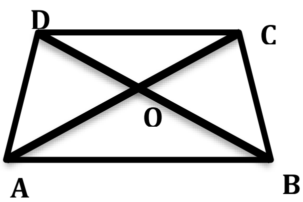
	Level-2
4.	Tania is very intelligent in maths. She always try to relate the concept of maths in daily life. One day she plans to cross a river and want to know how far it is to the other side. She takes measurements on her side of the river and make the drawing as shown below.  Q1. Which similarity criterion is used in solving the above problem ? (a) SAS similarity criterion (b) AA similarity criterion (c) SSS similarity criterion (d) None of these Q2. Consider the following statement : S1 : <ACB = <DCE; S2 : <BAC = <CDE Which of the above statement is/are correct. (a) S1 and S2 both (b) S1 (c) S2 (d) None Q3. What is the distance x across the river? (a) 96 ft (b) 48 ft (c) 24 ft (d) 16 ft Q4. What is the approximate length of AD shown in the figure? (a) 120 ft (b) 160 ft (c) 140 ft (d) 100 ft
5.	If AD and PM are medians of triangles ABC and PQR respectively where Δ ABC ~ Δ PQR, prove that AB / PQ = AD/PM
	Level 3 (4 marks)
6.	In the figure, if PQRS is a parallelogram, AB ‖ PS and PQ ‖ OC , then prove that OC ‖ SR. 
7.	In the figure, there are two points D and E on side AB of DABC such that AD = BE. If DP || BC and EQ || AC, then prove that PQ || AB. 
8.	In the given figure, AD = 3 cm, AE = 5 cm, BD = 4 cm, CE = 4 cm, CF = 2 cm, BF = 2.5 cm, then find the pair of parallel line and hence their lengths. 
9.	CD and GH are respectively the bisectors of ∠ ACB and ∠ EGF such that D and H lie on sides AB and FE of Δ ABC and Δ EFG respectively. If Δ ABC ~ Δ FEG, show that:

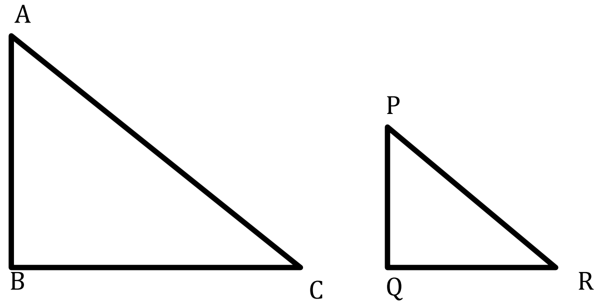
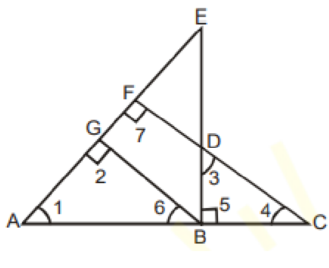
	(i) CD / GH = AC / FG (ii) Δ DCB ~ Δ HGE (iii) Δ DCA ~ Δ HGF

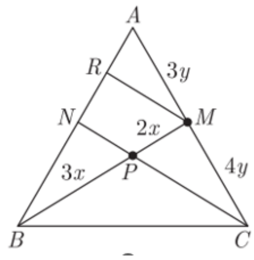
	Scoring Key
SL.NO.	ANSWERS
	SECTION A -LEVEL 1
1	Since the perimeters and two sides are proportional ∴ The third side is proportional to the corresponding third side. i.e., The two triangles will be similar by SSS criterion.
2	Yes, $\frac{PA}{AQ} = \frac{5}{12.5-5} = \frac{5}{7.5} = \frac{2}{3}$ $\frac{PB}{BR} = \frac{4}{6} = \frac{2}{3}$ Since $\frac{PA}{AQ} = \frac{PB}{BR} = \frac{2}{3}$ ∴ $AB \parallel QR$  Fig. 7.4
3	Since, ∠R = 180° – (∠P + ∠Q) = 180° – (55° + 25°) = 100° = ∠M ∠Q = ∠S = 25° (Given) ΔQPR ~ ΔSTM i.e., . ΔQPR is not similar to ΔTSM.
4	(c) ∠B=∠D
5	True
6	$(b) 3\sqrt{2} cm$ ΔABC ~ ΔADB, ΔABC ~ ΔBDC Then, ΔADB ~ ΔBDC . So , $\frac{AD}{BD} = \frac{DB}{DC} = \frac{AB}{BC}$
7	Since ΔABC ~ ΔDEF ∴ ∠A = ∠D = 47° ∠B = ∠E = 63° ∴ ∠C = 180° – (∠A + ∠B) = 180° – (47° + 63°) = 70° ∴ Given statement is true.
	LEVEL-2

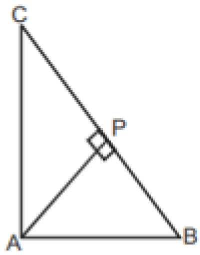
8	ΔABC ~ ΔDEF (Given) $\therefore \frac{AB}{DE} = \frac{BC}{EF}$ $\frac{AB}{2AB} = \frac{8}{EF}$ $(\because DE = 2AB)$ $\frac{1}{2} = \frac{8}{EF}$ $\therefore EF = 16$ cm  Fig. 7.6
9	$AB^2 = 2AC^2$ (Given) $AB^2 = AC^2 + AC^2$ $AB^2 = AC^2 + BC^2$ $(\because AC = BC)$ Hence AB is the hypotenuse and ΔABC is a right angle A. So, ∠C = 90° 
10	∵ The diagonals of rhombus bisect each other at 90°. ∴ In the right angle ΔBOC BO = 8 cm CO = 6 cm ∴ By Pythagoras Theorem $BC^2 = BO^2 + CO^2 = 64 + 36$ $BC^2 = 100$ BC = 10 cm
11	(b) Similar but not congruent
12	 By Pythagoras Theorem $AC^2 = AB^2 + BC^2 = (24)^2 + (10)^2$ $AC^2 = 676$ AC = 26 m ∴ The man is 26 m away from the starting point.
13	(b) 6 cm
14	Since ΔABC ~ ΔDEF. $\frac{\text{Perimeter of } \Delta DEF}{\text{Perimeter of } \Delta ABC} = \frac{DE}{AB}$ $\frac{25}{\text{Perimeter of } \Delta ABC} = \frac{6.5}{9.1}$ $\text{Perimeter of } \Delta ABC = \frac{25 \times 91}{65} = 35$ cm

15	 In ΔABC, we have DE ∥ BC, ∴ AD/DB = AE/EC [By Basic Proportionality Theorem] ⇒ x/(x−2) = (x+2)/(x−1) ⇒ x(x – 1) = (x – 2) (x + 2) ⇒ $x^2 – x = x^2 – 4$ ⇒ x = 4
	LEVEL-3
16	(c) 19cm
17	(c) 6 cm
18	$\Rightarrow \quad \frac{BD}{DA} = \frac{BE}{EC}$ (By BPT) ...(i) In ΔABL $DC \parallel AL$ $\Rightarrow \quad \frac{BD}{DA} = \frac{BC}{CL}$ (By BPT) ...(ii) From (i) and (ii) we get $\frac{BE}{EC} = \frac{BC}{CL} \Rightarrow \frac{4}{2} = \frac{6}{CL} \Rightarrow CL = 3$ cm 
19	ΔABC ~ ΔDEF (Given) therefore, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ So, $\frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$ Now, taking $\frac{2x-1}{18} = \frac{3x}{6x}$, we have $\frac{2x-1}{18} = \frac{1}{2}$  Fig. 7.26 ⇒ 4x – 2 = 18 ⇒ x = 5 ∴ AB = 2 × 5 – 1 = 9, BC = 2 × 5 + 2 = 12 CA = 3 × 5 = 15, DE = 18, EF = 3 × 5 + 9 = 24 and FD = 6 × 5 = 30 Hence, AB = 9 cm, BC = 12 cm, CA = 15 cm DE = 18 cm, EF = 24 cm, FD = 30 cm
20	(a) ΔFDE ~ ΔCAB
	SECTION B (2 MARKS)
	Level 1

1	$\frac{AB}{ML}=\frac{4.4}{11}=0.4$ $\frac{AC}{LN}=\frac{3.6}{9}=0.4$ $\frac{BC}{MN}=\frac{4}{10}=0.4$ $\Delta ABC \sim \Delta LMN$ (SSS) $\angle M=\angle B=50^0$.
2	Since $\angle APQ=\angle ABC$, $\angle AQP=\angle ACB$ Then, $\Delta APQ \sim \Delta ABC$ (AA) $\frac{AP}{AB}=\frac{PQ}{BC}=\frac{AQ}{AC}$ $\frac{3}{9}=\frac{AQ}{7.5}$ $AQ=\frac{3\times7.5}{9}=2.5\ cm$
3	$\frac{x+3}{3x+19}=\frac{x}{3x+4}$ $(x+3)(3x+4)=x(3x+19)$ x=2
4	$\frac{AX}{AB}=\frac{1}{4}$ $AX=1k, AB=4k$ $BX=AB-AX=4k-1k=3k$ $\frac{AX}{XB}=\frac{1k}{3k}=\frac{1}{3}$ $\frac{AY}{YC}=\frac{2}{6}=\frac{1}{3}$ $XC\ parallel\ to\ BC$
5	$\frac{x}{x+1}=\frac{x+3}{x+5}$ $x=3$
6	5 : 7
7	$\angle OAB=\angle OCD$, $\angle OBA=\angle ODC$, alternate interior angles. $\Delta OCD \sim \Delta OAB$ $\frac{OC}{OA}=\frac{OD}{OB}=\frac{CD}{AB}$ 
	LEVEL 2

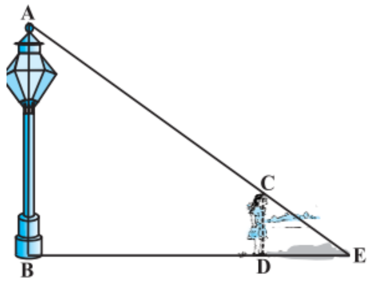
8	Let AB be the pole and PQ be the height of the woman. ∠ABC=∠PQR=90^0, ∠ACB=∠PRS ΔABC ~ ΔPQR (AA)  $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ $\frac{6}{1.5} = \frac{BC}{3}$ Length of shadow of the pole= BC=12 m.
9	10 m
10	CD=2 cm, PD= 5 cm.
11	$\frac{PB}{PD} = \frac{5}{10} = \frac{1}{2}, \frac{PC}{PE} = \frac{6}{12} = \frac{1}{2},$ ∠BPC=∠DPE(vertically opposite angles) ΔPBC ~ ΔPDE Using SAS similarity criteria.
12	$\frac{DA}{DE} = \frac{DB}{DW}$ $\frac{4}{12} = \frac{DB}{24}$ $DB\ = 8\ cm$
13	$\frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)} = \frac{AB}{DE}$, since$\Delta ABC \sim \Delta DEF$ Let AB= 20 cm $\frac{50}{70} = \frac{20}{DE}$ DE=28 cm
	LEVEL-3
14	∠2=∠5, ∠6=∠4 $\Delta ABG \sim \Delta DCB$ (AA) ∠1=∠3 ∠ABE=∠5 $\Delta ABE \sim \Delta DBC$ (AA) $\frac{BC}{BD} = \frac{BE}{BA}$ 

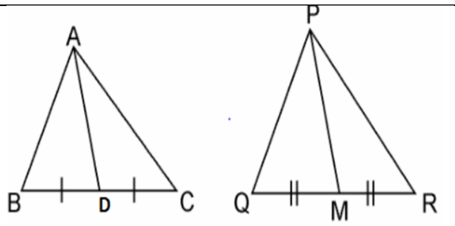
15	$\frac{AM}{MB} = \frac{AL}{LC}$ – – – – – – (1) $\frac{AL}{LC} = \frac{AN}{ND}$ – – – – – – (2) $\frac{MB}{AM} + 1 = \frac{ND}{AN} + 1$ $\frac{AB}{AM} = \frac{AD}{AN}$
16	ΔKNP ~ ΔKML Using AA similarity $\frac{KN}{KM} = \frac{PN}{LM}$. Then, $\frac{c}{b+c} = \frac{x}{a}$. That is, $x = \frac{ac}{b+c}$
17	Draw MR parallel to CN which meets AB at the point R.  $\frac{BN}{NR} = \frac{BP}{PM}$ Since PN || MR (BPT) $\frac{12}{NR} = \frac{3}{2}$ Then, NR=8 cm. $\frac{AR}{RN} = \frac{AM}{MC}$ Since RM || NC $\frac{AR}{8} = \frac{3}{4}$ Then, AR= 6 cm AN=AR+RN=6+8=14 cm.
18	In ΔABD, PQ|| AB . Then, $\frac{PQ}{AB} = \frac{DQ}{BD}$ ie, $\frac{z}{x} = \frac{DQ}{BD}$ ....(i) In ΔBCD, PQ||CD . Then, $\frac{PQ}{CD} = \frac{BQ}{BD}$ ie, $\frac{z}{y} = \frac{BQ}{BD}$ .... (ii) Adding (i) and (ii), $\frac{z}{x} + \frac{z}{y} = \frac{DQ}{BD} + \frac{BQ}{BD} = \frac{DQ+BQ}{BD} = \frac{BD}{BD} = 1$ Then , $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$
19	Since ΔABC ~ ΔPQR, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ Also, ∠A=∠P Since ∠A=∠P, $\frac{AB/2}{PQ/2} = \frac{AM}{PN} = \frac{AC}{PR}$ implies (a) ΔAMC ~ ΔPNR (SAS) (b) $\frac{CM}{RN} = \frac{AC}{PR} = \frac{AB}{PQ}$

20	$AC^2=BC^2 - AB^2$ $AC^2+AB^2=BC^2$. $\angle BAC=90^0$. $\Delta APB \sim \Delta CPA$ (Converse of BPT) $\frac{AP}{CP}=\frac{PB}{PA}$ $PA^2=PB.CP$ 
	Section C
	Level 1
1.	$\frac{7x-4}{3x+4}=\frac{5x-2}{3x}$ $3x^2-13x+4=0$ x=4,1/3 If x=1/3, 7x-4=-5/3<0, not possible. Therefore, x=4
2.	X=2
3.	Statement and proof of the theorem.
4.	In ΔABC, given as, DE || AC Then, BD/DA = BE/EC …………**(i) (BPT)** In ΔBAE, given as, DF || AE Then, BD/DA = BF/FE …………**(ii) (BPT)** From equation **(i)** and **(ii)**, we get BE/EC = BF/FE , Then BF/BE= FE/EC.
5.	In ΔPQR, ∠PQR = ∠PRQ ∴ PQ = PR ……………………….**(i)** QR/QT = QS/PR Using equation **(i)**, we get QR/QT = QS/PQ**……….(ii)** In ΔPQS and ΔTQR, by equation (ii), QR/QS = QT/QP , ∠PQS = ∠TQR ∴ ΔPQS ~ ΔTQR [By SAS similarity criterion]
	Level 2
6.	Since ΔABC ~ ΔDEF, $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=\frac{Perimeter(\Delta ABC)}{Perimeter(\Delta DEF)}$ $\frac{4}{6}=\frac{BC}{9}=\frac{AC}{12}=\frac{Perimeter(\Delta ABC)}{27}$ Perimeter(ΔABC)= 18 cm.

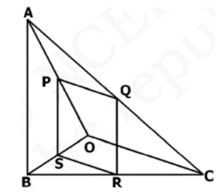
7.	$\frac{AD}{DB}=\frac{AE}{EC}$ $\frac{DB}{AD}+1=\frac{EC}{AE}+1$ $\frac{AB}{AD}=\frac{AC}{AE}$ ……….(i) Since ∠ADE=∠AED, AD=AE Then from(i), AB=AC and ΔABC is isosceles.
8.	Since AB || PQ, $\frac{OA}{AP}=\frac{OB}{BQ}$ ……………(i) (BPT) Since AC||PR, $\frac{OA}{AP}=\frac{OC}{CR}$ ……………(ii) (BPT) From (i) and (ii) $\frac{OB}{BQ}=\frac{OC}{CR}$ Then, BC || QR.
	Level 3
9.	∠ACB=∠APQ, ∠ABC=∠AQP, Then ΔABC ~ ΔAQP (AA) $\frac{AB}{AQ}=\frac{BC}{QP}=\frac{AC}{AP}$ $\frac{6.5}{AQ}=\frac{8}{4}=\frac{AC}{2.8}$ $AC=\frac{8\times 2.8}{4}=5.6\ cm$
10.	OB/OD = BE/AD ½ = 1.5/AD AD = 3 cm. As AD = BC, BC = 3 cm
11.	Given, △NSQ ≊ △MTR Then,by CPCT, ∠NQS = ∠MRT ie, ∠PRQ = ∠PQR ------------- (1) In △PST, ∠P + ∠1 + ∠2 = 180° ∠P + 2∠1 = 180° -------------------- (2) In △PQR, ∠P + ∠PQR + ∠PRQ = 180° ∠P + 2∠PQR = 180° -------------------- (3) Equating (2) and (3), ∠P + 2∠1 = ∠P + 2∠PQR ∠PQR = ∠1 So, ∠PST = ∠PQR --------- (4) In △PTS and △PRQ, ∠P = ∠P = common angle. From (4) ∠PST = ∠PQR

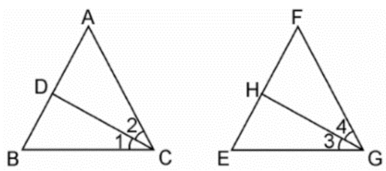
	Therefore, △PTS ~ △PRQ (AA)
12.	BD = 55/3 cm
13.	∠EAP=∠FBP, ∠APE=∠FPB, Then ΔAEP ~ ΔBFP (AA) So, $\frac{AE}{BF} = \frac{EP}{FP} = \frac{AP}{BP}$ ……………(i) ∠ECP=∠FDP, ∠CPE=∠FPD, Then ΔCEP ~ ΔDFP (AA) $\frac{EP}{FP} = \frac{CE}{DF}$ …………(ii) ∠ACP=∠BDP, ∠CAP=∠PBD, Then ΔACP ~ ΔBDP (AA) $\frac{AC}{BD} = \frac{CP}{DP} = \frac{AP}{BP}$ …………..(iii) From (i), (ii), (iii) , $\frac{AE}{BF} = \frac{CE}{DF} = \frac{AC}{BD}$
14.	$\frac{PC}{CQ} = \frac{RA}{AQ}$ (BPT) $\frac{PC}{15} = \frac{20}{12}$ Then, $PC = \frac{15\times20}{12} = 25\ cm$ In △PQR, CB || QR $\frac{PC}{CQ} = \frac{PB}{BR}$ (BPT) $\frac{25}{15} = \frac{15}{BR}$ Then, $BR = \frac{15\times15}{25} = 9\ cm$
15.	∠1+∠2+∠4=180^0. ∠1=90^0 - ∠2 ………….(i) Since TP perpendicular to PQ, ∠TPQ= 90^0. ∠2+∠3=90^0. ∠3=90^0 = ∠2 …………(ii) From (i) and (ii), ∠1=∠3 In ΔRQP and ΔPST, ∠1=∠3 , ∠4=∠5 ΔRQP ~ ΔPST (AA) $\frac{ST}{QP} = \frac{PS}{RQ}$ Then, ST.RQ= PS. PQ
	Section – D (4 mark questions)
	Level-1
1.	Height of the tree is 18 feet

<table>
<tr><td>2.</td><td>Let AB denote the lamp-post and CD the girl after walking for 4 seconds away from the lamp-post. DE is the shadow of the girl. Let DE be x metres.


Now, BD = 1.2 m × 4 = 4.8 m.
Note that in Δ ABE and Δ CDE, $\angle B = \angle D$ (Each is of 90° because lamp-post as well as the girl are standing vertical to the ground) and $\angle E = \angle E$ (Same angle) So, $\Delta ABE \sim \Delta CDE$ (AA similarity criterion)
Therefore, BE / DE = AB/ CD i.e., 4.8 + x/ x = 3.6 /0.9 (90 cm = 90/ 100 m = 0.9 m) i.e., 4.8 + x =4x i.e., 3x = 4.8 i.e., x = 1.6 So, the shadow of the girl after walking for 4 seconds is 1.6 m long.</td></tr>
<tr><td>3.</td><td>Q1. (d) AA
Q2. (c) 5 metres
Q3. (b) 3 metres
Q4. (d) 10 metres</td></tr>
<tr><td></td><td>Level - 2</td></tr>
<tr><td>4.</td><td>Q1. b) AA Similarity
Q2. a) S1 and S2 both
Q3. b) 48 ft
Q4. c) 140 ft</td></tr>
<tr><td>5.</td><td>We know that the corresponding sides of similar triangles are in proportion.


∴AB/PQ = AC/PR =BC/QR…………(i)
Also, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ ….(ii)
Since AD and PM are medians, they will divide their opposite sides.
∴ BD = BC/2 and QM = QR/2 ……………..………….(iii)
From equations (i) and (iii), we get
AB/PQ = BD/QM ……………………….(iv)
In ΔABD and ΔPQM, From equation (ii), we have $\angle B = \angle Q$
From equation (iv), we have, AB/PQ = BD/QM
∴ ΔABD ~ ΔPQM (SAS similarity criterion)
⇒AB/PQ = BD/QM = AD/PM</td></tr>
<tr><td></td><td>Level-3</td></tr>
</table>

<table>
<tr><td>6.</td><td>


$\frac{OP}{PA} = \frac{OS}{SB}$, Since AB||PS
$\frac{AP}{OP} = \frac{AQ}{QC}$, Since PQ|| OC
$\frac{OP}{AP} = \frac{QC}{AQ}$
$\frac{OS}{SB} = \frac{QC}{AQ}$ ……….(i)
Since PQRS is a parallelogram, QR||AB
Then, $\frac{CQ}{AQ} = \frac{CR}{BR}$ ………..(ii)
From (i) and (ii), $\frac{OS}{SB} = \frac{CR}{BR}$ Then, SR||OC
</td></tr>
<tr><td>7.</td><td>
In ΔABC , DP || BC, $\frac{AD}{DB} = \frac{AP}{PC}$ ……(i) (BPT)
EQ || AC, $\frac{BE}{EA} = \frac{BQ}{QC}$ (BPT)
$\frac{AD}{DB} = \frac{BQ}{QC}$ ……(ii) Since AD=BE, EA=DB
From (i) and (ii), $\frac{AP}{PC} = \frac{BQ}{QC}$ Then, PQ || AB (Converse of BPT)
</td></tr>
<tr><td>8.</td><td>
$\frac{EC}{EA} = \frac{4}{5}$ and $\frac{CF}{FB} = \frac{2}{2.5} = \frac{4}{5}$
$\frac{EC}{EA} = \frac{CF}{FB}$
In ΔABC , EF || AB (Converse of BPT)
$\frac{CE}{CA} = \frac{4}{4+5} = \frac{4}{9}$
$\frac{CF}{CB} = \frac{2}{2+2.5} = \frac{2}{4.5} = \frac{4}{9}$
$\frac{EC}{CA} = \frac{CF}{CB}$, ∠ECF=∠ACB
ΔCFE ~ ΔCBA (SAS)
$\frac{EF}{AB} = \frac{CE}{CA}$
$\frac{EF}{7} = \frac{4}{9}$
$$EF = \frac{28}{9}\ cm, AB = 7\ cm.$$
</td></tr>
<tr><td>9.</td><td>
From the given condition, ΔABC ~ ΔFEG.
∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE
Since, ∠ACB = ∠FGE
∴ ∠ACD = ∠FGH (Angle bisector)
And, ∠DCB = ∠HGE (Angle bisector)


</td></tr>
</table>

	In ΔACD and ΔFGH, $\angle A = \angle F$, $\angle ACD = \angle FGH$ $\therefore \Delta ACD \sim \Delta FGH$ (AA similarity criterion) $\Rightarrow CD/GH = AC/FG$ (ii) In ΔDCB and ΔHGE, $\angle DCB = \angle HGE$ (Already proved), $\angle B = \angle E$ (Already proved) $\therefore \Delta DCB \sim \Delta HGE$ (AA similarity criterion) (iii) In ΔDCA and ΔHGF, $\angle ACD = \angle FGH$ (Already proved), $\angle A = \angle F$ (Already proved) $\therefore \Delta DCA \sim \Delta HGF$ (AA similarity criterion

****************** ******************** ************** ************

UNIT 5 - TRIGONOMETRY

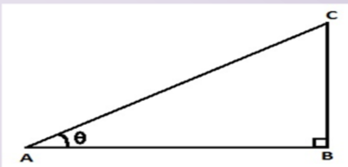
INTRODUCTION TO TRIGONOMETRY

Definition

- It is the branch of mathematics which deals with the study of relationships between the sides and angles of a triangle.

Trigonometric Ratios

- Trigonometric ratio of an acute angle of a right angled triangle :-
  Let ABC be right angled at B and <CAB = θ be an acute angle, then



- $\sin\theta = \frac{\text{opposite side of }\theta}{\text{hypotenuse}} = \frac{BC}{AC}$ and $\text{cosec}\theta = \frac{\text{hypotenuse}}{\text{opposite side of }\theta} = \frac{AC}{BC}$
- $\cos\theta = \frac{\text{adjacent side of }\theta}{\text{hypotenuse}} = \frac{AB}{AC}$ and $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side of }\theta} = \frac{AC}{AB}$
- $\tan\theta = \frac{\text{opposite side of }\theta}{\text{adjacent side of }\theta} = \frac{BC}{AB}$ and $\cot\theta = \frac{\text{adjacent side of }\theta}{\text{opposite side of }\theta} = \frac{AB}{BC}$

Relation between Trigonometric Ratios

Reciprocal Relation :-

- $\text{Sin}\,\theta = \frac{1}{\text{cosec}\theta} \Rightarrow \text{cosec}\theta = \frac{1}{\text{Sin}\,\theta} \Rightarrow \text{Sin}\,\theta\,.\,\text{cosec}\theta = 1$
- $\cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta} \Rightarrow \cos\theta\,.\,\sec\theta = 1$
- $\tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta} \Rightarrow \tan\theta\,.\,\cot\theta = 1$

MULTIPLE CHOICE QUESTIONS

SECTION A

1. If x= a sinϑ and y= a cosϑ, then $x^2 + y^2 =$
   a) a
   b) a^2
   c) 1
   d) 1/a
2. If sin ϑ =x and secϑ =y, then tan ϑ is equal to
   a) xy
   b) $\frac{x}{y}$
   c) $\frac{y}{x}$
   d) $\frac{1}{xy}$
3. If Cos A = $\frac{3}{5}$,find the value of 9 +9 $\tan^2$ A
   a) 9
   b) 16
   c) 25
   d) 3/5
4. If $0 \leq A,B \leq 90^0$ such that Sin A =$\frac{1}{2}$ and Cos B = $\frac{1}{2}$, A + B =
   a) 0°
   b) 60°
   c) 90°
   d) 30°
5. In a Δ ABC, right angled at B the value of Sin (A +C) is
   a) 0
   b) 1
   c) $\frac{1}{2}$
   d) $\frac{\sqrt{3}}{2}$
6. In Δ ABC, right angled at B,sin A = $\frac{7}{25}$,then the value of cos C is
   a) $\frac{24}{7}$
   b) $\frac{7}{24}$
   c) $\frac{24}{25}$
   d) $\frac{7}{25}$
7. If $\tan\theta = \sqrt{3}$, then the value of $\sec^2\theta + \operatorname{cosec}^2\theta$ is
   a) 1
   b) $\frac{40}{9}$
   c) $\frac{38}{9}$
   d) $5\frac{1}{3}$
8. If the length of the shadow of a man is equal to the height of the man, then the angle of elevation is
   a) 90°
   b) 60°
   c) 45°
   d) 30°
9. The value of $\sin^2 30^0 + \cos^2 45^0 + \cos^2 30^0$ is
   a) $\frac{1}{2}$ b)$\frac{2}{3}$ c)$\frac{3}{2}$ d)1

10. The value of (1-cos θ) (1+cos θ) cosec θ is

a) $cot\theta$
c) $cosec\theta$
b) $sin\theta$
d) $cos\theta$

VERY SHORT ANSWER QUESTIONS (1mark each)

1) Find the value of (Sin 30 +Cos 30) – (Sin60 + Cos 60)

2) If Sinϑ - Cos ϑ = 0, find the value of ϑ

3) ΔABC is right angled at C, and AC = √3 BC, prove that <ABC = 60^0

4) If 2 Sin 3x= √3, then find the value of x

5) If Sin A + Sin^2A=1 then find Cos^2 A +Cos^4 A

6) If tan (A – B) = $\frac{1}{\sqrt{3}}$ and tan (A + B) = $\sqrt{3}$,find the value of A and B

7) Evaluate $\frac{1-tan^2 45^0}{1+tan^2 45^0}$

8) If cos α = $\frac{1}{2}$ and tan β = $\frac{1}{\sqrt{3}}$, find sin ($\alpha + \beta$) where α and β are both acute angles.

ASSERTION AND REASONING

9) Statement-1 (Assertion): For any acute angle ϑ, the value of sin ϑ cannot be greater than1 Statement-2 (Reason): Hypotenuse is the longest side in any right-angled triangle.

10) Statement-1 (Assertion): For $0 \le \vartheta \le 90^o$, $sec\ x\ +\ cos\ x \ge 2$

11) Statement-2 (Reason): For any x > 0, $x\ +\ \frac{1}{x} \ge 2$

SHORT ANSWER TYPE QUESTIONS (2marks questions)
SECTION B

Q1. If tanβ=$\frac{24}{7}$, then the value of sinβ +cosβ is

Q2. If tan 3x = sin45^0cos$45^{\ 0}$ + sin30^0 then value of x is

Q3. In triangle ABC, right angled triangled at B, AB =5cm and <ACB=30^0,then the length of the side AC is

Q4. Given that the sin$\beta = \sqrt{3/2}$ and the cosα= 0,then the value of β - α is

Q5. In a triangle ABC, right angled at C if <A=30^0, AB=40 units find BC

Q6. If Θ is an acute angle and sin Θ = cos Θ ,then find the value of 3 tan^2 Θ + 2 sin^2Θ – .

Q7. If in a triangle ABC right angled at B , AB =6 units and BC=8 units then the value of the $sinA\ cosC + cosAsinC$

Q8. If sin (A – B) = $\frac{1}{2}$ and cos (A + B) = $\frac{1}{2}$,then find the value of A and B If x =2cos^2 α and y= 2sin^{2}α +1 then the value of x + y

Q9. If x =2 cos^2 α and y= 2 sin^{2}α +1 then the value of x + y

Q10. If cot Θ + $\frac{1}{\cot\Theta}$ = 2, then find the value of $cot^2\,\theta + \frac{1}{cot^2\,\theta}$

Q11. In triangle ABC right angled at B if ∠ A = ∠ C then find the value of sin A sin B + cos A cos B

Q12. Triangle PQR is right angled at Q, if PQ = 5 cm and RQ = 10 cm then find the value of sin P × cos P

Q13. If 2 sin ^{2} Θ – cos ^{2} Θ = 2, then find the value of Θ

Q14. The value of (sinα + cos α)^2 + (cosα – sin α)^2

Q15. In right triangle ABC, right angled at B, ∠ ACB= ∅, AB =2 cm and BC = 1 cm then the value of sin^2 ∅ + tan^2∅

Q16. If 4tan α = 3 find the value of $\frac{5sin a-3\cos a}{5\sin a+2cosa}$

Q17. The value of α and β if sin (α +2 β) = √3/2 and cos (α + 4β) = 0

Q18. If sin Θ + cos Θ = $\sqrt{3}$,then find the value of tan Θ + cot Θ

Q19. If 8 tan x=15 then find sin x – cos x

Q20. In triangle PQR right angled at Q, PQ= 3 cm and PR = 6cm Determine $\angle PRQ$

SHORT ANSWER TYPE QUESTIONS (3marks questions)
SECTION C

1. If b cos θ= a, then Prove that Cosecθ + Cotθ = $\sqrt{\frac{b+a}{b-a}}$

2. If $sin\,\theta = \frac{3}{5}$ evaluate $\frac{cosec\,\theta - cot\,\theta}{2\,cot\,\theta}$

3. In Δ ABC, right angled at B. If AC+BC = 25cm and AB=5cm. Determine the value of sin A, cos A and tan A.

4. Evaluate:$\frac{2\,cos^2\,60+3\,sec^2\,30-2\,tan^2\,45}{sin^2\,30+cos^2\,45}$

5. If sin(A+2B) =$\frac{\sqrt{3}}{2}$ and cos (A+4B) =0. A>B and (A+4B)≤90. Then find the value of A and B
6. Prove that $\frac{sin\,\theta}{1+cos\,\theta} + \frac{1+cos\,\theta}{sin\,\theta} = 2\,cosec\,\theta$
7. If $7\,sin^2\,\alpha + 3\,cos^2\,\alpha = 4$, then show that $tan\,\alpha = \frac{1}{\sqrt{3}}$
8. Prove that$\frac{cos^3\,\theta+sin^3\,\theta}{cos\,\theta+sin\,\theta} + \frac{cos^3\,\theta-sin^3\,\theta}{cos\,\theta-sin\,\theta} = 2$
9. Find the acute angle θ satisfying the equation: $sec^2\,\theta + tan^2\,\theta = 3$
10. Prove that $sec^4\,\theta - sec^2\,\theta = tan^4\,\theta + tan^2\,\theta$
11. If √3tan$\theta = 3\,sin\,\theta,\ then\ find\ the\ value\ of\ \ sin^2\,\theta - cos^2\,\theta$
12. If 4sin θ =3cos θ ,find the value of: $\frac{12\,sin\,\theta-7\,cos\,\theta}{8\,sin\,\theta+3\,cos\,\theta}$
13. In **Δ**ABC, right angled at B, if tan A=$\frac{1}{\sqrt{3}}$, find the value of sinAcosC+cosAsinC
14. If $\sqrt{5}\,tan\,\theta = 1$ find the value of $\frac{cosec^2\,\theta-sec^2\,\theta}{cosec^2\,\theta+sec^2\,\theta}$.
15. If sin2x= sin30° cos60°+sin60° cos30°, then find the value of x.

<u>LONG ANSWER TYPE QUESTIONS</u>(4 marks each)

1. Prove that: $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4(\sin^6\theta + \cos^6\theta) = 13$.
2. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, then find the value of $\cos^6\theta - 4\cos 4\theta + 8\cos^2\theta$.
3. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$
4. Prove that $\sec^2\theta - \frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta} = 1$
5. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$, prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.
6. RPQ is a right-angled triangle at Q. If PQ = 5 cm and RQ = 10 cm, find
   (i) $\sin^2 P$
   (ii) $\cos^2 R$ and tan R
   (iii) sin P x cos P
   (iv)$\sin^2 P - \cos^2 P$

7. If θ = 60°, show that

(i) $\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

(ii) $\tan\theta = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$

8. If sin (A+B) =1 and tan (A+B) = $\frac{1}{\sqrt{3}}$. Find the value of

(i) tan A + cot B

(ii) sec A + cosec B

9. Evaluate: 4 ($\sin^4 30° + \cos^4 60°$) – 3 ($\cos^4 45°$ - $\sin^4 90°$)

10. Prove that : $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta}$ = secθ cosecθ – 2 sinθ cosθ

CASE STUDY BASED QUESTIONS

CASE STUDY QUESTION 1

Doing swing ball in a cricket match turns the ball and can put the batsman in danger. Our two famous bowlers Ashwin and Akash, throws the ball at an angle of A and B respectively. The relation between A and B are such that $sin(A-B) = \frac{1}{2}$ and $cos(A+B) = 0), 0^\circ < A + B \leq 90^\circ$, $A > B$

1. What is the measure of $\angle A$?

   (a) 30° (b) 45° (c) 60° (d) 90°

2. What is the measure of $\angle B$?

   (a) 30° (b) 45° (c) 60° (d) 90°

3. Now on the bases of value of A and B derived find $cosec\ (A - B)$

   (a) 0 (b) 2 (c) $\sqrt{2}$ (d) $\frac{2}{\sqrt{3}}$

4. What is the value of $sec\ sec\ B$?

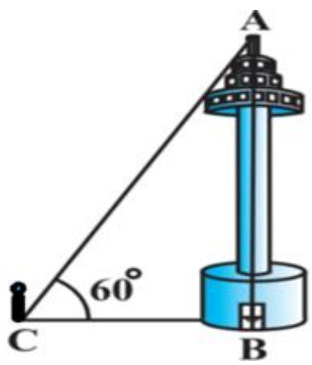
   (a) 0 (b) 1 (c) ∝ (d) $\frac{2}{\sqrt{3}}$

5. If $sin\theta = \frac{a}{b}$, then $cos\theta$ is equal to:

(a) $\frac{b}{\sqrt{b^2-a^2}}$ (b) $\frac{b}{a}$ (c) $\frac{a}{\sqrt{b^2-a^2}}$ (d) $\frac{\sqrt{b^2-a^2}}{b}$

CASE STUDY QUESTION 2

In the month of November, Akshay notices a tower built near his colony's playground. He sees that it is being held by a wire, attached to the top of the tower. The wire makes an angle of $60^\circ$with the ground. Using these Information find the answers to the following questions



1. What is the measure of $\angle CAB$?

(a) 15° (b)25° (c) 30° (d) 45°

2. What is the $sin\, sin\, ratio$ of $\angle CAB$?

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

3. What is the value of $cos\, cos\ \angle ACB$?

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

4. $sin^2\angle CAB + cos^2\angle CAB =$

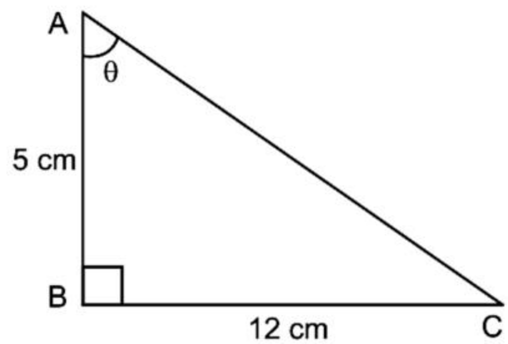
(a) 0 (b) 1 (c) 2 (d) 3

5. What is the value of $tan\, tan\, 90$?

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) not defined

CASE STUDY 3

Mohan, a class X student is a big foodie. Once his mother has made a sandwich for him. A thought has come into his mind by seeing a piece of sandwich. He thought if he increases the base length and height, he can eat a bigger piece of sandwich.



Answer the following questions accordingly:

1. If the length of the base is 12 cm and the height is 5 cm then the length of the hypotenuse of that sandwich is:

   (a) 17 cm (b) 7 cm (c) 169 cm (d) 13

2. 2.What will be the value of cosine of the angle between hypotenuse and the height of sandwich?

   (a) $\frac{5}{13}$cm (b) $\frac{12}{13}$ cm (c) $\frac{13}{5}$cm (d) $\frac{13}{12}$ cm

3. If he increases the base length to 15 cm and the hypotenuse to 17 cm, then the height of the sandwich is :

   (a) 7 cm (b) 8 cm (c) 32 cm (d) none of these

4. If the value of tan θ is √3, then sin- equals to:

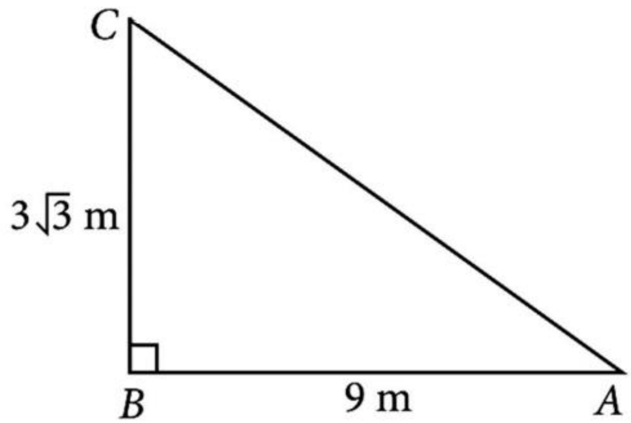
   (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) 1

5. The value of tan 45° + cot 45°

   (a) 1 (b) 2 (c) 3 (d) 4

CASE STUDY 4

Three friends Ashwin, Bhagath & Amal are playing hide and seek in a park. Ashwin, Bhagath hide in the shrubs and Amal have to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right-angled triangle, such that AB =9 m, BC= $3\sqrt{3}$ m and $\angle B$ =90°.Now answer the following questions



On the basis of above answer the following questions

1. The measure of $\angle A$ is

   (a) 30° (b) 45° (c) 60° (d) 90°

2. The measure of $\angle C$ is

   (a) 30° (b) 45° (c) 60° (d) 90°

3. The length of AC is

   (a) $8\sqrt{2}$ (b) $6\sqrt{3}$ (c) $4\sqrt{2}$ (d) $2\sqrt{3}$

4. $2A =$

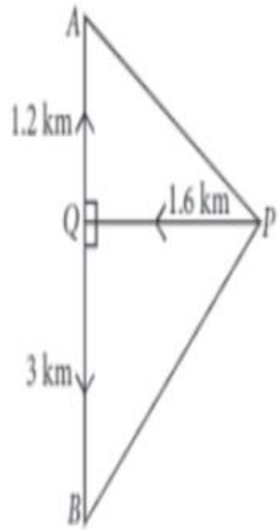
   (a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

5. $sin\, sin \left(\frac{C}{2}\right)$ =

   (a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

CASE STUDY 5

Two aeroplanes leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aeroplanes are 400 km/hr and 500 km/hr respectively. Considering PQ as runway and A and B are any points in the path followed by two planes



On the basis of above answer the following questions

1. Find $tan\, tan\, \theta$, if $\angle APQ = \theta$

   (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

2. Find the value of $cot\, cot\, B$

   (a) $\frac{3}{4}$ (b) $\frac{15}{4}$ (c) $\frac{3}{8}$ (b) $\frac{15}{8}$

3. Find the value of $tan\, tan\, A$

   (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

4. Find the value of sec A

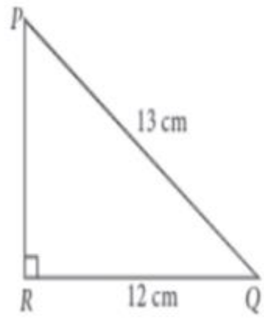
   (a) 0 (b) $\frac{5}{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

5. Find cosec B

   (a) $\frac{17}{8}$ (b) $\frac{8}{17}$ (c) $\frac{12}{5}$ (d) $\frac{5}{12}$

CASE STUDY 6

Raji a student of class10 has to made a project. She decides to make a bird house which is triangular in shape. She uses cardboard to make the bird house as shown in the figure. Considering the front side of bird house as a right-angled triangle PQR, right angled at R, answer the following questions



On the basis of above answer the following questions

1. If $\angle PQR = \theta$, the cos θ =

   (a) $\frac{12}{13}$ (b) $\frac{13}{12}$ (c) $\frac{12}{5}$ (d) $\frac{5}{12}$

2. Find the value of sec θ

   (a) $\frac{12}{13}$ (b) $\frac{13}{12}$ (c) $\frac{12}{5}$ (d) $\frac{5}{12}$

3. Find the value of $\frac{tantan\,\theta}{1+tan^2\theta}$

   (a) $\frac{60}{169}$ (b) $\frac{169}{60}$ (c) $\frac{12}{5}$ (d) $\frac{5}{12}$

4. The value of $cot^2\theta - cosec^2\theta$

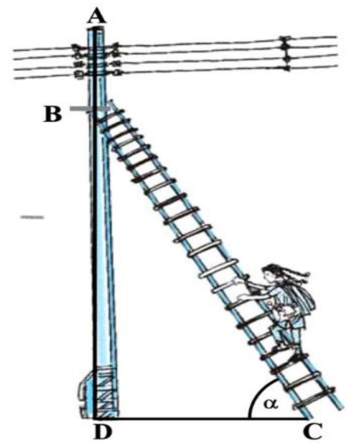
   (a) 0 (b) 1 (c) 2 (d) -1

5. The value of $sin^2\theta + cos^2\theta$

   (a) 0 (b) 1 (c) 2 (d) -1

CASE STUDY 7

Raj is an electrician in a village. One day power was not there in entire village and villagers called Raj to repair the fault. After thorough inspection he found an electric fault in one of the electric pole of height 5 m and he has to repair it. He needs to reach a point 1.3m below the top of the pole to undertake the repair work



On the basis of above, answer the following question

1. When the ladder is inclined at an angle of α such that $\sqrt{3}\tan\alpha + 2 = 5$ to the horizontal, find the angle **α**.

   (a) 30° (b) 45° (c) 60° (d) 90°

2. How far from the foot of the pole should he place the foot of the ladder? (Use 3 = 1.73)

   (a) 2.89 m (b) 2.14 m (c) 3 m (d) none of these

3. In the above situation, find the value of $sin \propto cos\frac{\propto}{2} - cos \propto sin\frac{\propto}{2}$

   (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) none of these

4. In the above situation if BD = 3 cm and BC = 6 cm. Find α

   (a) 45° (b) 30° (c) 60° (d) none of these

5. If 15 cot α = 8. The value of sin α is

   (a) $\frac{17}{15}$ (b) $\frac{15}{17}$ (c) $\frac{15}{8}$ (d) $\frac{8}{17}$

ANSWER KEY

MULTIPLE CHOICE QUESTIONS (1mark each)			
Question	Answer	Question	Answer
1	c	6	a
2	a	7	d

3	c	8	c
4	c	9	c
5	b	10	b

11. (Sin 30 +Cos 30) – (Sin 60 +Cos 60) = $\frac{1}{2} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} - \frac{1}{2}$ = 0

12. Sinθ -Cosθ =0,

Therefore Sinθ= Cos θ,θ =45

13. Tan B = $\frac{AC}{BC} = \frac{\sqrt{3}BC}{BC}$ =√3

Tan 60 = √3, therefore <B =60

14. 2 Sin 3x=√3, Sin 3x ==$\frac{\sqrt{3}}{2}$

Sin 60 = $\frac{\sqrt{3}}{2}$, 3x=60, therefore x=$\frac{60}{3}$ =20

15. Sin A = 1-Sin^2 A= Cos^2 A

Sin^2 A =Cos^4 A, ThereforeCos^2 A + Cos^4 A = Sin A + Sin^2 A = 1

16. tan (A – B) = $\frac{1}{\sqrt{3}}$, A – B = 30 ^{0} ………(1)

tan (A + B) = $\sqrt{3}$, A + B = 60 ^{0} ………(2)

Solving A = 45 ^{0} and B = 15 ^{0}

17. $\frac{1-tan^2 45^0}{1+tan^2 45^0} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

18. cos $\alpha = \frac{1}{2}$ ……… $\alpha = 60^0$

tan $\beta = \frac{1}{\sqrt{3}}$ ………. $\beta = 30^0$

Sin ($\alpha + \beta$) = Sin (60 ^{0} + 30 ^{0}) = Sin 90 ^{0} = 1

19. Both statements are true and statement-2 is the correct explanation for statement-1, because

sinθ = (Perpendicular / Hypotenuse) < 1

20. For any x > 0, we find that

$[\sqrt{x} - (1/\sqrt{x})]^2 \geq 0 \Rightarrow x + (1/x) - 2 \geq 0 \Rightarrow x + (1/2) \geq 2$

So, statement-2 is true. Since, sec x = (1 / cos x). Therefore,

$$\sec x + \cos x = \cos x + (1 / \cos x) \geq 2$$

So, statement-1 is also true and statement-2 is the correct explanation for statement-1.

Hence, option (a) is correct.

VERY SHORT ANSWER TYPE QUESTIONS (2 marks each)							
Que	Ans	Que	Ans	Qu	Ans	Que	Ans
1	31/25	6	3	11	$1/\sqrt{2}$	16	3/23
2	15°	7	1	12	2/5	17	α =30°&β=15°
3	10	8	A = 45°& B=30°	13	90°	18	1
4	30°	9	3	14	2	19	7/17
5	20 units	10	2	15	24/5	20	30°

SHORT ANSWER TYPE QUESTIONS (3 marks each)			
Que	Answer	Que	Answer
1	Proof	9	θ=45°
2	1/8	11	1/3
3	sin A= 12/13, cos A = 5/13, tan = 12/5	12	2/9
4	10/3	13	1
5	A= 30°, B= 15°	14	2/3
		15	X= 45°

<u>LONG ANSWER TYPE QUESTIONS</u> (4 marks each)

1. $(\sin\theta - \cos\theta)^4$

$= [(\sin\theta - \cos\theta)^2]^2$

$= [\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta]^2$

$= [1 - 2\sin\theta\cos\theta]^2$

$= 1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta$

Also $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

and $(\sin^6\theta + \cos^6\theta) = (\sin^2\theta)^3 + (\cos^2\theta)^3 = (\sin^2\theta + \cos^2\theta)\,[(\sin^2\theta)^2 - \sin^2\theta\cos^2\theta + (\cos^2\theta)^2]$

$= (1)\,[\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta]$

$= [(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta]$

$= 1 - 3\sin^2\theta\cos^2\theta$

$\therefore$ LHS

$= 3(1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta) + 6(1 + 2\sin\theta\cos\theta) + 4(1 - 3\sin^2\theta\cos^2\theta)$

$= 3 + 6 + 4 = 13 =$ RHS

Hence proved

2. $\sin\theta + \sin^3\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta\,(1 + \sin^2\theta) = \cos^2\theta$

$\Rightarrow \sin\theta\,(2 - \cos^2\theta) = \cos^2\theta$

Squaring both sides, we get

$\sin^2\theta\,(2 - \cos^2\theta)^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)(4 + \cos^4\theta - 4\cos^2\theta) = \cos^4\theta$

$\Rightarrow 4 + \cos^4\theta - 4\cos^2\theta - 4\cos^2\theta - \cos^6\theta + 4\cos^4\theta = \cos^4\theta$

$\Rightarrow \cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4.$

3. LHS $= m^2 - n^2 = (m + n)(m - n)$

$=(\tan\theta + \sin\theta + \tan\theta - \sin\theta)(\tan\theta + \sin\theta - \tan\theta + \sin\theta)$

$= 2\tan\theta \cdot 2\sin\theta$

$= 4.\tan\theta\sin\theta$

RHS $= 4\sqrt{mn}$

$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$

$= 4\sqrt{(tan^2\theta - sin^2\theta)}$

$= 4\sqrt{\left(\frac{sin^2\theta}{cos^2\theta} - sin^2\theta\right)}$

$= 4\sqrt{sin^2\theta\left(\frac{1}{cos^2\theta} - 1\right)}$

$= 4 \sin \theta \sqrt{sec^2\theta - 1} = 4. \sin \theta \tan \theta$

LHS = RHS

Hence proved

4. $\sec^2\theta - \frac{sin^2 \theta - 2sin^4 \theta}{2\, cos^4\theta - cos^2\theta}$

$= \sec^2\theta - \frac{sin^2 \theta\, (1-2sin^2 \theta)}{cos^2\theta(2cos^2\theta-1)}$

$= \sec^2\theta - \frac{sin^2 \theta\, (1-2sin^2 \theta)}{cos^2\theta(2-2sin^2 \theta-1)}$

$= \sec^2\theta - \tan^2\theta$

$=1$

5. $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$

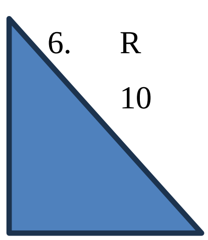
Square both expressions

$[\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta]^2 = 1$ ….(1)

$[\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta]^2 = 1$ …..(2)

Add both equations and simplify

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$



6. R

10 $5\sqrt{5}$

P

5

(i) $\sin P = \frac{10}{5\sqrt{5}}$

$\sin^2 P = \frac{4}{5}$

(ii) $\cos^2$ R and tan R = $\frac{4}{5}$, $\frac{1}{2}$

(iii) sin P x cos P = $\frac{2}{5}$

(iv) $\sin^2$ P – $\cos^2$ P = $\frac{3}{5}$

7. (i) $\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

$\sin 60 = \frac{\sqrt{3}}{2}$ $\frac{\tan\theta}{\sqrt{1+\tan^2\theta}} = \frac{\sqrt{3}}{2}$

(ii) $\tan\theta = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$

tan 60 = √3 = $\frac{\sin 60}{\cos 60}$ = tan 60 = √3

8. sin (A+B) =1

sin (A+B) = sin 90°

A+B = 90° ……..(1)

tan (A -B) = $\frac{1}{\sqrt{3}}$

tan (A -B) = tan 30°

A – B = 30° …….(2)

Solving equation (1) and (2) for A and B

We get A = 60° and B = 30°

tan A + cot B = tan 60° + cot 30° = √3 + √3 = 2√3

(ii) sec A – cosec B

= sec 60° - cosec 30°

= 2 – 2 =0

9. 4 $(sin^4 30° + cos^4 60°)$ – 3 $(cos^4 45° - sin^4 90°)$ = 2

10. $tan^3\theta$ /1+ $tan^2\theta$ + $cot^3\theta$ /1+ $cot^2\theta$

Ans. secθ cosecθ – 2 sinθ cosθ

ANSWERS

VERY SHORT ANSWER

Case based questions

Q.no 1 (1)	c
(2)	a
(3)	b
(4)	d
(5)	d
Q.no 2(1)	c
(2)	b
(3)	b
(4)	b
(5)	d
Q.no 3(1)	d
(2)	a
(3)	b
(4)	b
(5)	b
Q.no 4(1)	a
(2)	c
(3)	b
(4)	b
(5)	b
Q.no 5 (1)	a
(2)	d
(3)	b
(4)	b
(5)	a
Q.no 6 (1)	a
(2)	b
(3)	a
(4)	d
(5)	b
Q.no 7 (1)	c
(2)	b
(3)	c
(4)	b
(5)	b

SOME APPLICATIONS OF TRIGONOMETRY

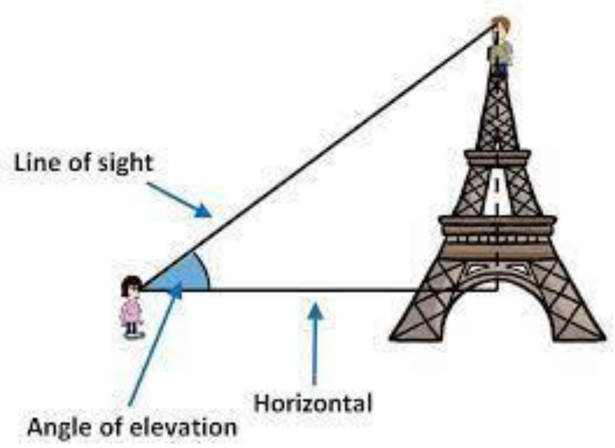
HEIGHTS AND DISTANCES: Trigonometry is used for finding the heights and distances of various objects, without measuring them.

Line of sight is the line drawn from the eye of the observer to the point on the object viewed by the observer.

Horizontal level is the horizontal line through the eye of the observer.

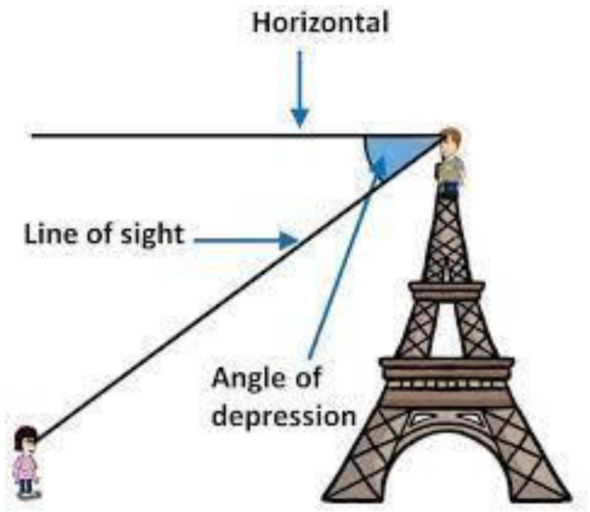
ANGLE OF ELEVATION

The angle of elevation is relevant for objects above horizontal level.
It is the angle formed by the line of sight with the horizontal level.



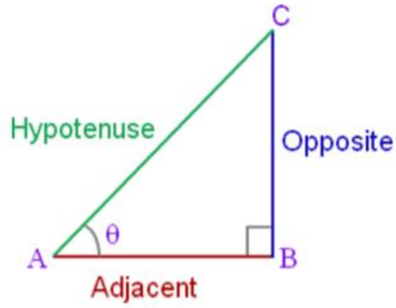
ANGLE OF DEPRESSION

The angle of depression is relevant for objects below horizontal level.
It is the angle formed by the line of sight with the horizontal level.



IMPORTANT POINTS TO REMEMBER:

In this right triangle ∠B = 90°. If we take ∠A as acute angle, then -



AB is the base, as the side adjacent to the acute angle.
BC is the perpendicular, as the side opposite to the acute angle.
AC is the hypotenuse, as the side opposite to the right angle.

Trigonometric ratios with respect to ∠A

RATIO	FORMULA	VALUE	ALTERNATIVE FORMULA	SHORT FORM
sin A	$\frac{opposite}{hypotenuse}$	$\frac{BC}{AC}$	$\frac{perpendicular}{hypotenuse}$	$\frac{P}{H}$
cos A	$\frac{adjacent}{hypotenuse}$	$\frac{AB}{AC}$	$\frac{base}{hypotenuse}$	$\frac{B}{H}$
tan A	$\frac{opposite}{adjacent}$	$\frac{BC}{AB}$	$\frac{perpendicular}{base}$	$\frac{P}{B}$
cosec A	$\frac{hypotenuse}{opposite}$	$\frac{AC}{BC}$	$\frac{hypotenuse}{perpendicular}$	$\frac{H}{P}$
sec A	$\frac{hypotenuse}{adjacent}$	$\frac{AC}{AB}$	$\frac{hypotenuse}{base}$	$\frac{H}{B}$
cot A	$\frac{adjacent}{opposite}$	$\frac{AB}{BC}$	$\frac{base}{perpendicular}$	$\frac{B}{P}$

RECIPROCAL RELATION BETWEEN TRIOGONOMETRIC RATIOS

sin A $=\frac{1}{Cosec\ A}$	cosec A $=\frac{1}{sin\ A}$	sin A. cosec A = 1
cos A $=\frac{1}{secsec\ A}$	sec A $=\frac{1}{cos\ A}$	cos A. sec A=1
tan A $=\frac{1}{Cot\ A}$	cot A $=\frac{1}{tan\ A}$	tan A. cot A= 1

QUOTIENT RELATION

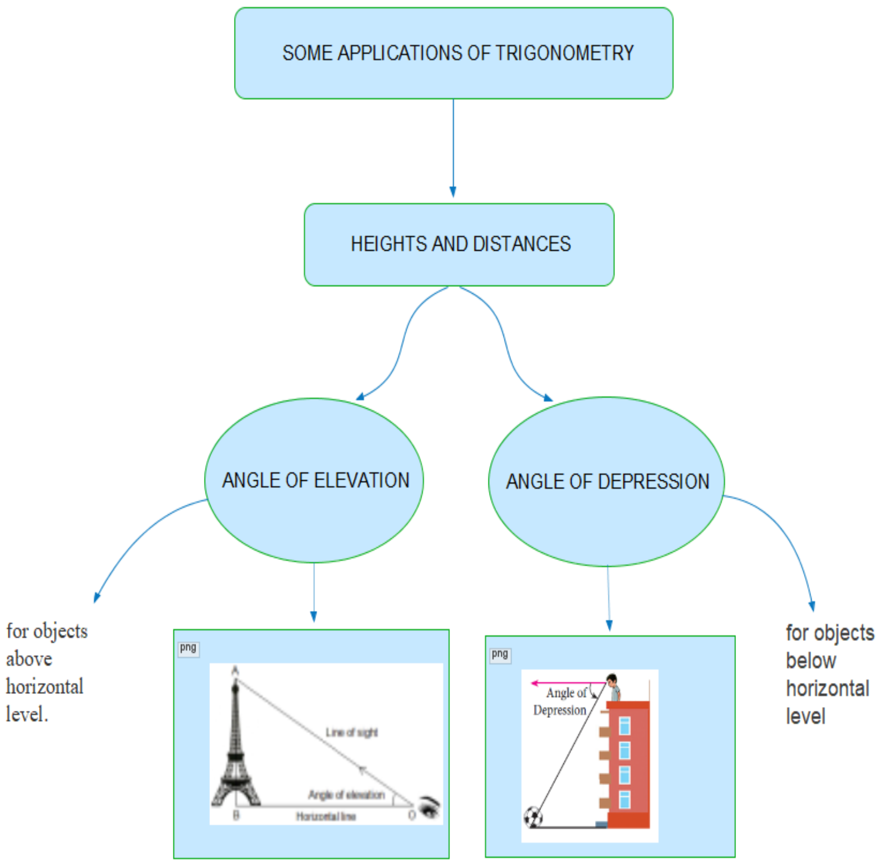
tan A $=\frac{sinsin\ A}{coscos\ A}$
cot A $=\frac{coscos\ A}{sinsin\ A}$

TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES

Trigonometry Table

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec}\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

MIND MAP



Choose the correct answer:(MCQ)

1. **Choose the correct answer:(MCQ)**

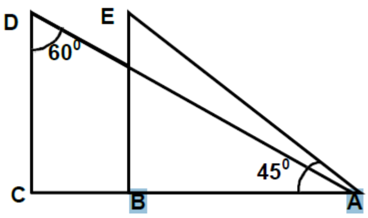
   1. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°.find the distance between the foot of the tower and the ball.

(a) $\frac{20}{\sqrt{3}}$m
(b) $20\sqrt{3}$ m
(c) $10\sqrt{3}$m
(d) 12 m

2. The angle of elevation of a ladder leaning against a wall is 60^0 and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.
(a) 10 m
(b) 19 m
(c) 20 m
(d) none of these

3. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$, what is the angle of elevation of the Sun?
(a) 30^0
(b) 60^0
(c) 45^0
(d) none of these

4. In the figure given below, what are the angles of depression from the observing positions D and E of the object A?



(a) $30^0, 45^0$
(b) $60^0, 45^0$
(c) $45^0, 60^0$
(d) none of these

5. If the angle of elevation of a tower from a distance of 100m from its foot is 600, then the height of the tower is
(a) $100\sqrt{3}$ m
(b) $200/\sqrt{3}$ m
(c) $50\sqrt{3}$ m
(d) $100/\sqrt{3}$ m

6. A tower is 50m high, its shadow ix 'x' metres shorter when the sun's altitude is 45^0 than when it is 30^0. Find the value of 'x'

7. A 1.5m tall boy stands at a distance of 2m from lamp post and casts a shadow of 4.5m on the ground. Find the height of the lamp post.
(a) 3 m
(b) 2.5 m
(c) 5 m
(d) none of these

8. The tops of two poles of height 20m and 14m are connected by a wire. If the wire makes an angle of 30^0 with horizontal, then the length of the wire is

(a) 12 m (b) 10 m (c) 8 m (d) 6 m

9. If the angles of elevation of a tower from two points distant a and b (a > b) from its foot and in the same straight line from it are 30^0 and 60^0, then the height of the tower is

(a) $(\sqrt{a} + b)$ m (c) $\sqrt{ab}$ m

(b) $(\sqrt{a} - b)$ m (d) $\sqrt{a}/b$ m

10. At some time of the day, the length of the shadow of a tower is equal to its height. Then, the sun's altitude at that time is:

(a) 30° (b) 60° (c) 90° (d) 45^0

II State whether True or False

1. If the length of the shadow of a tower is increasing, then the angle of elevation of the sun is also increasing.
2. If a man standing on a platform 3 metres above the surface of a lake observes a cloud and its reflection in the lake, then the angle of elevation of the cloud is equal to the angle of depression of its reflection.
3. The angle of elevation of the top of a tower is 30°. If the height of the tower is doubled, then the angle of elevation of its top will also be doubled.
4. If the height of a tower and the distance of the point of observation from its foot, both, are increased by 10%, then the angle of elevation of its top remains unchanged.

III Fill in the blanks

1. The …………………… is the line drawn from the eye of an observer to the point in the object viewed by the observer.
2. The …………………………. of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level.
3. The ………………………… of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level.
4. The …………………….. of an object or the ………………….. distance between two distant objects can be determined with the help of trigonometric ratios.

IV Very short answer questions

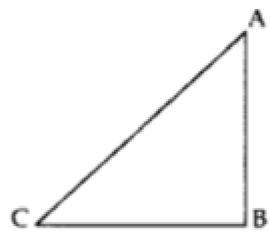
1. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9m from the base of the tower and in the same straight line with it are 60° and 30° respectively. Find the height of the tower.
2. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find x:y.

V SHORT ANSWER QUESTIONS (TWO MARKS)

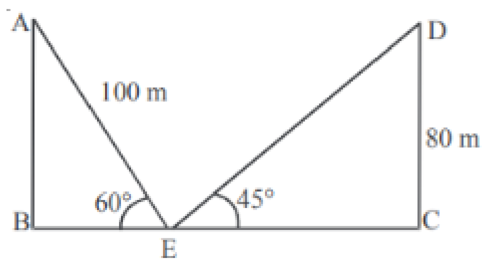
LEVEL 1

1. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then calculate the height of the wall.

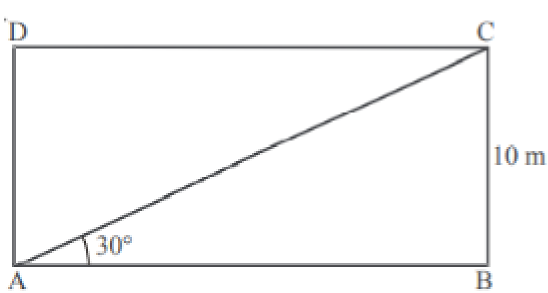
2. In the given figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the Sun's altitude.



3. The string of a kite is 100 m long and it makes an angle of 60° with the horizontal. Find the height of the kite, assuming that there is no slack in the string.
4. A tree 12 m high, is broken by the storm. The top of the tree touches the ground making an angle 30°. At what height from the bottom the tree is broken by the storm?
5. In the figure, find the value of BC.

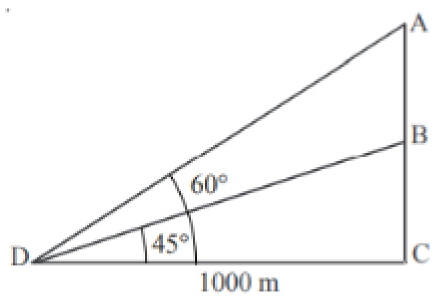


6. Find the angle of elevation of a point which is at a distance of $10\sqrt{3}$ m from the base of a tower 30m high.
7. The height of the tower is 15 m. What is the length of its shadow when sun's altitude is 45°?
8. A 1.5 m tall boy stands at a distance of 2m from lamp post and casts a shadow of 4.5 m on the ground. Find the height of the lamp post?
9. The tops of two poles of height 20m and 14 m are connected by a wire. Find the length of the wire if it makes an angle of 30° with horizontal?
10. In the given figure, find the perimeter of rectangle ABCD.



LEVEL 2

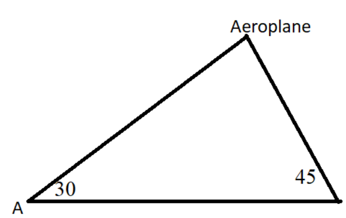
11. In the figure, find the value of AB.



12. If the shadow of a tower 30 m long, when the Sun's elevation is 30°. What is the length of the shadow, when Sun's elevation is 60°?
13. From a point on the ground, which is 21 m from the foot of a tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.
14. In figure, AB is a 6m pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If AD=2.54 m, find the length of the ladder. (Use $\sqrt{3}$=1.73)
15. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30°. Find the height of tower.
16. The angle of depression from the top of a tower 12 m high, at a point on the ground is 30°. Then find the distance of the point from the top of the tower.

LEVEL 3

17. The top of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of line joining their feet, then find x : y.
18. A vertical stick 10 cm long casts a shadow 8 cm long. At the same time, a tower casts a shadow 28 m long. Determine the height of the tower.
19. Stations A and B are $3(1+\sqrt{3})$ km apart. Each station sights an aeroplane at an angle of 30° and 45° as shown in figure. Find the altitude of the aeroplane.



20. From the vertex of a tower the angle of depression of a point 120 m away from the foot of the tower is 60°. Find the height of the tower.

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

LEVEL 1

1. Find the angle of elevation of the sun when the shadow of a pole h metres high is $\sqrt{3}$ h metres long.
2. Two pillars of equal heights are on either side of a road, which is hundred metres wide. The angles of elevation of the tops of the pillars are 60° and 30° at a point on the road between the pillars. Find the position of the point between the pillars?
3. From a point on the ground, the angles of elevation of the bottom and top of a water tank kept on the top of the 30 m high building are 30° and 45° respectively. Find the height of the water tank?
4. From the top of a multi-storeyed building, 90m high, the angles of depression of the top and the bottom of a tower are observed to be 30° and 60° respectively. Find the height of the tower?

LEVEL 2

5. Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the base of the lighthouse are in the same straight line. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45° . If the height of the lighthouse is 200m, find the distance between the two ships.
6. From the top of a 300 metre high light-house, the angles of depression of two ships, which are due south of the observer and in a straight line with its base, are 60° and 30° .Find their distance apart?
7. A Statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal? (Use $\sqrt{3} = 1.73$)

LEVEL 3

8. A peacock is sitting on the top of a tree. It observes a serpent on the ground making an angle of depression of 30° . The peacock with the speed of 300 metre/ minute catches the serpent in 12 seconds. What is the height of the tree?
9. An aero plane, at an altitude of 1200 m, finds that two ships are sailing towards it in the same direction. The angles of depression of the ships as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two ships?
10. From a balloon vertically above a straight road, the angles of depression of two cars at an instant are found to be 45° and 60°. If the cars are 100 m apart, find the height of the balloon.
11. The angle of elevation of the top of a tower from certain point is 30°. If the observer moves 20 metres towards the tower, the angle of elevation of the top increases by 15°. Find the height of the tower.

12. The angle of elevation of the top of a tower from two points distant s and t from its foot are complementary. Find the height of the tower.
13. The shadow of a tower standing on a level plane is found to be 50 m longer when Sun's elevation is 30° than when it is 60°. Find the height of the tower.
14. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the other tower.
15. The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 m vertically above the first, its angle of elevation is 45°. Find the height of the tower.

Long Answer Type Questions (5 MARKS)

LEVEL 1

1. A person standing on the bank of a river observes that angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 30m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.
2. At a point on a level ground, the angle of elevation α of a vertical tower is found to be such that $\tan \alpha = 5/12$. On walking 192m towards the tower, the angle of elevation becomes β such that $\tan \beta = 3/4$. Find the height of the tower.
3. A boy whose eye level is 1.3m from the ground, spots a balloon moving with wind in a horizontal line at some height from the ground. The angle of elevation of the balloon from the eyes of the boy at any instant is 60°. After 12 seconds, the angle off elevation reduces to 30°. If the speed of wind at that moment is $29\sqrt{3}$ m/s, then find the height of the balloon from the ground.
4. Two pillars of equal height stand on either side of the roadway which is 150m wide. From a point on the roadway between the pillars, the elevations of the top of the pillars are 60° and 30°. Find the height of the pillars and the position of the point.
5. The angle of elevation of the top of the building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60m high, find the height of the building.

LEVEL 2

6. From the top of the building, 100m high, the angles of depression of the top and bottom of a tower are observed to be 45° and 60° respectively. Find the height of the tower. Also find the distance between the foot of the building and the bottom of the tower.
7. The angles of elevation and depression of the top and bottom of a lighthouse from the top a 60° high building are 30° and 60° respectively. Find
   (i) The difference between the heights of the lighthouse and the building
   (ii) The distance between the lighthouse and the building.
8. The angle of elevation of the top of the hill at the foot of the tower is 60° and the angle of depression from the tower of the foot of the hill is 30°. If the tower is 50m high, find the height of the hill.

HOT QUESTIONS

9. A man standing on the deck of the ship, which is 16m above the water level, observes the angle of elevation of the top of the clip as 60° and the angle of depression of the base of the cliff as 30°. Calculate the distance of the cliff from the ship and height of the cliff.
10. If the angle of elevation of a cloud from a point 'h' meters above a lake is α and angle of depression of its reflection in the lake is β, prove that distance of the cloud from the point of observation is $\frac{2h secsec\,\alpha}{tantan\,\beta\, -\, tantan\,\alpha}$

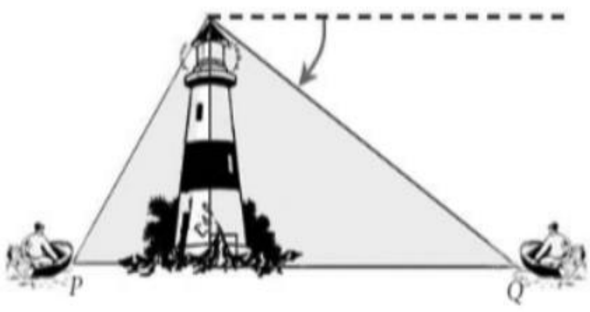
Case Study 1

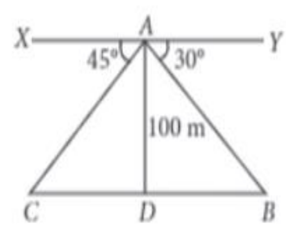
A group of students of class x visited India Gate on an education trip. The teacher and students had interest in History as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919.The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 meters) in height.

I. What is the angle of elevation if they are standing at a distance of 42m away from the monument?
II. They want to see the tower at an angle of 60^0.So they want to know the distance where they should stand and hence find the distance.
III. When the altitude of the sun is at 60^0, find the height of a vertical tower that casts a shadow of 20m length.
IV. What is the angle of elevation of the sun when the ratio of the height of the tower to its shadow is 1:1

CASE STUDY 2:
LIGHT HOUSE

A boy is standing on the top of light house. He observed that boat P and boat Q are approaching to light house from opposite directions. He finds that angle of depression of boat P is 45^0 and angle of depression of boat Q is 30^0. He also knows that height of the light house is 100m.

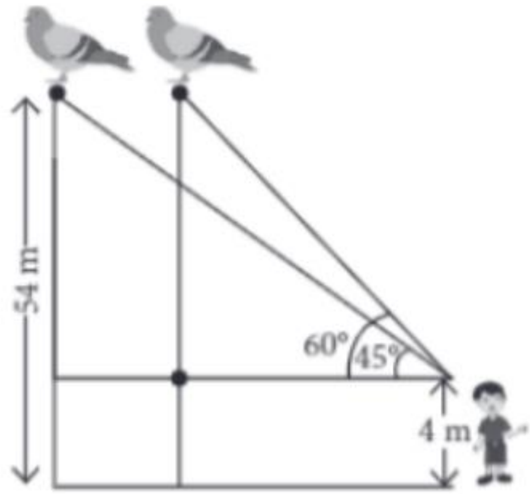




Based on the above information, answer the following questions.

I. What is the measure of $\angle ACD$?
II. If $\angle YAB = 30^0$, then $\angle ABD$ *is also* 30^0, why?
III. Find the distance between the boat P and the light house.
IV. Find the distance between the two boats**.**

CASE STUDY 3:



A boy 4 m tall spots a pigeon sitting on the top of a pole of height 54m from the ground. The angle of elevation of the pigeon from the eyes of boy at any instant is 60^0. The pigeon flies away horizontally in such a way that it remained at a constant height from the ground. After 8 seconds, the angle of evaluation of the pigeon from the same point is 45^0. Based on the above information answer the following questions (take $\sqrt{3} = 1.73$)

I. Find the distance of first position of the pigeon from the eyes of the boy
II. Find the distance between the boy and the pole.
III. (a) How much distance the pigeon covers in 8 seconds?

OR

(b) Find the speed of the pigeon.

Case Study 4

A flagstaff stands on the top of a 5-meter-high tower. From a point on the ground the angle of elevation of the top of the flag staff is 60° and from the same point the angle of elevation of the top of the tower is 45°. (CBSE 2023- Basic

Based on the above, answer the following questions:

I. What is the distance of the point from the foot of the tower?
II. What is the height of the flagstaff?
III. (a) If at some other point, the top of tower's angle of elevation is 30°, then find the distance of this new point from the foot of the tower.

OR

(b) Find the distance between the top of the tower and the point at which the angle of elevation of the top of tower is 30°

Case Study 5

A TV Tower stands vertically on the ground. From a point ‘A’ on the ground, the angle of elevation of top of the tower (point ‘B’) is 60°. There is a point ‘C’ on the tower which is 78 m (approx.) above the ground. The angle of elevation of the point C from the point ‘A’ is found to be 30°

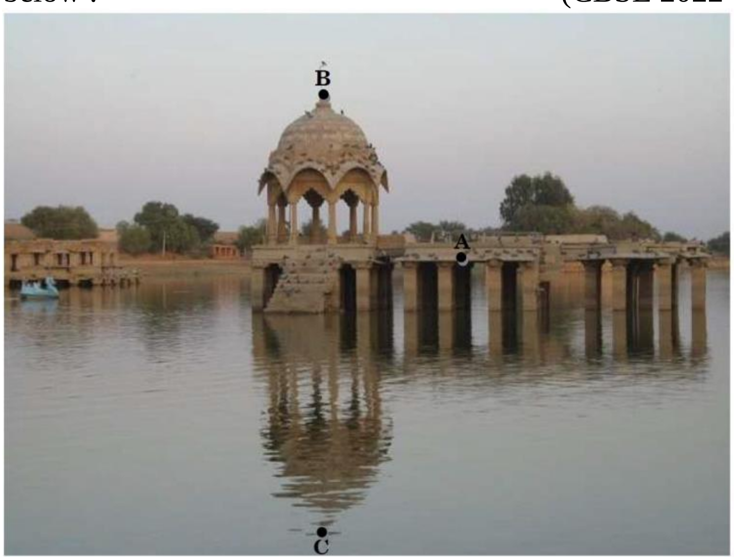
(CBSE 2022

Based on the above information answer the questions

I. Draw a well – labelled figure, based on the information given
II. Find the height of the tower
III. Find the distance of the tower from the point A

Case Study 6

Gadisar Lake is located in the Jaisalmer district of Rajasthan. It was built by the King of Jaisalmer and rebuilt by Gadsi Singh in 14th century.The lake has many Chhatris. One of them is shown below : (CBSE 2022



Observe the picture. From a point ‘A’, $h\ m$ above from water level, the angle of elevation of top of Chhatri (point B) is 45° and angle of depression of its reflection in water (point C) is 60°. If the height ofChhatri above water level is (approximately) 10 m

I. Draw a well-labelled figure based on the above information.
II. Find the height (h) of the point A above water level.(Use 3 = 1·73)

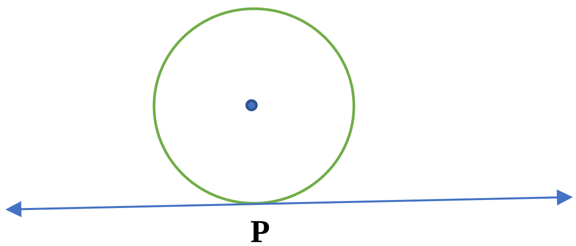
CHAPTER 10

Circles

Important Concepts

Tangent to a circle

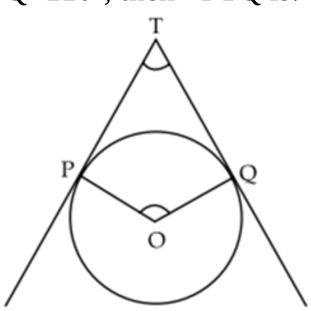
A tangent to a circle is a line that intersects the circle at only one point



* **There is only one tangent at a point on a circle**
* **There are exactly two tangents to a circle through a point lying outside the circle.**
* **The tangent at any point of a circle is perpendicular to the radius through the point of contact.**
* **The length of tangents drawn from an external point to a circle are equal.**

Multiple Choice Questions

1. In Fig. if from an external point T, TP and TQ are two tangents to a circle with centre O so that <POQ=110^0, then <PTQ is:

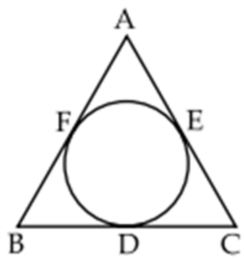


a) 60^0
b) 70^0
c) 80^0
d) 90^0

2. From a point P which is at a distance of 13cm from the centre O of a circle of radius 5cm, the pair of tangents PQ and PR to the circle are drawn. What are the lengths (in cm) of tangents PQ and PR?

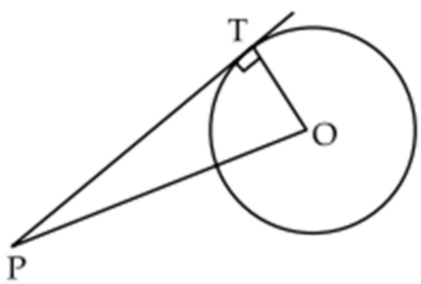
   a) 13,12
   b) 13,13
   c) 12,12
   d) 12,18

3. In the fig. if the semi perimeter of $\Delta ABC = 23cm$, then AF + BD + CE is:



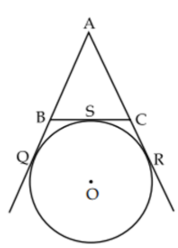
   a) 46cm
   b) 11.5cm
   c) 23cm
   d) 34.5cm

4. In the fig. PT is a tangent to a circle with centre O. If PT = 30cm and diameter of circle is 32cm, then the length of the line segment OP will be:



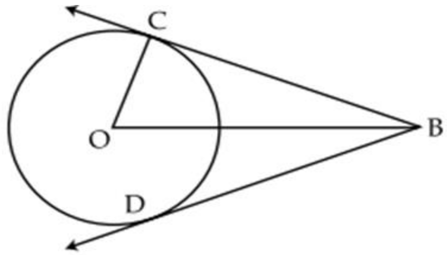
   a) 68cm
   b) 34cm
   c) 17cm
   d) 34.8cm

5. In fig. AQ, AR and BC are tangents to a circle with centre O, If AB = 7cm, BC= 5cm,AC = 5cm, then the length of tangent AQ is:



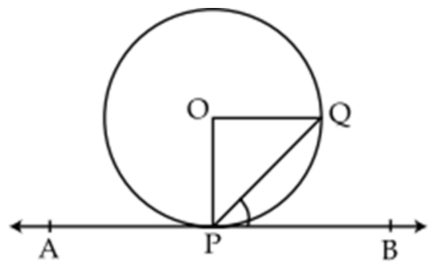
   a) 5cm
   b) 7cm
   c) 8.5cm
   d) 17cm

6. In Fig. if OC =9cm, and OB = 15cm, then find BC+BD



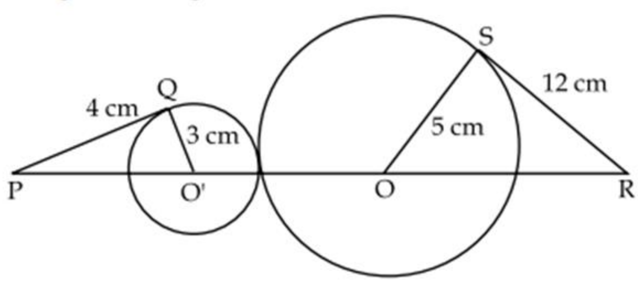
a) 18cm
b) 12cm
c) 24cm
d) 36cm

7. APB is a tangent to a circle with centre O, at point P. If <QPB=50^0, then the measure of <POQ is:



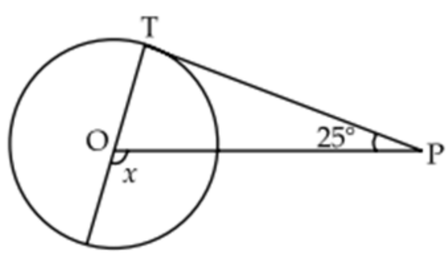
a) 120^0
b) 100^0
c) 140^0

8. In fig. the length of PR is:



a) 20cm
b) 26cm
c) 24cm
d) 28cm

9. In fig. PT is a tangent to a circle with centre O and <TPO = 25^0, then the measure of x is:

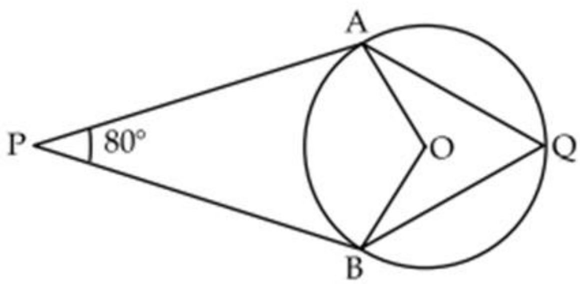


a. 120^0

b. 125^0
c. 110^0
d. 115^0

10. Maximum number of common tangents that can be drawn to two circles intersecting at two distinct points is:
    a) 1
    b) 2
    c) 3
    d) 4

11. In the fig. O is the centre of the circle. If PA and PB are tangents to the circle, then <AQB is equal to:

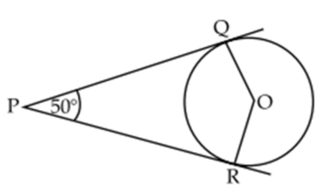


    a) 100^0
    b) 80^0
    c) 70^0
    d) 50^0

12. A line which is perpendicular to the radius of the circle through the point of contact is:
    a) Tangent
    b) Chord
    c) segment
    d) normal

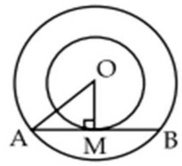
13. Number of tangents to a circle which are parallel to a secant is:
    a) 1
    b) 2
    c) 3
    d) Infinite

14. In the given quadrilateral, OQPR, <QOR is equal to:



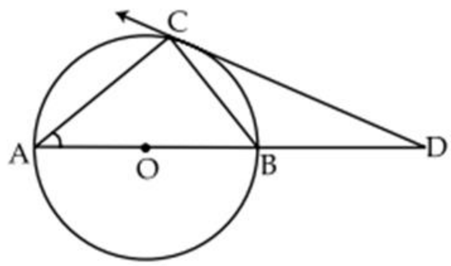
    a) 120^0
    b) 130^0
    c) 145^0
    d) 110^0

15. In fig. if OA = 5cm, OM = 3cm, the length of chord AB (in cm) is:



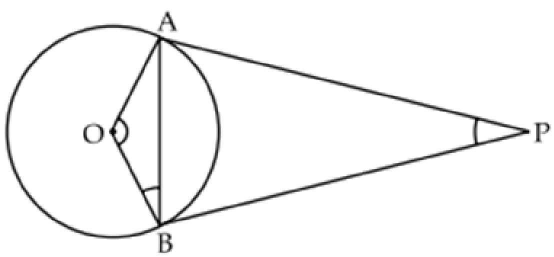
a) 8
b) 10
c) 6
d) 4

16. In the fig. AB is a diameter and AC is a chord of a circle such that <BAC = 30^0. If DC is a tangent, then ΔBCD is:



a) Equilateral
b) Isosceles
c) Right angled triangle
d) Acute angled

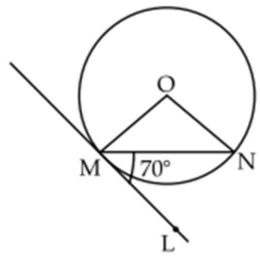
17. Two tangents are drawn from an external point P (as given in fig.) such that <OBA = 10^0. Then <BPA is:



a) 10^0
b) 20^0
c) 30^0
d) 40^0

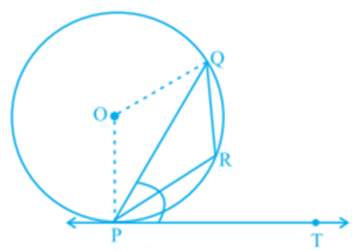
18. If two tangents inclined to each other at an angle 60^0 are drawn to a circle of radius 3cm, then the length of tangent is equal to:
a) √3 cm
b) 2√3cm
c) 23 cm
d) 3√3 cm

19. In fig. O is the centre of the circle, MN is a chord and the tangent ML at the point M makes an angle 70^0 with MN then <MON is equal to:



a) 120^0 c) 140^0
b) 90^0 d) 70^0

20. The distance between two parallel tangents to a circle of radius 5cm is:
   a) 10cm c) 12cm
   b) 11cm d) 14cm
21. If the circumference of a circle increases from 4π to 8π, then its area will become
   a) half c) 4 times
   b) 2 times d) does not change
22. In Fig, PQ is a chord of a circle and PT is the tangent at P such that ∠QPT = 60°. Then ∠PRQ is equal to



   a) 135^0 c) 120^0
   b) 150^0 d) 110^0
23. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to
   a) 323 cm c) 3 cm
   b) 6 cm d) 33 cm

24. There is a circle with centre O. Manu wants to draw a tangent RS to the circle. What is the number of points at which the line RS will meet the circle?
   a. 0 c. 2
   b. 1 d. 3

ONE MARK QUESTIONS (OTHER THAN MCQs)

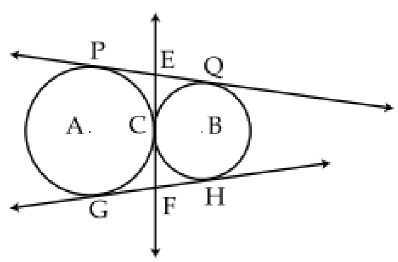
25. Tangent to a circle intersects the circle at ……….. point(s)?

26. The tangent at any point of circle is perpendicular to the ………through the point of contact.
27. The lengths of tangents drawn from an external point to a circle are not equal. (true/false)
28. The common point of a tangent to a circle with the circle is called------------.
29. If diagonal of a cyclic quadrilateral are the diameters of a circle through the vertices of a quadrilateral, then quadrilateral is a-------------.
30. Given three non collinear points, then the number of circles which can be drawn through these three points are?
31. PQ is a tangent drawn from an external point P to a circle with centre O and QOR is the diameter of the circle. If ∠POR= 120^0, what is the measure of ∠OPQ?

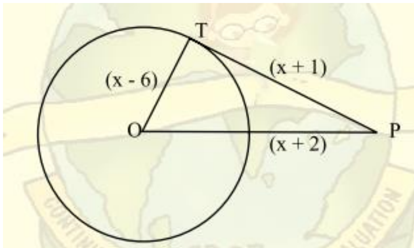
Very Short Answer Questions

LEVEL-I

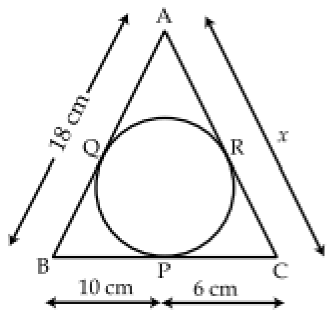
1. Prove that the line segments joining the points of contact of two parallel tangents is a diameter of the circle.
2. Two concentric circles have centre O, OP= 4cm, OB = 5cm. AB is a chord of the outer circle and tangent to the inner circle at P. Find the length of AB.
3. Two tangents PA and PB are drawn to a circle with centre O such that $< APB$ = $120^{0.}$ Prove that OP=2AP
4. In fig. two circles touch each other externally at C. Prove that the common tangent at C bisects the other two tangents



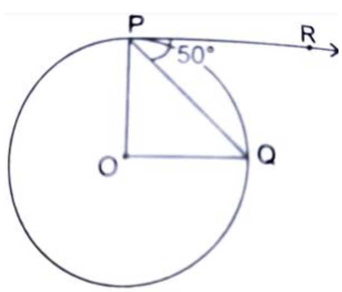
5. Find the actual length of sides of ΔOTP



6. In fig. all three sides of the triangle touch the circle. Find the value of x (CBSE 2020).

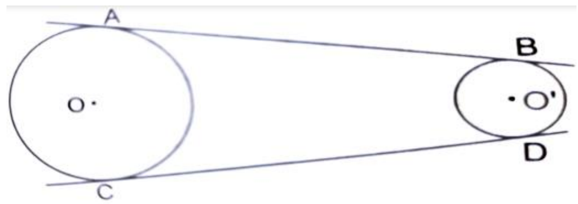


7. Two tangents PR and PQ are drawn from external point P to a circle with centre O. Prove that PROQ is a cyclic quadrilateral.
8. Prove that tangents drawn at the ends of a chord make equal angles with the chord
9. Two concentric circles are of radii 7 cm and r cm respectively, where r > 7. A chord of the larger circle, of length 48 cm, touches the smaller circle. Find the value of r
10. If O is the centre of a circle, PQ is chord and the tangent PR at P makes an angle 50^0 with PQ. Find ∠POQ

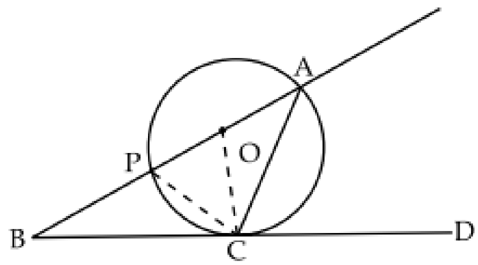


LEVEL 2

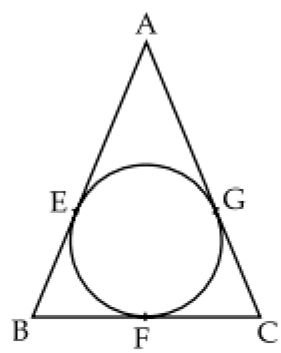
1. In the fig. AB and CD are common tangents to two circles of unequal radii. Prove AB = CD



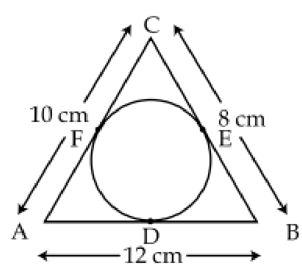
2. O is the centre of the circle and BCD is a tangent to it at C. Prove that $< BAC + < ACD = 90^0$



3. In the isosceles triangle ABC in fig. AB = AC, show that BF = FC

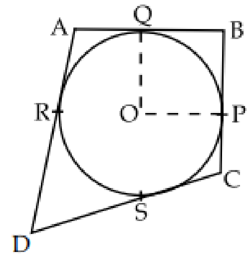


4. In the fig. a circle is inscribed in a ΔABC with sides AB = 12cm, BC = 8 cm and AC = 10cm. Find the lengths of AD, BE and CF

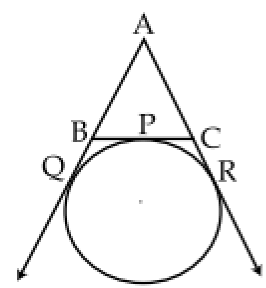


<u>LEVEL 3</u>

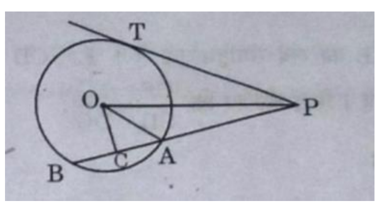
1. In fig. circle is inscribed in a quadrilateral ABCD in which $< B = 90^0$. If AD = 23cm, AB = 29cm, and DS = 5cm, find the radius 'r' of the circle



2. In fig. the circle touches the side BC of a triangle ABC at the point P and AB and AC produced at Q and R. Show that AQ = $\frac{1}{2}(perimeter\ of\ \Delta ABC)$



3. In fig.PT is a tangent to a circle with centre at O. OC is perpendicular to the chord AB. Prove that PA.PB $= PC^2 - AC^2$ (CBSE 2023)

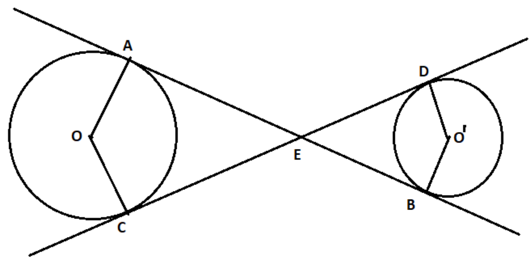


Short Answer Questions

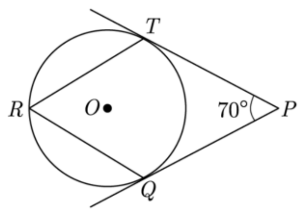
LEVEL I

1. If an angle between two tangents drawn from a point P to a circle of radius 'a' and centre O is 60°, then prove that AP = a√3.

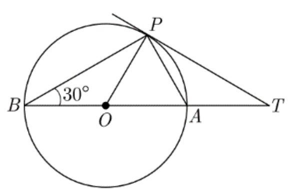
2. In the figure common tangents AB and CD to two circles with centre O and ‘O^I intersects at E. Prove that AB = CD.



3. In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, find ∠TRQ.

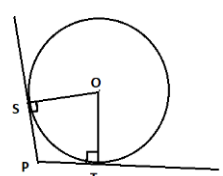


4. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If ∠PBO = 30°, what is the measure of ∠PTA?

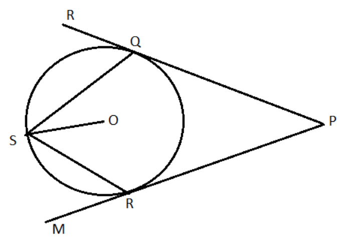


LEVEL II

1. In figure tangent segments PS and PT are drawn to a circle with centre O such that $\angle SPT$ = 120°. Prove that OP = 2PS.

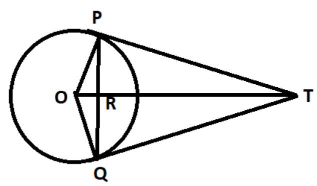


2. In fig. 3, PQ and PR are tangents to the circle with centre O and S is a point on the circle such that ∠SQR= 50° and ∠SRM = 60°. Find ∠QSR.

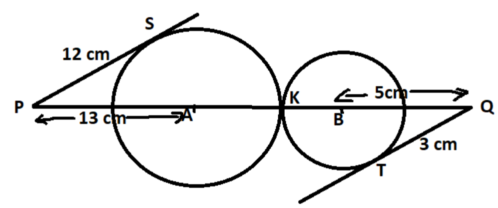


3. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ=2∠OPQ.

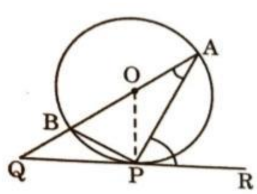
4. In fig, PQ is a chord of length 8 cm of a circle of radius 5 cm, the tangents at P and Q intersect at a point T. Find the length TP.



5. In fig, two circles with centres A and B touch each other externally at K. find the length of segment PQ. (Given PA=13 cm, BQ=5 cm ,
PS=12 cm AND QT=3 cm)



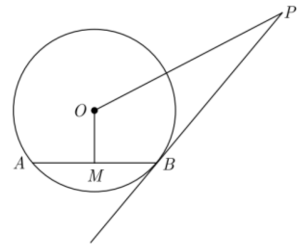
6. In the given fig.O is the centre of the circle and
QPR is a tangent to it at P. Prove that $\angle QAP + \angle APR = 90^0$



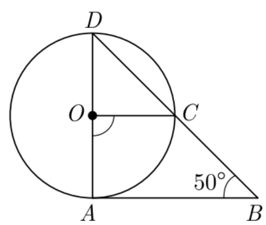
(CBSE 2023)

LEVEL III

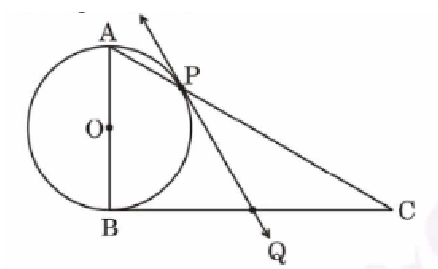
1. PB is a tangent to the circle with centre O to B. AB is a chord of length 24 cm at a distance of 5 cm from the centre. It the tangent is of length 20 cm, find the length of PO.



2. In the given figure, AD is a diameter of a circle with centre O and AB is a tangent at A. C is a point on the circle such that DC produced intersects the tangent at B and ∠ABC = 50°. Find ∠AOC.



3. In figure a triangle ABC with $<B=90^{o}$ is shown. Taking AB as diameter ,a circle has been drawn intersecting AC at the point P. Prove that the tangent drawn at the point P bisects BC (CBSE2022)



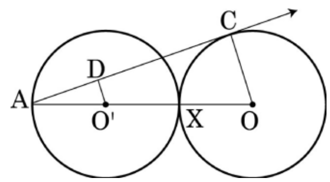
Long Answer Questions

LEVEL I

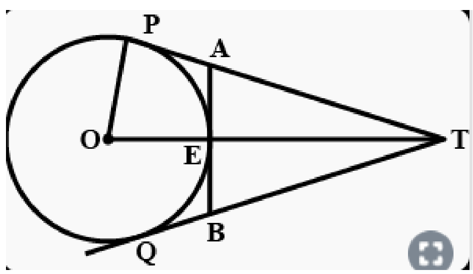
1. Prove that the lengths of tangents drawn from an external point to a circle are equal.
2. Prove that opposites sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

LEVEL II

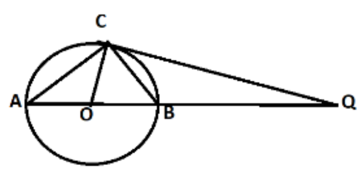
1. In fig, two equal circles with centres O and O^I , touch each other at X. OO^I produced meet the circle with centre O^I at A. AC is tangent to the circle with centre O, at the point C. O^ID is perpendicular to AC. Find the value of $\frac{DO^{\iota}}{CO}$.



2. n fig, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13cm and OT intersect circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.

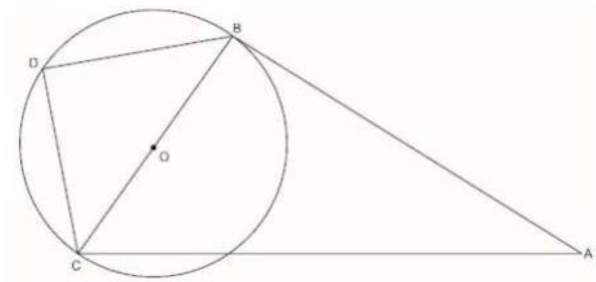


3. In fig AB is the diameter of a circle with centre O and QC is a tangent to the circle at C. If $\angle CAB=30°$, find $\angle CQA$ and $\angle CBA$.



LEVEL III

1. The figure below represents a circle with centre O and diameter 12cmIn triangle DBA, $\angle DBC = \angle BCD$ and $\angle A = 50°$.



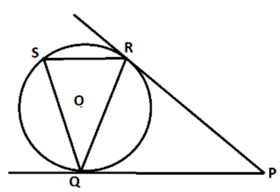
   (a) What is the measure of $\angle DCA$?

   (b) Dhruv said that, "The quadrilateral DBAC is a cyclic quadrilateral." Is Dhruv correct? Give a reason to support your answer.

   (c) In triangle BAC, the length of side CA = 2.5 times OB. What is the length of side BA?

2. The radius of the in-circle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.

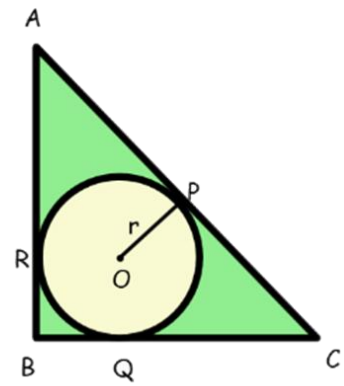
3. In fig, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.



CASE STUDY BASED QUESTIONS

CASE STUDY-1(PLAYGROUND)

A playground is in the shape of a triangle with right angle at B, AB = 3m and BC = 4m. A pit was dig inside it such that it touches the walls AC, BC and AB at P, Q and R, respectively such that AP = x m.



Based on the above information, answer the following questions.

i. The value of AR =

(a) 2x m (b) x / 2 m (c) x m (d) 3x m

ii. The value of BQ=

(a) 2x m (b) (3 − x) m (c) (2 − x) m (d) 4x m

iii. The value of CQ=

(a) (4 + x) m (b) (5 − x) m (c) (1 + x) m (d) Both (b) and (c)

iv. Which of the following is correct?

(a) Quadrilateral AROP is a square (b) Quadrilateral BROQ is a square

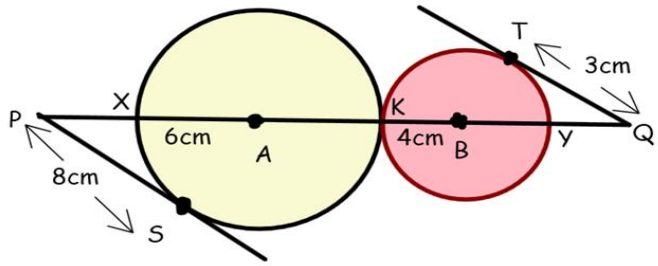
(c) Quadrilateral CQOP is a square (d) None of the above

v. Radius of the pit is

(a) 1 m (b) 3 m (c) 4 m (d) 5 m

CASE STUDY – 2 (CIRCLE DRAWING)

A student draws two circles that touch each other externally at point K with centres A and B and radii 6 cm and 4cm, respectively as shown in the figure



Based on the above information, answer the following questions.

i. The value of PA =
(a) 10 cm (b) 5 cm (c) 13 cm (d) Can't be determined

ii. The value of BQ=
(a) 4 cm (b) 5 cm (c) 6 cm (d) 18 cm

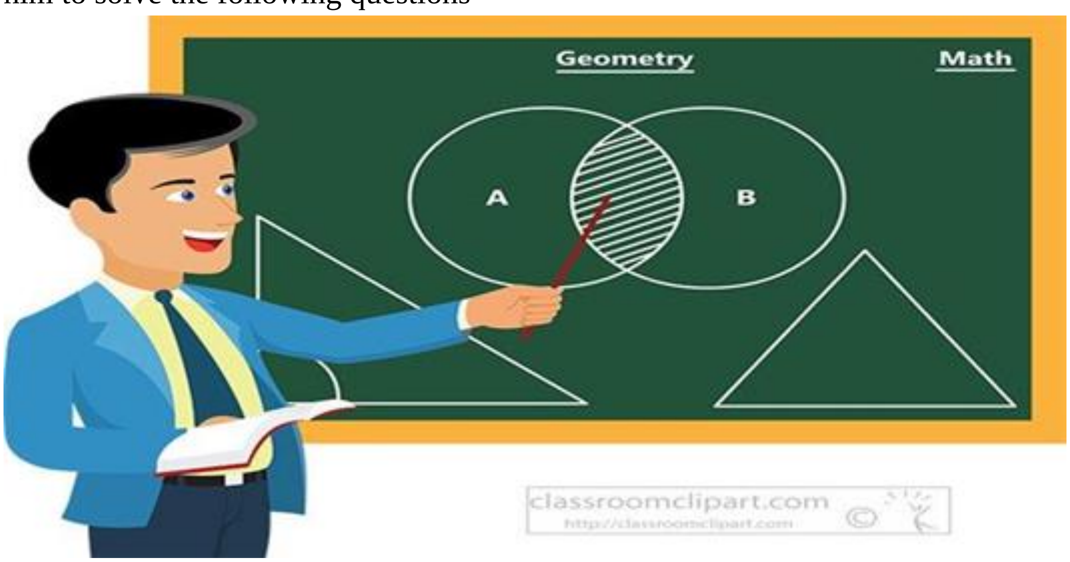
iii. The value of PK =
(a) 13 cm (b) 15 cm (c) 16 cm (d) 18 cm

iv. The value of QY =
(a) 2 cm (b) 5 cm (c) 1 cm (d) 3 cm

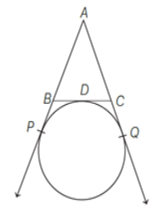
v. If two circles touch externally, then the number of common tangents can be drawn is
(a) 1 (b) 2 (c) 3 (d) None of these

CASE STUDY – 3

Kuldeep loves geometry. So, he was curious to know more about the concepts of circles. His grandfather is a mathematician. So, he reached to his grandfather to learn something interesting about tangents and circles. His grandfather gave him knowledge on circles and tangents and ask him to solve the following questions

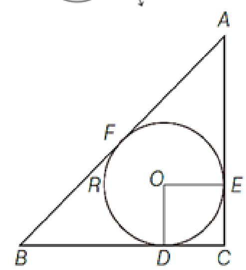


i. In the given figure, AP, AQ and BC are tangents to the circle such that AB = 7 cm, BC = 4 cm and AC = 9 cm. Find AP



b. (a) 12 cm (b) 15 cm
c. (c) 13 cm (d) 10 cm

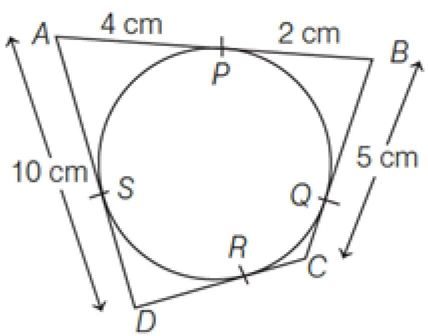
ii. A circle of radius 3 cm is inscribed in a right angled ΔBAC such that BD = 9 cm



and DC= 3 cm Find the length of AB.

(a) 6 cm (b) 12 cm
(c) 15 cm (d) 10 cm

iii. In the given figure, what is the length of CD?

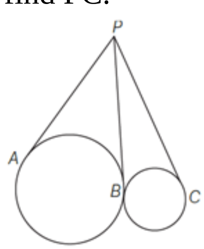


a. (a) 11 cm (b) 9 cm
b. (c) 7 cm (d) 13 cm

iv. If PA and PB are two tangents to a circle with centre O from an external point P such that ∠OPB = 50°, then find ∠BPA

a. (a) 60° (b) 50° (c) 120° (d) 100°

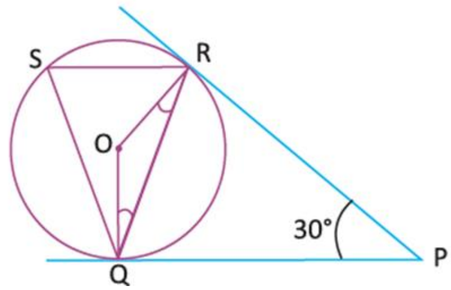
v. In the given figure, P is an external point from, which tangents are drawn to two externally touching circles. If PA =11 cm, then find PC.

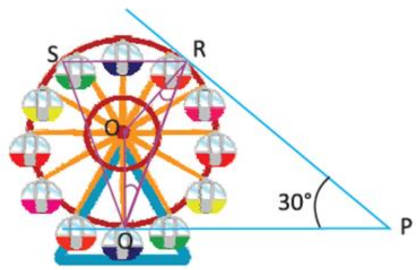


(a) 3.5 cm (b) 4 cm
(c) 11 cm (d) Can’t be determined

CASE STUDY-4

A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passengers carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity. After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.





i. In the given figure find ∠ROQ.
(A) 60° (B) 100° (C) 150° (D) 90°

ii. Find ∠RQP.
(A) 75° (B) 60° (C) 30° (D) 90°

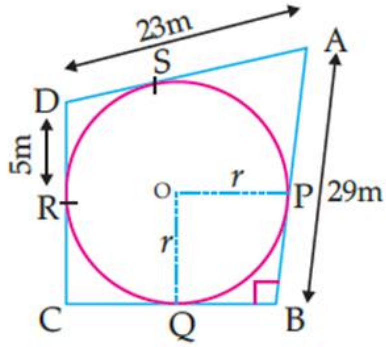
iii. Find ∠RSQ.
(A) 60° (B) 75° (C) 100° (D) 30°

iv. Find ∠ORP.
(A) 90° (B) 70° (C) 100° (D) 60°

v. If PQ=40m and OQ=30m then PO=
(A) 50m (B) 60m (C) 70m (D) 80m

CASE STUDY-5

ABCD is a playground. Inside the playground a circular track is present such that it touches AB at point P, BC at Q, CD at R and DA at S.



1. If DR = 5 m, then DS is equal to:

(A) 6 m (B) 11 m (C) 5 m (D) 18 m

2. The length of AS is:

(A) 18 m (B) 13 (C) 14 m (D) 12 m

3. The length of PB is:

(A) 12 m (B) 11 m (C) 13 m (D) 20 m

4. What is the angle of OQB?

(A) 60° (B) 30° (C) 45° (D) 90°

5. What is the diameter of given circle?

(A) 22 m (B) 33 m (C) 20 m (D) 30 m

CASE STUDY- 6

Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff. The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.

1. Find the length of AD

a) 7 b) 8 c) 5 d) 9

2. Find the Length of BE

a) 8 b) 5 c) 2 d) 9

3. Find the length of CF

a) 9 b) 5 c) 2 d) 3

4. If radius of the circle is 4cm, Find the area of ΔOAB

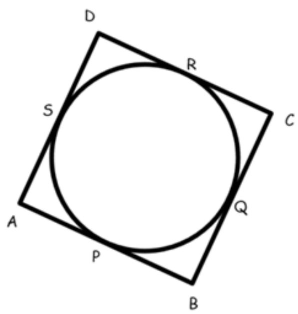
a) 20 b) 36 c) 24 d) 48

5. Find area of ΔABC

a) 50 b) 60 c) 100 d) 90

CASE STUDY-7

In a park, four poles are standing at positions A, B, C and D around the fountain such that the cloth joining the poles AB, BC, CD and DA touches the fountain at P, Q, Rand S respectively as shown in the figure. Based on the above information, answer the following questions.



i. If O is the Centre of the circular fountain, then ∠OSA =
(A) 60° (B) 90° (C) 45° (D) None of these

ii. Which of the following is correct?

(A) AS=AP (B) BP=BQ (C) CQ=CR (D) All of these

iii. If DR = 7 cm and AD = 11 cm, then AP =

(A) 4cm (B) 18cm (C) 7cm (D) 11cm

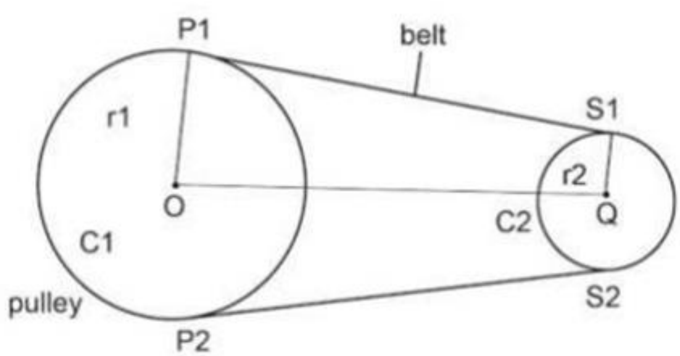
iv. If O is the Centre of the fountain, with ∠QCR = 60°, then ∠QOR

(A) 60° (B) 120° (C) 90° (D) 30°

v. Which of the following is correct?
(A) AB +BC=CD+DA (B) AB+ AD= BC + CD
(C) AB+CD = AD +BC (D) All of these

CASE STUDY- 8

Given below is the diagram of a pair of pulleys



C1 and C2 are two pulleys attached with a belt. O and Q are the centres of C1 and C2, respectively. P1 and P2 are points of contact, where the belt meets C1.S1 and S2 are points of contact, where belt meets C2

i. Identify the common tangents to the two circles (pulleys)?

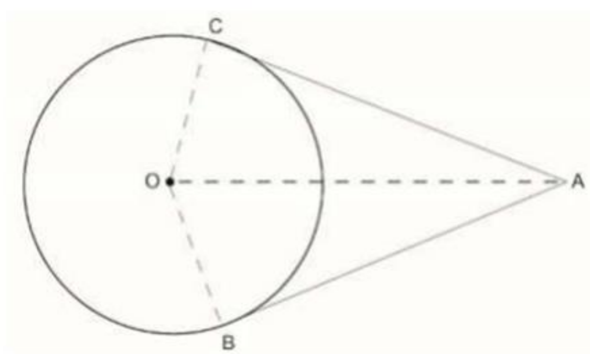
Ankit joins the centre of the two pulleys and observes line segments P1S1 and P2S2 when extended meet at a point X.

ii. What is the length OX when the diameter of C1 is 30cm, diameter of C2 is 10cm and length of OQ is 100cm?

iii. Which line segment is equal to the length P1S1?

a) OQ b) P1S1 c) QX d) XS2

Given below is the diagram of a pair of pulleys. The length of AC is 12 cm and radius is 5cm

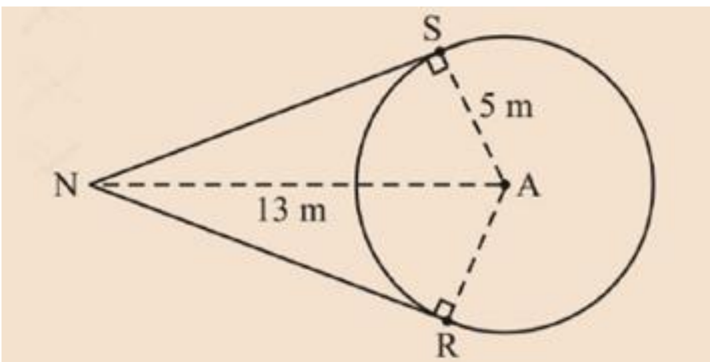


iv. What is the perimeter of the triangle ABO?

v. If in the given fig. $\angle CAB = 20°$, what is the measure of $\angle AOC$?

CASE STUDY -9

In an international school in Hyderabad organized an Interschool Throwball Tournament for girls just after the pre-board exam. The throw ball team was very excited. The team captains Anjali directed the team to assemble in the ground for practices. Only three girls Priyanshi, Swetha and Aditi showed up. The rest did not come on the pretext of preparing for pre-board exam. Anjali drew a circle of radius 5 m on the ground. The centre A was the position of Priyanshi. She marked a point N, 13 m away from centre A as her own position. From the point N, she drew two tangential lines NS and NR and gave positions S and R to Swetha and Aditi. Anjali throws the ball to Priyanshi, Priyanshi throws it to Swetha, Swetha throws it to Anjali, Anjali throws it to Aditi, Aditi throws it to Priyanshi, Priyanshi throws it to Swetha and so on.



i. What is the measure of ∠NSA?

I. a) 30 b) 45 c) 60 d) 90

ii. Find the distance between Swetha and Aditi

II. a) 8m b) 12m c) 15m d) 18 m

iii. How far does Priyanshi have to throw the ball towards Aditi?

III. a) 18m b) 15m c) 12m d) 8 m

iv. If ∠SNR is equal to , then which of the following is true?

IV. a) ∠ANS=90- b) ∠SAN=90- c) ∠RAN=90- d) ∠RAS=180-

v. If ∠SNR is equal to , then ∠NAS id equal to ?

V. a) 90- (θ/2) b) 180- 2θ c) 90- d) 90+

CASE STUDY-10

People of village want to construct a road nearest to the circular village Parli. The road

cannot pass through the village. But the people want the road should be at the shortest distance from the center of the village. Suppose the road start from point O which is outside the circular village and touch the boundary of the circular village at point A such that OA = 20 m. And also, the straight distance of the point O from the center C of the village is 25 m

i. Find the shortest distance of the road from the centre of the village

a) 15m b) 14m c) 13m d) 12m

ii. Which method should be applied to find the shortest distance?

a) Concept of tangent to a circle b) Pythagoras theorem

c)Both a and b d) None of these

iii. If a point is inside the circle, how many tangents can be drawn from that point

a) 0 b) 1 c) 2 d) 3

iv. Number of common tangents can be drawn to two circles which do not intersect

a) 2 b) 3 c) 4 d) 1

v. If we draw two tangents at the end of the diameter, these tangents are always

a) Parallel b) perpendicular c) coincident d) None of these

ANSWERS

MULTIPLE CHOICE QUESTIONS

QN. NO	CORRECT OPTION	QN. NO	CORRECT OPTION	QN. NO	CORRECT OPTION
1	B	11	D	21	C

2	C	12	A	22	C
3	C	13	B	23	D
4	B	14	B	24	C
5	C	15	A		
6	C	16	B		
7	B	17	B		
8	B	18	D		
9	D	19	C		
10	B	20	A		

ONE MARK QUESTIONS (OTHER THAN MCQs

QUES	ANSWER	QUES	ANSWER
24	One Point	29	150°
25	Radius	30	Parallelogram
26	False	31	Only one
27	7cm	32.	30°
28	Point of contact	33	10cm

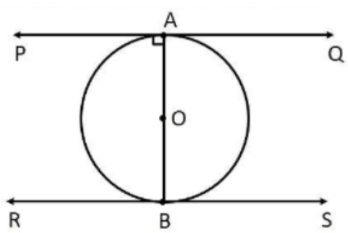
Very Short answer Questions

1. Consider the circle with centre at O
   PQ & RS are two parallel tangents to it touching at A and B respectively.
   Join OA and OB
   Now OA perpendicular to PQ (∴ radius is perpendicular to tangent)
   and OB perpendicular to RS
   ∴OA∥OB
   But OA and OB pass through O
   ∴AB is straight line through centre
   ∴AB is a diameter
2. $<OCD= 90^0$ (∵ radius is perpendicular to tangent at the point of contact)
   $<OCA +<ACD = 90^0$

<OAC + < ACD = 90^0 (∵ OC = OA , <OCA=<OAC)
<BAC + < ACD = 90^0

3. AS = AP ,,,,,,,,,,,,,,,,,,,,,,(i) (Length of tangents drawn from an external point to a circle are equal)
DS=DR……………..(ii)
CQ=CR…………….. (iii)
BQ=BP………………(iv)
Adding (i), (ii) ,(iii) and (iv) we get
AS + DS+ CQ + BQ = AP + DR + CR + BP
AD +BC = AB + CD

4.



Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.

Radius drawn to these tangents will be perpendicular to the tangents.

Thus, OA ⊥ PQ and OB ⊥ RS

∠OAP = 90º

∠OAQ = 90º

∠OBR = 90º

∠OBS = 90º

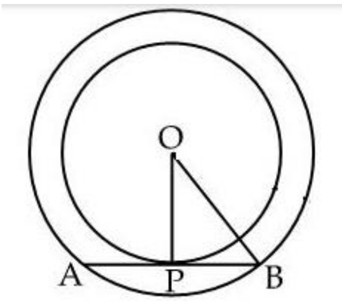
It can be observed that

∠OAP = ∠OBS (Alternate interior angles)

∠OAQ = ∠OBR (Alternate interior angles)

Since alternate interior angles are equal, lines PQ and RS will be parallel.

5.



OP = 4 cm, OB = 5 cm

We know that the radius is perpendicular to the tangent at the point of contact.

$\therefore \angle OPB = 90^\circ$

In right triangle OPB,

$OB^2 = OP^2 + PB^2$

$(5)^2 = (4)^2 + PB^2$

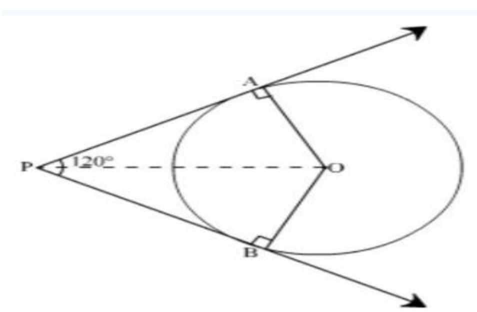
$PB^2 = 25 - 16 = 9$

PB = 3 cm

We know that perpendicular from the centre to the chord bisect the chord.

- AB = 2PB = 6 cm

6



In ΔOAP and ΔOBP,

OP = OP (Common)

∠OAP = ∠OBP (90°) (Radius is perpendicular to the tangent at the point of contact)

OA = OB (Radius of the circle)

∴ ΔOAP is congruent to ΔOBP (RHS criterion)

∠OPA = ∠OPB = 120°/2 = 60° (CPCT)

In ΔOAP,

Cos ∠OPA = cos 60° = AP/OP

Therefore, 1/2 =AP/OP

Thus, OP = 2AP

Hence, proved.

7. AB= AC (given)

ie AE +BE = AG + GC

BE = GC (Length of tangents drawn from an external point to a circle are equal)

BF = CF (∵ BE = BF and GC = CF)

8. Let AD= x cm

BD = 12 – x

BE = 12 – x

CE = 8 – (12 – x)

CE = x – 4 …….. (i)

AF = x

CF = 10 – x --------(ii)

From (i) and (ii) , we get

x- 4 = 10-x

x = 7 cm

AD = 7 cm

BE = 5cm

CF = 3cm

9. OPBQ is a square

Let AQ = x

So BQ = 29 -x , BP = 29 – x

AQ = AR = x , DR = DS = 23-x

i.e. 23-x = 5 gives x = 18 units

Radius of the circle = 29-x = 29-18 = 11cm

10. PE = CE = EQ (lengths of tangents from an external point to a circle are equal)

GF= CF = FH

Therefore, CF bisects PQ and GH

11. AQ = AB +BQ = AB + BP

AR = C R+ AC = CP + AC

AQ + AR = AB + BP + CP + AC

2AQ = AB + BC + AC

AQ = ½ (perimeter of triangle ABC)

12. $(x+2)^2 = (x+1)^2 + (x-6)^2$

$x^2 - 14x + 33 = 0$

$(x-11)(x-3) = 0$

x= 11

so OT = 5 units ,TP = 12 units , OP = 13 units

13. BP=BQ= 10cm

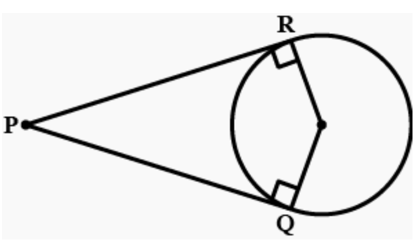
AQ = AQ = 8cm

CR = CP = x – 8 cm

x-8 = 6cm

there fore x = 14cm

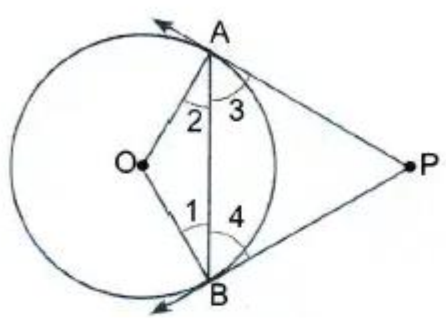
14.



Given : Tangents PR and PQ from an external point P to a circle with centre O.
To prove : Quadrilateral QORP is cyclic.
Proof : RO and RP are the radius and tangent respectively at contact point R.
$\therefore \angle PRO = 90^\circ$

Similarly ∠PQO=90°
In quadrilateral OQPR, we have
$\angle P+\angle R+\angle O+\angle Q=360^0$
$\Rightarrow \angle P+\angle 90^0+\angle O+\angle 90^0=360^0$
$\Rightarrow \angle P+\angle O=360^0-180^0=180^0$
These are opposite angles of quadrilateral QORP and are supplementary.
∴ Quadrilateral QORP is cyclic, hence, proved.

15.



Given: - A circle with centre O, PA and PB are tangents drawn at ends A and B on chord AB.

To prove: - ∠**PAB**=∠**PBA**
Construction: - Join OA and OB
Proof: - In △AOB, we have
OA=OB (Radii of the same circle)
∠OAB=∠OBA (Angles opposite to equal sides)
∠OAP=∠OBP=90 (∵Radius ⊥Tangent)
⇒∠PAB=∠PBA
Hence proved.

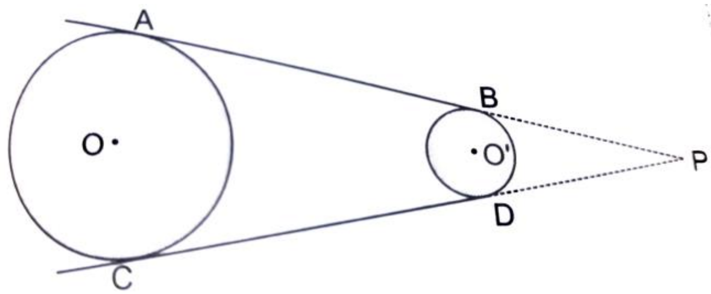
16. AOQ=2 ∠ABQ
So $\angle ABQ=58/2=29^0$
$\angle ATQ=180^0-\angle TAB-\angle ABT$
$=180^0-90^0-29^0$
$=61^0$

17. R = √(72+242)
=√(49+576)
=√625 =25cm

18. PA = PB
∠PAB= ∠PBA

∴ ΔPAB is an equilateral triangle.

Hence AB = PA = 5cm



19. Extend AB and CD to meet at P PA

= PC, PB = PD

PA – PB = PC – PD

AB = CD

20. RPQ= 50^0

OPQ = 40^0

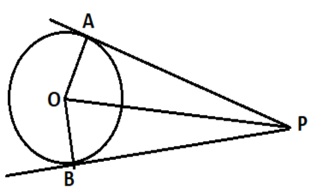
OP=OQ

∴ OPQ = OQP = 40^0

POQ = 100^0

Short Answer Questions

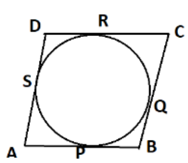
1. ΔAOP≅ ΔBOP, ∠APO=30° , use tan 30 in ΔAOP



2. AE=EC and DE=BE (lengths of tangents are equal)

AB=AE+EB = EC+DE=CD

3. AP=AS , BP=BQ, RC=CQ , DR=DS



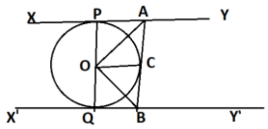
AB+DC=AP+PB+DR+RC=AS+BQ+DS+CQ =(AS+DS)+(BQ+CQ)=AD+BC

AB+AB=AD+AD

2AB=2AD ⇒ AB=AD ⇒ ABCD is a rhombus

4. ΔAPO≅ ΔACO and ΔOBC≅ ΔOBQ

∠AOP=∠AOC and ∠BOC=∠BOQ , use POQ as straight angle.

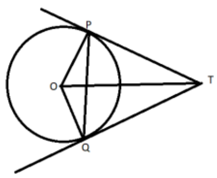


5. $\Delta PSO \cong \Delta PTO \Rightarrow \angle OPS = \angle OPT = 60°$

   Use cos 60° in ΔPSO

6. QSR=70°

7. PTQ= 180- POQ=180-(180-2 OPQ)=2 OPQ



8. $TP = \frac{20}{3}$ (PR=4cm , OR=3cm , $\Delta POR \sim \Delta TOP$ by AA criteria , use side proportionality)

9. PQ= 27 cm

10. Here, O is the centre of circle.

    PQ and PT are tangents to the circle from a point P

    R is any point on the circle. RT and RQ are joined.

    $\angle TPQ = 70°$

    Now,

    Join TO and QO

    $\angle TOQ = 180° - 70° = 110°$

    Here, OQ and OT are perpendicular on QP and TP.

    $\angle TOQ$ is on the centre and $\angle TRQ$ is on the rest part.

    $\angle TRQ = 1/2 \angle TOQ = 1/2(110°) = 55°$

    Therefore, $\angle TRQ = 55°$

11. Given, BOA is a <u>diameter of a circle</u>

$\angle OPT = 90°$, $\angle BPA = 90°$

$\angle PBA + \angle PAB + \angle BPA = 180°$

From the figure,

$\angle PBA = 30°$

$30° + \angle PAB + 90° = 180°$

$120° + \angle PAB = 180°$

$\angle PAB = 180° - 120°$

$\angle PAB = 60°$

We know that $\angle PAB = \angle OAP = 60°$

From the figure,

OP = OA = OB = radius

In triangle OPA,

$\angle OPA = \angle OAP$

Also, $\angle OPT = \angle OPA + \angle APT$

$90° = 60° + \angle APT$

$\angle APT = 90° - 60°$

$\angle APT = 30°$

Therefore, the measure of angle APT is equal to 30°

12. Joint OT.

Let it meet PQ at the point R.

Then ΔTPQ is isosceles and TO is the angle bisector of $\angle PTO$.

[$\because TP=TQ=$ Tangents from T upon the circle]

$\therefore OT \perp PQ$

$\therefore$OT bisects PQ.

PR=RQ=4 cm

Now, $OR^2=OP^2-PR^2=5^2-4^2$

OR=3 cm

Now, ∠TPR+∠RPO=90∘(∵TPO=90∘)

=∠TPR+∠PTR(∵TRP=90∘)

∴∠RPO=∠PTR

∴ Right triangle TRP is similar to the right triangle

PRO. [By A-A Rule of similar triangles]

∴ ⇒TP=20/3 cm.

13. Now join OB .
In right angle triangle OMB
$OB^2 = OM^2 + MB^2$ ....(i) (by pythagoras theorem)
we have , OM = 5cm and MB = 12cm
Put the given value in equation (i)
∴ $OB^2 = (5)^2 + (12)^2$

= 25 +144=169

$OB = \sqrt{169} = 13$c

we have , $OP^2 = OB^2 + PB^2$

∴ $OP^2 = (13)^2 + (20)^2$ (length of tangent = 20 cm given)

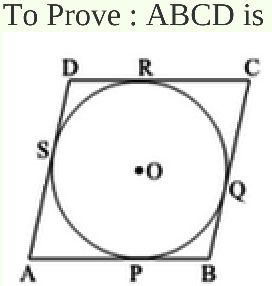
= 169 +400 = 509

$OP = \sqrt{509}$ =22.5 cm

Hence, the length of PO is 22.5 cm.

14. Given AB is tangent to the circle at A and OA is radius, OA ⊥ AB
In ΔABD
< DAB + < ABD + < ADB=180
90 + 50+ < ADB = 180
< ADB = 40
< AOC = < OCD + < ODC = 40 + 40 = 80

15. Given ABCD be a parallelogram circumscribing a circle with centre O.
To Prove : ABCD is a rhombus.

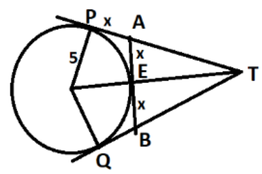


We know that the tangents drawn to a circle from an exterior point are equal is length.
∴ AP = AS, BP = BQ, CR = CQ and DR = DS.

AP+BP+CR+DR = AS+BQ+CQ+DS
(AP+BP) + (CR+DR) = (AS+DS) + (BQ+CQ)
∴ AB+CD=AD+BC
or 2AB=2AD (since AB=DC and AD=BC of parallelogram ABCD)
∴ AB=BC=DC=AD
Therefore, ABCD is a rhombus.

Long Answer Questions

1. Proof of theorem 10.2
2. $\Delta ADO^{I} \sim \Delta ACO \Rightarrow \frac{AO^{\iota}}{AO} = \frac{DO^{\iota}}{CO} \Rightarrow \frac{r}{3r} = \frac{DO^{\iota}}{CO}$
3. The other two sides are 13 cm and 15 cm. (Hint: use area of triangle)
4. ∠RQS=30°
6. ∠CQA=30° , ∠CBA=60°



7. AB = 6.6 cm (PT=12 cm, $TA^2=TE^2+EA^2 \Rightarrow (12-x)^2=64+x^2$)
8. i) 85°

   ii) No, opposite angles of a cyclic quadrilateral are supplementary

   iii) 1.5 times OB = 18cm

CASE STUDY-1 (PLAYGROUND)					
QUESTION	I	II	III	IV	V
ANSWER	(C) x m	(b)(3 − x) m	(d) Both b and c	(b) Quadrilateral BROQ is a square	(a) 1 m
CASE STUDY-2 (CIRCLE DRAWING)	(a) 10 cm	(b) 5 cm	(c) 16 cm	(c) 1 cm	(c) 3
ANSWER	(d) 10 cm	(c) 15 cm	(b) 9 cm	(d) 100°	(c) 11 cm
CASE STUDY-4 (FERRIS WHEEL)	(C) 150°	(A) 75°	(B) 75°	(A) 90°	(A) 50m

CASE STUDY-5 (PLAYGROUND)	(C) 5 m	(A) 18 m	(B) 11 m	(D) 90°	(A) 22 m
CASE STUDY-6 (SPORTS DAY T-SHIRT)	a) 7	b) 5	d) 3	c) 24	b) 60
CASE STUDY-7 (PARK)	(B) 90°	(D) All of these	(A) 4cm	(B) 120°	(C) AB+CD = AD +BC
CASE STUDY- 8	P1S1 and P2S2	150cm	(b) P2S2	30cm	140^0
CASE STUDY- 9	d) 90	b) 12m	c) 12m	d) ∠RAS=180-- θ	a) 90- (θ/2)
CASE STUDY-10	a) 15m	c) both a and b	a) 0	c) 4	a) Parallel

Areas related to Circles

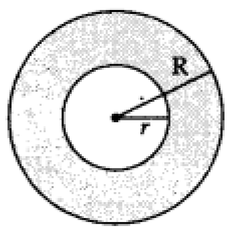
MCQ and CCT Questions

Summary

Circumference of a circle = $2\pi r$

Area of a circle = πr^2 [where r is the radius of a circle]
Area of a semicircle = $\pi r^2 / 2$

Area of a circular path or ring:



Let 'R' and 'r' be the radii of two circles
Then area of shaded part = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R + r)(R - r)$

Minor arc and Major Arc: An arc length is called a major arc if the arc length enclosed by the two radii is greater than a semi-circle.

If the arc subtends angle 'θ' at the centre, then the

Length of minor arc = $\frac{\theta}{360} \times 2\pi r = \frac{\theta}{180} \times \pi r$

Length of major arc = $(\frac{360-\theta}{360}) \times 2\pi r$

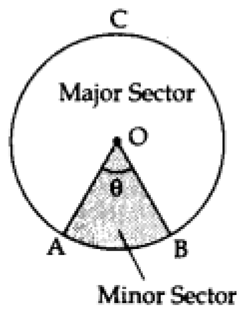
Sector of a Circle and its Area

(i) Sector is the region of the circle enclosed by the two radii and the arc between the two radii A sector is called a minor sector if the minor arc of the circle is part of its boundary.

O*A*B is minor sector.

Area of minor sector = $\frac{\theta}{360}(\pi r^2)$

Perimeter of minor sector = $2r + \frac{\theta}{360}(2\pi r)$



(ii) A sector is called a major sector if the major arc of the circle is part of its boundary.

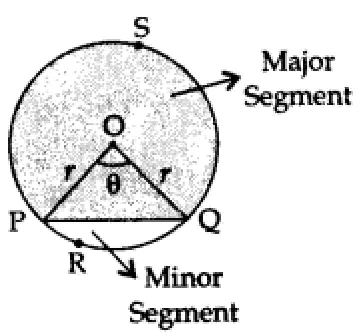
OACB is major sector

Area of major sector = $= (\frac{360-\theta}{360}) \times (2\pi r^2)$

Perimeter of major sector = $2r + (\frac{360-\theta}{360}) \times 2\pi r$

v. The sum of the arc lengths of major and minor sectors of a circle is equal to the circumference of the circle.

Minor Segment: The region enclosed by an arc and a chord is called a segment of the circle. The region enclosed by the chord PQ & minor arc PRQ is called the minor segment.



Area of Minor segment = Area of the corresponding sector – Area of the corresponding triangle

$$= \left[\frac{\theta}{360}\pi r^2 - \frac{1}{2}r^2 \sin\theta\right]$$

$$= \frac{1}{2}r^2\left[\frac{\theta}{180}\pi - \sin\theta\right] \quad \text{or} \quad \frac{1}{2}r^2\left[\frac{\theta}{180}\pi - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$$

Major Segment: The region enclosed by the chord PQ & major arc PSQ is called the major segment.

Area of major segment = Area of a circle – Area of the minor segment. [OR Area of major sector + Area of triangle]

$$= \pi r^2 - \frac{\theta}{360}\pi r^2 + \frac{1}{2}r^2 \sin\theta = r^2\left[\pi - \frac{\theta}{360}\pi + \frac{\sin\theta}{2}\right]$$

The sum of the areas of the major and minor sectors of a circle is equal to the area of the circle.

- Some useful results to remember;
- Angle described by minute hand in 60 minutes (1HOUR) = 360
- Angle described by minute hand in 1 minute = 6^0 (minute hand rotates through an angle of 6^0 in 1 minute)
- Angle described by the hour hand in 12 hours = 360
- Angle described by the hour hand in 1 hour = $\frac{360}{12}$ = 30
- Angle described by the hour hand in 1 minute = $\frac{30}{60} = \frac{1}{2}$

(hour hand rotates through($1/2)^0$ in 1 minute.

SECTION A

MCQ QUESTIONS AND VSA (1 Mark)

Q1. If θ is the angle in degrees of a sector of a circle of radius r units, then the area of the sector is

(a.) $\frac{\Pi r^2\theta}{360}$ (c.) $\frac{2\pi r\theta}{360}$

(b.) $\frac{\pi r^2\theta}{180}$ (d.) $\frac{2\pi r\theta}{180}$

Q2. Area of the largest triangle inscribed in a semi-circle of radius r units is

(a.) r^2 sq. units (c.) $2r^2$ sq. units

(b.) $\frac{1}{2}$ r^2 sq. units (d.) $\sqrt{2}\, r^2$ sq. units

Q3. If the circumference of a circle and the perimeter of a square are equal, then the

(a.) Area of the circle = Area of the square

(b.) Area of the circle > Area of the square

(c.) Area of the circle < Area of the square

(d.) We cannot definitely say about the relation between area of the circle and the square

Q4. Radii of two circles are 4 cm and 3 cm respectively. There is another circle, which is having area equal to the sum of the areas of two circles whose radii are known. Find the diameter (in cm) of the third circle.

(a.) 5
(c.) 0
(b.) 7
(d.) 14

Q5. Which ratio is denoted by a constant known as π

(a.) $\frac{Diameter}{Circumference}$
(c.) $\frac{Circumference}{Diameter}$
(b.) $\frac{Area}{Circumference}$
(d.) $\frac{Area}{Diameter}$

Q6. The minute hand of a clock is 14 cm long . The area described by it on the face of the clock in 5 minutes is

(a.) 51.33 cm^2
(c.) 21.15 cm^2
(b.) 15.33 cm^2
(d.) 12.35 cm^2

Q7. Find area of the largest circle that can be drawn inside a rectangle with length a cm and breadth b cm. (a > b).

(a.) $\frac{a^2\pi}{4}$ cm^2
(c.) $\frac{b^2\pi}{4}$ cm^2
(b.) $\frac{b^2\pi}{2}$ cm^2
(d.) $\frac{a^2\pi}{2}$ cm^2

Q8. The ratio of areas of two circles whose ratio of circumferences is in the ratio of 3 : 1 will be

(a.) 3 : 1
(c.) 1 : 9
(b.) 1 : 3
(d.) 9 : 1

Q9. Area of a square is same as area of a circle. What will be the ratio of their perimeters?

(a.) 1 : 1
(c.) $2 : \sqrt{\pi}$
(b.) $\pi : \sqrt{2}$
(d.) None of these

Q10. A display board is in the shape of a circle. While designing the board, if diameter of the board is increased by 40% from the previous design, then the area will be increased by

(a.) 40%
(c.) 96%
(b.) 80%
(d.) 45%

Q11. Find circumference of a circle whose area is 314 cm2. (Given π = 3.14)

Q12. State the following statement is "True" or "False".

"If the perimeter and area of a circle are numerically equal, then the radius of the circle is 2 units".

Q13. Find the area of a sector of a circle of radius 28 cm and central angle 45°. (Take $\pi = \frac{22}{7}$)

Q14. If the perimeter of a semi-circular protractor is 66 cm, find the length of the straight-line part of the protractor. (Take $\pi = \frac{22}{7}$)

Q15. Area of a sector is one- twelfth that of the complete circle. Find the angle of the sector.

Q16. An arc of a circle of length 5π cm bounds a sector whose area is 20π cm^2. Find the radius of the circle.

Q17. A chord of a circle of radius 10 cm subtends right angle at the centre of the circle. What will be the area of the corresponding major sector. (Given π = 3.14)

Q18. Rear wheel of a motorcycle is of radius 35 cm. It is assumed that the speed of the motorcycle fully depends on the rpm of the rear wheel and no loss of energy. How many revolutions per minute (rpm) must the wheel make so as to keep a constant speed of 66 km/hr. (Take $\pi = \frac{22}{7}$)

Q19. Find the area of a sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.

Q20. Say the following statement is "True" or "False". Write the reason for your answer.

"Area of a segment of a circle is less than the area of its corresponding sector."

SECTION B

SHORT ANSWER QUESTIONS (2 MARKS)

Q1. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

Q2. Find the diameter of the circle whose area is equal to the sum of the areas of two circles of diameters 20cm and 48cm.

Q3. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is 1256 cm^2.

Q4. A race track is in the form of a ring whose inner circumference is 352m and the outer circumference is 396m.Find the width of the track.

Q5. A bicycle wheel makes 5000 revolutions in moving 11km.Find the diameter of the wheel.

Q6. A wheel has diameter 84cm.Find how many complete revolutions it must take to cover 792m.

Q7. A car travels 1 km distance in which each wheel makes 450 complete revolutions. Find the radius of its wheels.

Q8. The perimeter of a sector of a circle of radius 5.2cm is 16.4cm. Find the area of the sector.

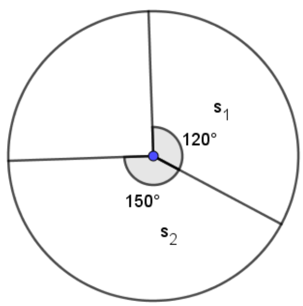
Q9. An arc of a circle is of length 5π cm and the sector it bounds has an area of $20\pi\ cm^2$.Find the radius of the circle.

Q10. The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand 9 A.M and 9.35 A.M.

Q11. The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 2 days.

Q12. If the perimeter of a sector of a circle of radius 6.5cm is 29cm, find its area.

Q13. Find the ratio of the areas of two sectors S_1 andS_2.



Q14. Find the area of a sector whose perimeter is four times its radius r units.

Q15. If the area of a circle inscribed in an equilateral triangle is given as 48π square units, then what is the perimeter of the triangle?

Q16. It is given that the area of a circle is equal to the sum of the areas of two circles of diameters 10cm and 24 cm.Then find the diameter of the larger circle.

Q17. A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60^0 at its centre. Find the radius of the circle.

Q18. Find the ratio of area of the circle circumscribing a square to the area of a circle inscribed in the square.

Q19. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the minor segment.

Q20. A ceiling fan has 3 wings. Find the length of the arc described between two consecutive wings, where length of each wing is 0.98 m.

SECTION C

SHORT ANSWER QUESTIONS (3 MARKS)

Q1. Calculate the perimeter of an equilateral triangle, if it is inscribed in a circle with area 154 cm^2.

Q2. In a circle of radius 21 cm, an arc subtends an angle of 60°at the centre. Find the area of sector formed by the arc.

Q3. A square is inscribed in a circle. Calculate the ratio of area of circle to that of square.

Q4. The chord of a circle of radius 10 cm subtends a right angle at its centre. Find the length of the chord. (Given $\pi = 3.14$)

Q5. The difference between circumference and radius of a circular field is 37 m. Find the area of the field. (Hint: $\pi = \frac{22}{7}$)

Q6. Four poles are erected at four corners of a rectangular field of dimensions 80 m by 50 m. Vasanthi tethered a cow at one corner of the field with a rope. After tying the length of rope from pole to cow is 7 m and Rajan tethered a buffalo at another pole of the same field and the length of rope from pole to animal is 6.3 m.

Answer the following questions.

i. How much area of the rectangular field did the cow graze?

ii. Find the ratio of grazing areas of the field by the cow and buffalo.

Q7. Diameter of a garden roller is 1.4 m. Find the cost of painting both circular faces of the roller at the cost of ₹ 120 per sq. m. (Take $\pi = \frac{22}{7}$)

Q8. Two circles touch externally. The sum of their areas is 130π cm^2. Distance between their centres is 14 cm, Find radius of each circle.

Q9. A square of diagonal 18 cm is inscribed in a circular plate. The square portion is cut using a LASER cutter and taken out. Find the area of the remaining portion of the circular plate.

Q10. A car has two wipers which do not overlap. Each wiper has a blade length of 25 cm and sweeps through an angle of 115°. What will be the total area of the glass wiped at each sweep of the wiper blades?

Q11. The difference between the radii of the smaller circle and the larger circle is 7 cm and the difference between the areas of the two circles is 1078 sq.cm. Find the radius of the smaller circle.

Q12. The central angles of two sectors of circles of radii 7cm and 21 cm are respectively 120^0 and 40^0.Find the areas of the two sectors as well as the length of the corresponding arcs. What do you observe?

Q13. In a circle with centre O and radius 5cm,AB is a chord of length$5\sqrt{3}$ cm. Find the area of sector AOB.

Q14. A chord AB of a circle of radius 10 cm makes a right angle at the centre of the circle. Find the area of the minor and major segments.

Q15. If the difference between the circumference and area of a circle is37 cm, find its area.

SECTION D

SHORT ANSWER QUESTIONS (4 MARKS)

Q1. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60cm, calculate the speed per hour with which the boy is cycling.

Q2. Two circles touch externally. The sum of their areas is 130πsq.cm and the distance between the centres is 14 cm. Find the radii of the circles.

Q3. Two circles touch internally. The sum of their areas is $116\ \pi\ cm^2$ and the distance between their centres is 6cm. Find the radii of the circles.

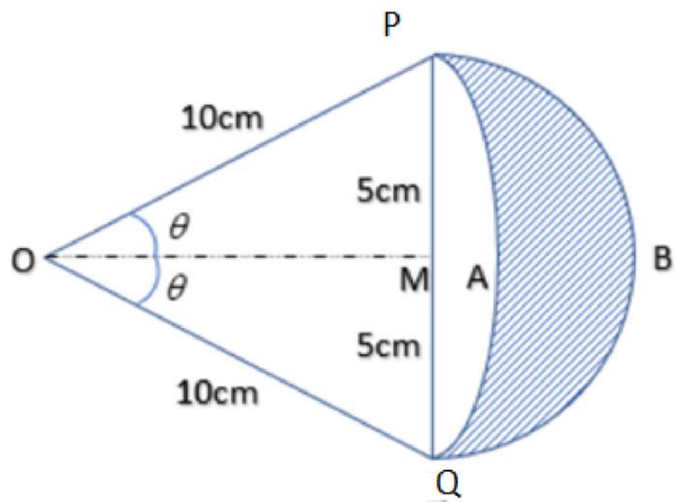
Q4. Find the difference of the areas of a sector of angle 120^0 and its corresponding major sector of a circle of radius 21 cm.

Q5. A chord of a circle of radius 10cm subtends a right angle at the centre.find

(1) area of the minor sector (2) area of the minor segment

(2) area of the major sector (4) area of the major segment

Q6. The figure given below shows two arcs A and B. Arc A is part of the circle with centre O and radius OP. Arc B is part of the circle with centre M and radius PM, where M is the midpoint of PQ. Show that the area enclosed by the two arcs is equal to 25 $[\sqrt{3} - \frac{\pi}{6}]\ cm^2$.



Q7. Find the difference of the areas of two segments of a circle formed by a chord of length 5cm subtending an angle of 90^0 at the centre.

Q8. Find the area of the segment of a circle of radius 15 cm, when the angle of the corresponding sector is 120^0.

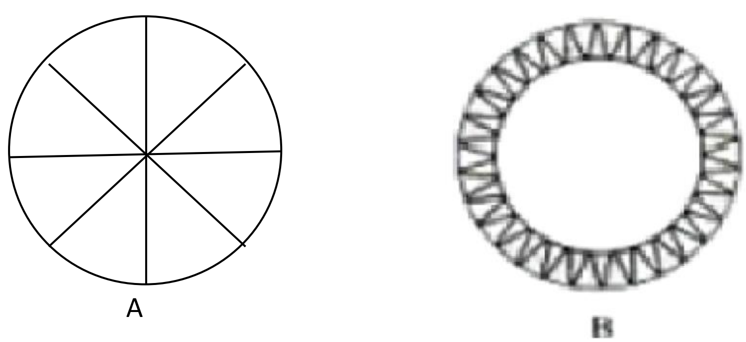
Q9. Find the area of the minor segment of a circle of radius 42 cm, if length of the corresponding arc is 44 cm.

Q10. The inner circumference of a circular track is 220 m.The track is 7m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of Rs.2 per metre.

CASE STUDY BASED QUESTIONS

CASE STUDY 1

Q1. A brooch is a small piece of jewellery which has a pin at the back so it can be fastened on a dress, blouse or coat. Designs of some brooch are shown below. Observe them carefully.



Design A; Brooch A is made with silver wire in the form of a circle with diameter 28 mm.The wire is used for making 4 diameters which divide the circle into 8 equal parts.

Design B; Brooch B is made up of 2 colours.Gold and Silver.Outer part is made with gold.The circumference of silver part is 44 mm and the gold part is 3 mm wide everywhere.

Refer to Design A

(i) Find the total length of silver wire required

(ii) Find the area of each sector of the brooch

(iii) REFER TO DESIGN B; Find the circumference of the outer part (golden)

(iv) A boy is playing with Brooch B; He makes revolutions with it along its edge. How many complete revolutions must it take to cover 80π mm?

CASE STUDY 2

In a Jewellery work shop, a brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig .

Q1. What is the radius of the circle?

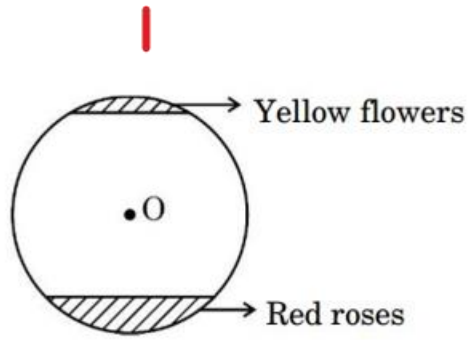
Q2. What is the circumference of the brooch?

Q3. What is the total length of silver wire required ?

Q4. What is the area of each sector of the brooch?

CASE STUDY 3

Flower beds look beautiful growing in gardens. One such circular park of radius ‘r’ m, has two segments with flowers. One segment which subtends an angle of 90° at the centre is full of red roses, while the other segment with central angle 60° is full of yellow-coloured flowers. (fig given below) (CBSE 2023



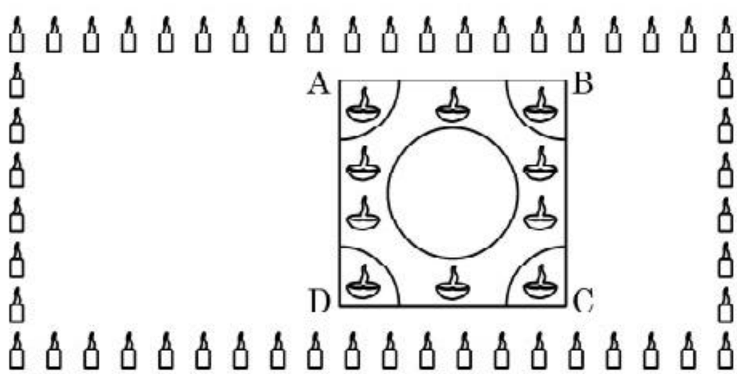
It is given that the combined area of the two segments (of flowers) is $256\frac{2}{3}\,sq\ m.$

Based on the above, answer the following questions:

i. Write an equation representing the total area of the two segments in terms of ‘r’.
ii. Find the value of ‘r’.
iii. Find the area of the segment with red roses.
iv. Find the area of the segment with yellow flowers

CASE STUDY 4

Inter school competitions was organized by one of the reputed schools of Odizza. The theme of the Rangoli competitions was Diwali celebration where students were supposed to make mathematical designs. Students from various schools participated and made beautiful Rangoli designs. One such design is given below. (CBSE 2023



Rangoli is in the shape of square marked as ABCD, side of square being 40cm. At each corner of a square , quadrants of circle of radius 10cm is drawn (in which diyas are kept).

Also a circle of diameter 20 cm is drawn inside the square.

i. What is the area of aquare ABCD?
ii. Find the area of circle.

iii. If the circle and the 4 quadrants are cut off from the square ABCD and removed, then find the area of the remaining portion of square ABCD.

iv. Find the combined area of 4 quadrants and the circle, removed

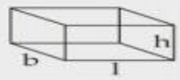
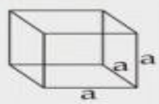
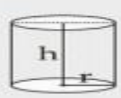
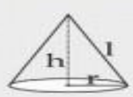
Short answer (1mark)			
1	A	11	62.8 cm
2	A	12	True
3	B	13	308cm^2
4	C	14	21cm
5	C	15	30
6	A	16	8cm
7	B	17	235.5cm^2
8	D	18	500rpm
9	C	19	8.7cm^2
10	C	20	false
SHORT ANSWER (2 MARKS)			
1	3.92 cm	11	1910.85 cm
2	d = 52 cm	12	52 sq.cm
3	800 sq.cm	13	4:5
4	7 m	14	r^2 sq.units
5	70 cm	15	48$\sqrt{3}$ units
6	300	16	26cm.
7	35.35 cm	17	$\frac{60}{\pi}$ cm
8	15.6 sq.cm	18	2:1
9	r = 8 cm	19	285.5
10	183.3 sq.cm	20	2.05 m

SHORT ANSWER (3 MARKS)					
1	423cm	6	38.5sq.cm, 100:81	11	r = 21 cm
2	231 sq.cm	7	Rs.369.6	12	154/3,154,44/3,44/3arc lengths of 2 circles of diff. Radii may be same but areas need not be equal.
3	Π :2	8	11cm and 3 cm	13	$\frac{25\pi}{3} cm^2$
4	102 cm	9	92.57 sq.cm	14	28.5 cm^2,285.5cm^2
5	157 sq.cm	10	1254.96 sq.cm	15	154cm^2

LONG ANSWER (4 MARKS)			
1	15.84 km/hr	6	$25\left[\sqrt{3} - \frac{\pi}{6}\right] cm^2$
2	11cm,3 cm	7	$\frac{25}{4}(\pi + 2)$
3	10 cm, 4cm	8	$75\pi - \frac{225}{4}\sqrt{3}\ cm^2$
4	462cm^2	9	160 sq.cm approx.
5	78.5,28.5,235.5,285.5	10	Rs.528

CASE STUDY 1	(1) 200M	(2) 77mm^2	(3) 82.2 mm	(4) 2
CASE STUDY 2	(1)35/2	(2) 110 mm	(3) 285 mm	(4) 385/4
CASE STUDY 2	(1) $\frac{1}{4}\pi r^2 - \frac{1}{2}r^2 + \frac{1}{6}\pi r^2 - \frac{\sqrt{3}}{4}r^2 = 256\frac{2}{3}$	(2)) $r^2 = \frac{770}{3} \rightarrow r = 26.1\ cm\ (approx)$	(3) 194.63 sq m	(4) $\frac{1}{6}\pi r^2 - \frac{\sqrt{3}}{4}r^2$
	1600 cm^2	$\frac{2200}{7}\ or\ 314.28cm^2$	$\frac{6800}{7}\ or\ 971.43cm^2$	$\frac{4400}{7}\ or\ 628.57cm^2$

MENSURATION : SURFACE AREA AND VOLUME

Name of the solid	Figure	Volume	Laterial/Curved Surface Area	Total Surface Area
Cuboid		lbh	2lh + 2bh or 2h(l+b)	2lh+2bh+2lb or 2(lh+bh+lb)
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3}\pi r^2 h$	πrl	$\pi rl + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$2\pi r^2+\pi r^2$ or $3\pi r^2$

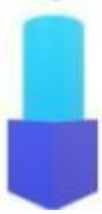
SURFACE AREAS AND VOLUMES OF COMBINATIONS OF SOLIDS

T. S. A. = T. S. A. (Cube) + C. S. A. (Cone) − Base Area (Cone)

Volume = Volume (Cube) + Volume (Cone)

T. S. A. = T. S. A. (Cube) + C. S. A. (Hemisphere) − Base Area (Hemisphere)

Volume = Volume (Cube) + Volume (Hemisphere)



T. S. A. = T. S. A. (Cube) + T. S. A. (Cylinder) − Base Area (Cylinder)

Volume = Volume (Cube) + Volume (Cylinder)

T. S. A. = C. S. A. (Cylinder) + C. S. A. (Cone) + Base Area (Cylinder)

Volume = Volume (Cone) + Volume (Cylinder)

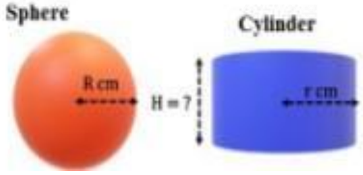
T. S. A. = C. S. A. (Cone) + C. S. A. (Hemisphere)

Volume = Volume (Cone) + Volume (Hemisphere)

CONVERSION OF SOLID FROM ONE SHAPE TO ANOTHER

Example -1:

Sphere Cylinder



Volume = V_1 Volume = V_2

$V_1 = V_2$

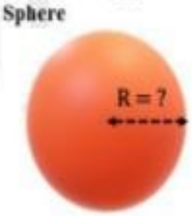
$\Rightarrow \frac{4}{3}\pi(R)^3 = \pi(r)^2 H$

Example -2:

Cone

Volume = V_1

Sphere



Volume = V_2

$V_1 = V_2$

$\therefore \frac{1}{3} \times \pi \times r^2 \times h = \frac{4}{3} \times \pi \times R^3$

Surface areas and volumes of combinations of solids of any two of the following:

cubes, cuboids, spheres, hemispheres and right circular cylinders/cones.

Deleted Topics

► Conversion of Solid from One Shape to Another

Problems involving converting one type of metallic solid into another and other mixed problems. (Problems with combination of not more than two different solids).

► **Frustum of a cone (Total surface area and volume of Frustum of a cone)**

SURFACE AREA AND VOLUMES

MULTIPLE CHOICE QUESTIONS(1 Mark each)

SECTION A

LEVEL 1

Q1. The ratio of the total surface area of a solid hemisphere to the square of its radius is

a) $2\pi : 1$
c) $4\pi : 1$
b) $3\pi : 1$
d) $1 : 4\pi$

Q2. Two cubes each of volume $8cm^3$ are joined end to end , then the surface area of the resulting cuboid is

a) $80cm^2$
c) $40cm^2$
b) $64cm^2$
d) $8cm^2$

Q3. The radius of a sphere is r cm. The sphere is divided into two equal parts. The whole surface area of two parts will be:

a) $8\pi r^2$
c) $4\pi r^2$
b) $6\pi r^2$
d) $3\pi r^2$

Q4. If the radius of the base of a right circular cylinder is halved, keeping the height same, the ratio of the volume of the reduced cylinder to that of original cylinder is

a) 2 : 3
c) 1 : 4
b) 3 : 4
d) 4 : 1

Q5. The surface area of the two spheres are in the ratio 1 : 2. The ratio of their volumes is :

a) $\sqrt{2}: 1$
b) $1 : 2\sqrt{2}$
c) 1 : 8
d) 1 : 4

Q6. If the areas of three adjacent faces of a cuboid are X, Y and Z respectively, then the volume of cuboid is :

a) XYZ
b) 2XYZ
c) $\sqrt{XYZ}$
d) $\sqrt{2XYZ}$

Q7. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Ratio of their volumes is

a) 27 : 20
b) 20 : 27
c) 9: 4
d) 4 : 9

LEVEL 2

Q8. The radius of a wire is decreased to one third. If the volume remains the same, the length will become

a) 3 times
b) 6 times
c) 9 times
d) 27 times

Q9. The ratio of the volumes of two spheres is 8 : 27 . If r and R are the radii of spheres respectively then (R – r) : r is :

a) 1 : 2
b) 1 : 3
c) 2 : 3
d) 4 : 9

Q10. The circumference of the edge of a hemispherical bowl is 132 cm. When π is taken as **22/7** the capacity of the bowl in cm^3 is :

a) 2772
b) 924
c) 19404
d) 9702

OBJECTIVE QUESTIONS (1 MARK)

Q11. The surface area of a sphere is same as the curved surface area of a right circular cylinder whose height and diameter are 12 cm each. Find the radius of the sphere.

a) 4 cm
b) 8cm
c) 5cm
d) 6cm

Q12. Find the volume of the greatest sphere that can be cut from a cylindrical log of wood of base radius 1 cm and height 5 cm.

a) $\frac{4}{3}\pi$
c) $\frac{1}{3}\pi$
b) $\frac{2}{3}\pi$
d) $\frac{3}{4}\pi$

Q13. Find the curved surface area of a right circular cone of height 15 cm and base diameter 16 cm.

a) $163\ \pi cm^2$
c) $136\ \pi cm^2$
b) $256cm^2$
d) $136cm^2$

Q14. A cone and a hemisphere have equal bases and equal volumes. What is the ratio of their heights?

a) 1:2
c) 2:3
b) 2:1
d) 3:2

LEVEL 3

Q15. Find the volume of a right circular cylinder of base radius 7 cm and height 10 cm .

a) 1240 cm^3
c) 1450cm^3
b) 1405cm^3
d) 1540cm^3

Q16. If h, c and V respectively are the height, curved surface area and volume of a cone then find $3\pi Vh^3 - c^2h^2 + 9V^2 =$ ………………

a) 0
c) 5
b) 2
d) 6

Q17. How many bags of grain can be stored in a cubic granary 12m x 6m x 5m , if each bag occupies a space of 0.48 m^3 ?

a) 580
c) 750
b) 570
d) 740

Q18. The volume of two cubes are in the ratio 8 : 64, then find the ratio of their surface areas .

a) 2:3
c) 3:2
b) 4:9
d) 9:4

Q19. A cylinder and a cone are of same base radius and of same height. What is the ratio of their volumes?

a) 3:1
c) 2:3
b) 1:3
d) 3:2

Q20. Find the Total Surface Area of a hemispherical solid having radius 7 cm.

a) 624
b) 642
c) 426
d) 462

SHORT ANSWER TYPE QUESTIONS – 2 MARKS

SECTION-B

LEVEL 1

Q21. Two cubes each of volume27cm^3 are joined end to end to form a solid. Find the surface area of the solid.

Q22. 22.Two cubes each of side 4cm are joined end to end. Find the volume of the resulting solid.

Q23. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

Q24. If the total surface area of a solid hemisphere is 462 cm^2, find its radius.

Q25. A wallpaper, 312m long and 25cm wide is required to cover the walls of a room. Length of the room is 7m and its breadth is twice its height. Determine the height of the room.

Q26. The surface area of a sphere is 616cm^2. Find its radius.

Q27. The radii of 2 cylinders are in the ratio 3:5 and their heights are in the ratio 2:3. What is the ratio of their curved surface areas.

Q28. 28.The base radii of 2 right circular cones of the same height are in the ratio 3:5. Find the ratio of their volumes.

Q29. The circumference of the base of a 9m high wooden solid cone is 44m. Find its volume.

Q30. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9cm.

LEVEL 2

Q31. A toy is in the form of a cone mounted on a hemi-sphere of same radius. The diameter of the base of the conical part is 7cm and the total height of the toy is 14.5cm. find the volume of the toy.

Q32. The TSA of a solid cylinder is 231cm^2. If its CSA is $\frac{2}{3}$of its TSA. Find its radius and height.

Q33. The length of a hall is 20m and width is 16m. the sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height of the hall.

Q34. A cone and a cylinder of same radius 3.5cm have same CSA. If height of the cylinder is 14cm then find the slant height of the cone.

LEVEL 3

Q35. A circus tent is cylindrical up to a height of 3m and conical above it. If the diameter of the base is 105m and the slant height of the conical part is 53cm, find the total canvas required in making the tent.

Q36. A bird-bath in a garden is in the shape of a cylinder with a hemi-spherical depression at one end. The height of the hollow cylinder is 1.45m and its radius is 30cm. find the TSA of the bird-bath.

Q37. A tent is in the shape of a cylinder of diameter 20m and height 2.5cm, surmounted by a cone of equal base and height 7.5m. find the capacity of the tent.(take $\pi = 3.14$)

Q38. A vessel in the shape of a hollow hemi-sphere mounted by a hollow cylinder. The diameter of the hemi-sphere is 14cm and the total height of the vessel is 13cm. find the inner surface area of the vessel.

Q39. A conical vessel whose inner radius is 10cm and height 48cm is full of water. Find the volume of water in it.

Q40. Fifty circular plates each of radius 7cm and thickness 0.5cm are placed one above another to form a solid right circular cylinder. Find its TSA.

SHORT ANSWER TYPE QUESTIONS – 3 MARKS

SECTION-C

LEVEL 1

Q41. A toy is in the form of a cone mounted on a hemisphere of same radius 7 cm. If the total height of the toy is 31 cm, find its total surface area.

Q42. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.

Q43. A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them is 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Q44. A solid cylinder of radius r and height h is placed over another cylinder of same height and radius. Find the total surface area of the shape so formed

Q45. An ice - cream cone consists of a cone surmounted by a hemisphere. The radius of the hemisphere is 3.5 cm and height of the ice - cream cone is 12.5 cm. Calculate the volume of the ice – cream in the cone.

LEVEL 2

Q46. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder

Q47. The radius and height of a solid right circular cone are in the ratio of 5 : 12. If its volume is 314 cm^3, find its total surface area. (Use $\pi = 3.14$)

Q48. A cone of maximum size is carved out from a cube of edge 14 cm. Find the surface area of the solid left out after the cone is carved out.

Q49. The largest possible cylinder is cut out from a wooden cube of edge 8 cm. Find the volume of wood remaining in the cube.

Q50. A semi-circular sheet of paper of diameter 28 cm is bent into an open conical cup. Find the depth and capacity of the cup.

LEVEL 3

Q51. A solid sphere of diameter 14 cm is cut into two halves by a plane passing through the centre. Find the combined surface area of the two hemispheres so formed.

Q52. If the radius of the base of a right circular cylinder is halved, keeping the height the same, find the ratio of the volume of the reduced cylinder to that of the original cylinder.

Q53. From a solid cylinder of height 14 cm and base radius 7 cm, two identical conical holes from each end of radius 2.1 cm and height 4 cm are drilled out. Find the volume of the remaining solid.

Q54. A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with a conical end of length 9 cm. Determine the capacity of the tank.

Q55. A rocket is in the form of a cylinder, closed at the lower end, and has a cone attached to its top. If each one has a radius 20 cm and height 21 cm, find the surface area of the rocket.

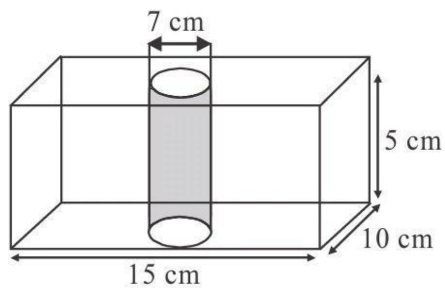
LONG ANSWER QUESTIONS (5 MARKS)

LEVEL 1

Q56. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

Q57. A tent is in the shape of a right circular cylinder up to a height of 300 cm and conical above it. The total height of the tent is 1350 cm and radius of its base is 1400cm. Find the cost of cloth required to make the tent at the rate of Rs.80 per square metre. (Take π= 22/7)

Q58. From a cuboidal solid metallic block of dimensions 15cm x 10cm x 5cm a cylindrical hole of



diameter 0.07m is drilled out. Find the surface area of the remaining block. (π= 22/7)

LEVEL 2

Q59. A hollow cylindrical pipe is made up of copper. It is 21 dm long. The outer and inner diameters of the pipe are 10cm and 6cm respectively. Find the volume of copper used in making the pipe (π= 22/7)

Q60. A circus tent is in the form of a right circular cylinder with a right circular cone above it. The

i. diameter and the height of the cylindrical part of the tent are 126m and 12m respectively. The total

ii. the height of the tent is 28m. Find the total cost of the tent if the canvas used costs Rs.30 per sq.m.

Q61. A right circular cylinder and a cone have equal bases and equal heights. If their curved surface areas are in the ratio 8:5, show that the ratio between the radius of their bases to their heights is 3:4

LEVEL 3

Q62. A metallic cylinder has radius 3cm and height 5cm. To reduce its weight, a conical hole is drilled in the cylinder. The conical hole is drilled in the cylinder. The conical hole has a radius of 3/2 cm. and its depth is 8/9cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in the conical shape.

Q63. A rectangular sheet of paper 30cm x 18cm can be transformed into the curved surface of a right circular cylinder in two ways either by rolling the paper along its length or by rolling it along its breadth. Find the ratio of the volume of the two cylinders thus formed.

Q64. The internal and external diameters of a hollow hemispherical vessel are 12cm and 16cm respectively. If the cost of painting 1 sq.cm of the surface area is Rs. 5, find the total cost of painting the vessel all over. ($\pi = 3.14$)

Q65. The sum of the radius of the base and height of a solid right circular cylinder is 37cm. if the total surface area of the solid cylinder is 1628 sq.cm, find the volume of the cylinder. ($\pi= 22/7$)

ASSERTION REASONING QUESTIONS

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c) Assertion (A) is true but reason (R) is false.

d) Assertion (A) is false but reason (R) is true.

1. Statement A (Assertion): Total Surface area of the top is the sum of the curved surface area of the hemisphere and the curved surface area of the cone.

   Statement R(Reason) : Top is obtained by joining the plane surfaces of the hemisphere and cone together.

2. Assertion (A): In a right circular cone, the cross- section made by a plane parallel to the base is a circle.

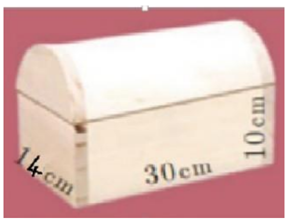
   Reason (R): If the volume and the surface area of a solid hemisphere are numerically equal, then the diameter of the hemisphere is 9 units.

3. Assertion (A): If the volumes of two spheres are in the ratio 64: 27, then the ratio of their surface areas is 4:3

Reason (R): If the surface areas of two spheres are in the ratio 16:9, then the ratio of their volumes is 64:27.

CASE STUDY BASED QUESTIONS

Case study question 1 An antique box and its dimensions excluding the stand is given below.



1. What is the volume of the jewellery box?

   a) $(l \times b \times h) + \pi r^2 h$
   b) $(l \times b \times h) + \frac{1}{2}\pi r^2 h$
   c) $2(lb \times bh \times lh) + \pi r^2 h$
   d) $(l \times b \times h) + 2\pi r h$

2. How much brass will be needed to plate the curved surface of the dome as shown in figure?

   a) 1320cm^2
   b) 220 cm^2
   c) 440 cm^2
   d) 660 cm^2

3. How many sheets of dimensions 14cm x 30cm x 2cm can be placed in the box?

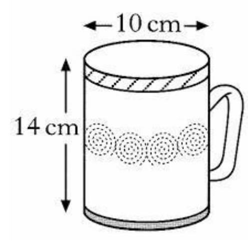
   a) 10
   b) 5
   c) 2
   d) 15

4. Considering the thickness of the box to be negligible, how much velvet cloth will be needed to cover the cuboidal inner area?

   a) 1720 cm^2
   b) 880 cm^2
   c) 1300 cm^2
   d) 1580 cm^2

Case study question- 2

During Covid times people prefer using homogenized milk, UHT Processed and aseptically packed in an exceptional six layer, tamper-proof Tetra Packaging with 0% bacteria and 100% pure health. This new six layer interfere proof, prevents air and freshness, light and bacteria from entering the pack. As an effect, the milk stays fresh and pure for a minimum of 180 days until opened, even without refrigeration.The 500ml milk is packed in cuboidal containers of dimensions 15 x 8 x 5 . These milk packets are then packed in cuboidal cartons of dimension 30x 32 x 15.(All dimensions are in **cm)**



Based on the above given information answer the following questions

1) Find the total surface area of a milk box.

a) $1890cm^2$ c) $470cm^2$

b) $400\ cm^2$ d) $600\ cm^2$

2) How many milk packets can be filled in a carton?

a) 12 c) 20

b) 24 d) 8

3) How much milk will the cup contain?

a) 1200 L c) 11 L 10 ml

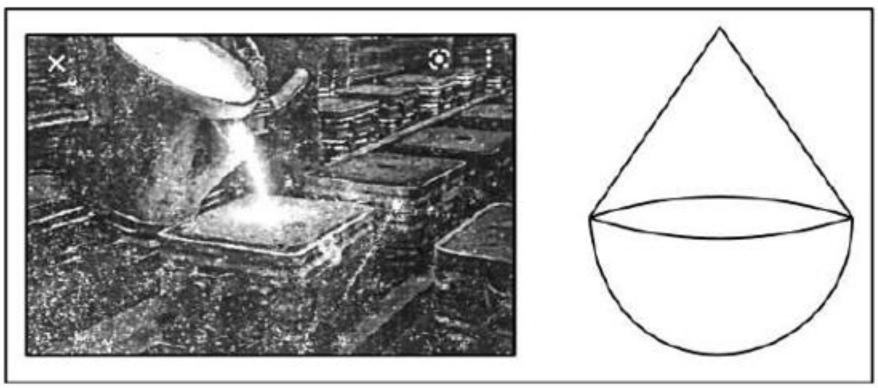
b) 1 L 100ml d) 100 L

4) How much cardboard is needed to make the carton if 10% of wastage is taken into account.

   a) 3310 cm^2
   b) 2100 cm^2
   c) 4200 cm^2
   d) 3969 cm^2

Case study question- 3

A company deals in casting and moulding of metal on orders received from its clients.
In one such order, company is supposed to make 50 toys in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of hemisphere. If the radius of the base of the cone is 21 cm and height is 28 cm, (CBSE 2022



a) Find the volume of 50 toys
b) Find the ratio of the volume of hemisphere to the volume of the cone.

Case study question- 4

John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions: (CBSE 2022

Cake: Cylinderical shape with diameter 24 cm and height 14 cm.

Cap: Conical shape with base circumference 44cm and height 24 cm.

Based on the above information, answer the following questions:

i. How many square cm paper would be used to make 4 such caps?

ii. The bakery shops sells cakes by weight(0.5 kg, 1 kg, 1.5 kg …). To have the required dimensions, how much cake would they order, if 650 cm^3 equals 100 g of cake?

SECTION-A ANSWER KEY

Question	Answer	Question	Answer
1	b) $3\pi : 1$	11	6 cm
2	c) 40 cm^2	12	$\frac{4}{3}\pi$
3	b) 6πr2	13	136π
4	c) 1 : 4	14	2 : 1
5	b) $1 : \mathbf{2\sqrt{2}}$	15	1540 cm^3
6	c) $\sqrt{XYZ}$	16	0
7	b) 20 : 27	17	750
8	c) 9 times	18	4 : 9
9	a) 1 : 2	19	3 : 1
10	c) 19404	20	462

SECTION-B

1. 90cm^2	6) 7cm	11) 231cm^3	16) 3.3m^2
2) 128cm^3	7) 2:5	12) 3.5cm, 7cm	17) 1570m^2
3) 9units	8) 9:25	13) 8.8m	18) 572cm^2
4) 7cm	9) 462cm^3	14) 28cm	19) 5024cm^2
5) 3m	10) 190.93cm^3	15) 9735m^2	20) 1408cm^2

SECTION-C

1. 858 cm^2	6. 4620 cm^3	11. 924 cm^2
2. 854 $\frac{6}{7}$ cm^2	7. 266.9 cm^2	12. 1: 4
3. 166.83 cm^3	8. 1365.2 cm^2	13. 2119.04 cm^3
4. $2\pi r^2$ + 4πrh sq. units	9. 109.8 cm^3	14. 29106 cm^3
5. 205.33 cm^3	10. 718$\frac{2}{3}$ cm^3	15. 5720 cm^2

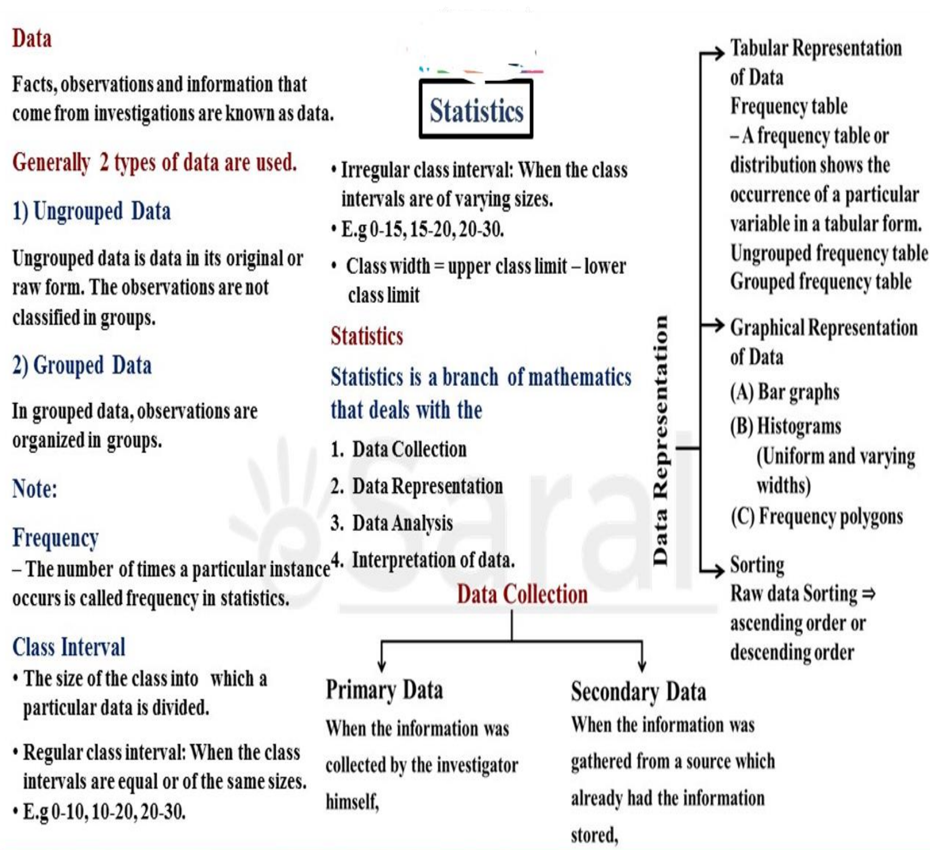
SECTION-D

1. 1:$\sqrt{3}$	6) 3:4
2. Rs.82720	7) 133: 2
3. 583sq.cm	8) 5:3
4. 10560cub.cm	9) Rs. 3579.60

5.Rs.528660	10) 4620 cub.cm
CASE STUDY-1	**CASE STUDY-2**
1. (b) $(l \times b \times h) + \frac{1}{2}\pi r^2 h$	1 b)470cm^2
2 (d) 660 cm^2	2. c) 24
3 (b) 5	3. c) 1 L 100ml
4 (c) 1300 cm^2	4. a) 3310 cm^2
CASE STUDY 3	CASE STUDY 4
a) Volume of 1 toy = 32340cm^3 Volume of 50 toys = 1617000 cm^3 b) Ratio = 3:2	$l = 25cm; Paper\ required$ $= 2200cm^2$ V of cake = 6336 cm^3 = 6500 cm^3 (approx.) 650 cm^3 = 100g ⇒ 6500cm^3 =1kg cake should be ordered

STATISTICS

MIND MAP



ARITHMETIC MEAN

- Direct Method $\underline{x} = \dfrac{\Sigma \quad f_i x_i}{\Sigma \quad f_i}$
- Assumed Mean Method $\underline{x} = a + \dfrac{\Sigma \quad f_i d_i}{\Sigma \quad f_i}$
- Step Deviation Method $\underline{x} = a + \dfrac{\Sigma \quad f_i u_i}{\Sigma \quad f_i}$

MODE

COMPUTATION OF MODE FOR A CONTINOUS FREQUENCY DISTRIBUTION

Algorithm

1. Obtain the continuous frequency distribution
2. Determine the class of maximum frequency either by inspection or by grouping method
3. This class is called the modal class
4. Obtain the values of the following from the frequency distribution table

$$l = lower\ limit\ of\ the\ modal\ class$$

$$f_1 = frequency\ of\ modal\ class$$

$h = \ width(size)$ of the modal class

f_0
$= \ frequency\ of\ the\ class\ preceding\ \ the\ modal\ class$

$f_2 = frequency\ of\ the\ class\ following\ \ the\ modal$ class

$$\text{Mode} = \ l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)\ h$$

MEDIAN OF GROUPED DATA

Algorithm

1. Obtain the frequency distribution
2. Prepare the cumulative frequency column
3. Obtain n = $(\sum \quad f_i)$ and $\frac{n}{2}$
4. See the cumulative frequency just greater than (nearer to) $\frac{n}{2}$ and determine the corresponding class. This class is known as *median class*
5. Obtain the values of the following from the frequency distribution table

$$l = lower\ limit\ of\ the\ median\ class$$

$$f = frequency\ of\ median\ class$$

$$h = \ width(size)\ of\ the\ median\ class$$

$$cf = \ cumulative\ frequency\ of\ the\ class\ preceding\ the\ median\ class$$

Substitute the values in the following formula

$$Median \ = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \text{h}$$

THE EMPIRICAL RELATIONSHIP BETWEEN THE THREE MEASURES OF CENTRAL TENDENCY

$$3\,median = \ mode\ + \ 2\,m$$

MULTIPLE CHOICE QUESTIONS (1 MARK)

SECTION A

Level-1

I. Choose the correct answer from the following: (1 mark each)

1) Which is the empirical relation between Mean ,Median and Mode

a) 3Mean =Mode +2Median
b) 3Median=Mode +2Mean
c) 2Median= Mode +3Mean
d) 3Median=Mode -2Mean

2) For the following distribution,

Class Interval	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The sum of lower limits of Median class and Modal class is

3) If Mode of data 64, 60, 48, x, 43,48,43,34 is 43 then x+3 is
a) 44
b) 45
c) 46
d) 48

Level-2

4) The Arithmetic Mean of 1,2, 3,4, ….. n is
a) $\frac{n+1}{2}$,
c) $\frac{n-1}{2}$
b) $\frac{n}{2}$
d) $\frac{n}{2}$ +1

5) If Arithmetic Mean of $x, x+2\,, x+4\ and\ x+6$ is 5. Then find the value of x
a) 3
c) 1
b) 2
d) 5

6) The algebraic sum of deviation of frequency distribution from its mean is
a) 0
c) -1
b) 1
d) 2

LEVEL-3

7) Mean of the following distribution is 2.5. Find the value of ‘y’

Variable x	1	2	3	4	5
Frequency y	4	5	y	1	2

a) 3
b) 4
c) 5
d) 2

8) If Median of data $16, 18, 20, 24 - x,\ 20 + 2x,\ 28, 30, 32$ is 24then x is

a) 4
b) 18
c) 16
d) 20

9) If Mean of first n natural number is $\frac{5n}{9}$ then n is

a) 5
b) 4
c) 9
d) 10

10) The Mean of five number is 15. If we include one more number, the mean of 6 numbers become 17. The included number is

a) 24
b) 25
c) 26
d) 2

II. Answer the following (1 mark each)

LEVEL-1

1. Find the median of first 9 prime numbers.
2. The mean and median of the data are 14 and 15. Find the value of mode.
3. Find the lower limit of the modal class:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	13	7	6

4. Find the frequency of class 30-40.

More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

5. Find the class mark of the class 10-25.

Level-2

6. If $u_i = \frac{xi-25}{10}$, $\sum f_iu_i$ =20 and $\sum f_i$ =100, then find mean.
7. If arithmetic mean of 7, 8, x, 11, 14, is x, find x.
8. Find the mean of first n odd natural numbers

Level-3

9. If the difference of mode and median of the data is 24, then find the difference of median and mean

ASSERTION REASONING QUESTIONS

DIRECTION : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

e) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
f) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
g) Assertion (A) is true but reason (R) is false.
h) Assertion (A) is false but reason (R) is true.

Q1. Assertion(A) - For a moderately asymmetric distribution,

Mode – Median = 2 (Median-Mean)

Reason (R) - For a symmetric distribution, Mean = Median = Mode

Q2. Assertion (A) - The mean of 1, 4, 7, 10, ……..301 is 151.

Reason (R) - The mean of the series $a, a+d, a+2d, \ldots\ldots\ldots\ldots a+2nd, is\ a+nd.$

Q3. Assertion (A) – If the number of runs scored by 11 players of a cricket team of India are 5,19,42,11,50,30,21,0,52,36,27 then median is 30.

Reason (R) - Median= ((n+1)/2)th value if n is odd.

Q4. Assertion (A) - If the value of mode and mean is 60 and 66 respectively.,then the value of median is 64.

Reason (R) – Median = 1/2 (mode + 2mean)

Short Answer Question(2 Marks questions)
SECTION B

Level-1

1. Find the mode of the fallowing data:

CI	1-3	3-5	5-7	7-9	9-11
F	7	8	2	2	1

2. From the following distribution, find the lower limit of the median Class:

CI	85-89	90-94	95-99	100-104	105-109
F	10	12	11	5	30

3. Find the mean of first five odd multiples of five.
4. Find the mean, if $d_i = x_i - 25$; $\sum f_i d_i = 20$; $\sum f_i = 100$.
5. Find the mean of the Following distribution:

Class	3-5	5-7	7-9	9-11	11-13
frequency	5	10	10	7	8

6. If the mean of first n natural numbers is 15, then find n.
7. Find the mode of following frequency distribution:

Class	10-20	20-30	30-40	40-50	50-60
Frequency	15	10	12	17	4

8. Find the mean of the data using an empirical formula when it is given that mode is 50.5 and median is 45.5.

Level-2

9. Find the value of x, if the mode of following distribution is 45.

CI	0-20	20-40	40-60	60-80	80-100
F	5	10	x	6	3

10. An inter house cricket match was organised by a school. Distribution of run made by the students is given below. Find the median runs scored.

Runs Scored	0-20	20-40	40-60	60-80	80-100
No. of Students	4	6	5	3	4

Level-3

11. Given below is a cumulative frequency distribution showing the marks secured by 50 students:

Marks	Below 20	Below 40	Below 60	Below 80	Below 100
No. of Students	17	22	29	37	50

Form frequency distribution table for the data.

12. The mean and median of 100 observations are 50 and 52 respectively. The value of the largest observation is 100. It was later found that it is 110 not 100. Find the true mean and median.

13. If the median of a series exceeds the mean by 3, find by what number the mode exceeds its mean?

14. Find the value of p, if the arithmetic mean of the following distribution is 25:

CI	0-10	10-20	20-30	30-40	40-50
F	5	8	15	p	6

15. Find the x and y from the following cumulative frequency distribution:

Class	0-8	8-16	16-24	24-32	32-40
frequency	15	x	15	18	9
Cumulative frequency	15	28	43	y	70

16. Calculate the median from the fallowing data:

CI	0-10	10-20	20-30	30-40	40-50
F	5	15	30	8	2

17. In a frequency distribution, if a = assumed mean =55, $\sum f_i$ =100, h=10 and $\sum f_i u_i$=-30 then find the mean of the distribution.

18. Change the following distribution into a less than type distribution table:

CI	200-300	300-400-	400-500	500-600	600-700
F	12	18	35	20	15

19. Change the following distribution into a more than type distribution table.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

20. For the following distribution, find the modal class:

Marks	Below10	Below20	Below30	Below40	Below50	Below60
No. of Students	3	12	27	57	75	80

SHORT ANSWER TYPE QUESTION (3 MARKS)

SECTION C

Level-1

1. Data regarding the height of students of class X is given Find the average height of students of the class

Height (in cm)	150-156	156-162	162-168	168-174	174-180
Number of students	4	7	15	8	6

2. Find the median for the following frequency distribution

Class	0-6	6-12	12-18	18-24	24-30
Frequency	1	4	9	3	3

3. Find the mode of the following distribution

Class	0-10	10-20	20-30	30-40	40-50
Frequency	8	12	10	11	9

4. The median of the data is 46.Find the value of p and q, if total frequency is 230

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	p	65	q	25	18

5. The mean of the following data is18.75.Find p

Class marks	10	15	P	25	30
Frequency	5	10	7	8	2

Level-2

6. 6.Find the mean of the following data.

Class	Less than 20	Less than 40	Less than 60	Less than 80	Less than 100
Frequency	15	37	74	99	120

.

7. Find the mean of the following distribution by Assumed Mean Method:

CI	10-20	20-30	30-40	40=50	50-60	60-70	70-80	80-90	90-100
Frequency	8	7	12	23	11	13	8	6	12

Level-3

8. Find mean and mode of the given data. Also find median using Empirical Formula.

Class	20-30	30-40	40-50	50-60	60-70

Frequency	25	40	42	33	10

9. The mean of the following distribution is 48 and the sum of all frequencies is 50. Find the missing frequencies

Class	20-30	30-40	40-50	50-60	60-70
Frequency	8	6	x	11	y

10. The median of the distribution given below is 14.4. Find the values of 'x' and 'y', if the sum of frequency is 20.

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	x	5	y	1

LONG ANSWER TYPE QUESTIONS (4 MARKS)

SECTION D

Level-1

1. If mode of the following distribution is 55, then find the value of x.

Class	0-15	15-30	30-45	45-60	60-75	75-90
Frequency	10	7	x	15	10	12

2. If the median of the distribution is 28.5, find the values of x and y

Class	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	20	15	y	5	60

3. The daily wages of 110 workers obtained in a survey are tabulated below. Compute the mean daily wages and modal daily wages of these workers.

Daily wages (Rs)	100-120	120-140	140-160	160-180	180-200	200-220	220-240
No. of workers	10	15	20	22	18	12	13

Level-2

4. The mean of the following distribution is 53. Find the missing frequencies f1 and f2

Classes	0-20	20-40	40-60	60-80	80-100	Total
Frequency	15	f1	21	f2	17	100

5. The lengths of 40 leaves of plant are measured correct to the nearest millimetre and the data is given

Length in mm	118-126	127-135	136-144	145-153	154-162	163-171	172-180
Number of leaves	3	5	9	12	5	4	2

Find the median length of leaves

6. Find the missing frequency x of the following data if its mode is 240 rupees

Expenditure	0-100	100-200	200-300	300-400	400-500
Number of families	140	230	270	x	150

Level-3

7. The following distribution gives the monthly consumption of electricity of 68 consumers of a locality. As Mr. Syam always saves electricity by switching off after usage, his family belongs to 65-85

Monthly consumption	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	8	4

Find the mean and mode of the data. Also find the median using empirical formula

8. Find the values of x and y if the median of the following data is 31

Class	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	X	6	Y	6	5	40

9. Mode of the distribution is 65 and sum of frequencies is 70.find x and y

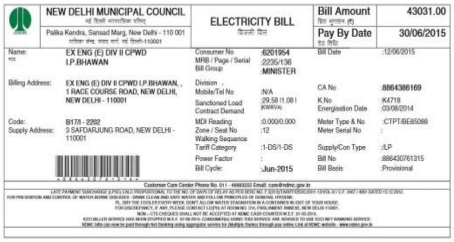
Class	0-20	20-40	40-60	60-80	80-100	100-120	120-140	140-160
Frequency	8	11	X	12	Y	9	9	5

10. Find mode of the following distribution

Class	25-35	35-45	45-55	55-65	65-75	75-85
Frequency	7	31	33	17	11	1

CASE STUDY QUESTIONS

1. Electricity energy consumption is the form of energy consumption that uses electric energy. Global electricity consumption continues to increase faster than world population, leading to an increase in the average amount of electricity consumed per person (per capita electricity



consumption).

A survey is conducted for 56 families of a Colony A. The following table gives the weekly consumption of electricity of these families.

Weekly consumption (in units)	0-10	10-20	20-30	30-40	40-50	50-60
No. of families	16	11	19	6	4	0

i. Find the difference between upper limit of the modal class and lower limit of median class.

ii. calculate mean of the data.

2. An electric scooter manufacturing company wants to declare the mileage of their electric scooters. For this, they recorded the mileage (km/ charge) of 50 scooters of the same model. Details of which are given in the following table.

Mileage (km/charge)	100-120	120-140	140-160	160-180
Number of scooters	7	12	18	13

Based on the case given, answer the following

i. Find the average mileage.
ii. Find the average of maximum number of scooters.

3. Electric buses are becoming popular now adays. These buses have the electricity stored in a battery. Electric buses have a range of approximately 280km with just one charge. These

buses are superior to diesel buses as they reduce brake wear and reduce pollution. Transport department of a city wants to buy some electric buses for the city. So, the department wants to know the distance travelled by existing public transport buses in a day.

The following data shows the distance travelled by 50 existing public transport buses in a day

Daily distance travelled in km	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of Buses	12	14	8	6	10

a) Find the Median distance travelled by a bus.
b) Find the Mean (average) distance travelled by a bus.

4.

An agency has decided to install customized playground equipment's at various colony parks. For that they decided to study the age group of children playing in a park of the particular colony The classification of children according to their ages, playing in a park is shown in the following table

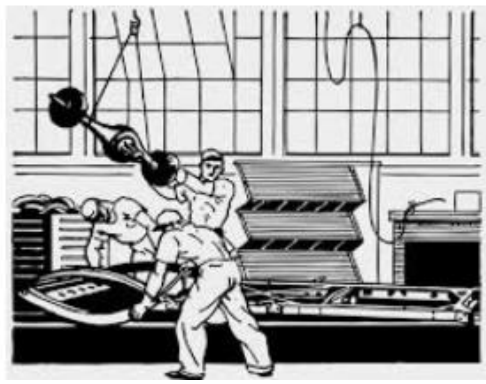
Age group of children (in years)	6-8	8-10	10-12	12-14	14-16
Number of children	43	58	70	42	27

Based on the above information answer the following

i. In which age group, will the maximum number of children belong?

ii. Find the mode of the ages of children playing in the park

5. As the demand of the products grew, a manufacturing company decided to hire more employees. For which they want to know the mean time required to complete the work for a worker.



The following table shows the frequency distribution of the time required for each worker to complete the work

Time(in hours)	15-19	20-24	25-29	30-34	35-39
Number of workers	10	15	12	8	5

Based on the above information answer the following

i. Find mean time required to complete the work for a worker (in hrs)

ii. If a worker works for 8 hours in a day , then the approximate time required to complete the work for a worker is (in days)

6. Transport department of a city wants to buy some electric buses for the city for which they wants to analyse the distance travelled by existing public transport buses in a day

The following shows the distance travelled by 60 existing public transport buses in a day

Daily distance travelled(km)	200-209	210-219	220-229	230-239	240-249
No of buses	4	14	26	10	6

Based on the above information answer the following questions

i. Find median of the distance travelled
ii. If the mode of the distance travelled is 223.78 km, find the mean of the distance travelled by the bus

7. A group of 71 people visited to a museum on a certain day. The following table shows their ages

Age in years	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50	Less than 60
No. of persons	3	10	22	40	54	71

Based on the above information answer the following

i. Find the median age of the persons visited the museum
ii. If the price of the ticket for the age group 30-40 is ₹30 .then the total amount spent by this age group is

Answer key (1 mark questions)

1 3Median=Mode +2Mean

2. 25

3. 46

4. $\frac{n+1}{2}$

5. 2

6. 0

7. 4

8. 4

9. 9

10. 27

II

1. Median of 2, 3, 5, 7 , 11, 13, 17, 19, 23 = 11

2. Mode= 3x15 – 2x14 = 17

3. Lower limit of 20-30 is 20.

4 Frequency= 51-48=3

5. Class mark= ½(10+25) =17.5

6. Mean= a+ h x $\frac{\sum fiui}{\sum fi}$ =27

7. $\frac{7+8+x+11+14}{5}$ =x ; x=10

8. Mean = $\frac{1+3+5+\cdots.+(2n-1)}{n}$ = n

9. Mode-Median=24
   Mode=24+Median
   3Median-2Mean=Mode
   3Median-2Mean=24+Median
   3Median-Median=24+2Mean
   Median-Mean=24/2=12

Answer key (2 marks)

1. Mode-3.28
2-lower limit of median class-99.50
3-Mean-25
4-Mean-25.2
5- Mean-8.15
6. n=29
7. Mode- 42.77
8. Mean=43
9. x=12
10. Median= 44 runs
11.

Marks	number
0-20	17
20-40	5
40-60	7
60-80	8
80-100	13

12. Mean= 50.10
    Median=52
13. Mode exceeds mean by 9
14. p=6
15. x=13 , y=61
16. Median=23.33
17. Mean=52
18.

Less than 300	12
Less than 400	30
Less than 500	65
Less than 600	85
Less than 700	100

19.

More than 0	100
More than 10	95
More than 20	80
More than 30	60
More than 40	37
More than 50	20
More than 60	9

20. Modal Class = 30-40

Assertion & Reasoning Answers

Qn 1-(b), Qn 2-(a), Qn 3-(a), Qn 4-(c)

ANSWER KEY (3 MARKS)

1) $\sum fi$= 40 Mean =165+($\frac{5}{40}$ x 6)=165.75

2) Median=12+$\frac{5X6}{9}$ = 15.3

3) Mode= 10 + $\frac{12-8}{24-8-10}$ x 10 =10+6.6666= 16.67

4) 150+ p+q = 230

p+q= 80 …….(1)

On applying formulae for median p=34 q=46

5)) 18.75 = $\frac{460+7p}{32}$

p=20

6)

class	frequency	x_i	di = $\frac{xi-50}{20}$	fidi
0-20	15	10	-2	-30
20-40	22	30	-1	-22
40-60	37	50	0	0
60-80	25	70	1	25
80-100	21	90	2	42
	∑fi=120			∑fidi=15

Mean= A + $\frac{\sum fidi}{\sum fi}$ x h = 50 + $\frac{15}{120}$x20 =50+2.5 =52.5

7)

CLASS INTERVAL	FREQUENCY	xi	di = $\frac{xi-55}{10}$	fidi
10-20	8	15	-4	-32

20-30	7	25	-3	-21
30-40	12	35	-2	-24
40-50	23	45	-1	-23
50-60	11	55	0	0
60-70	13	65	1	13
70-80	8	75	2	16
80-90	6	85	3	18
90-100	12	95	4	48
	∑fi =100			∑fidi = -5

Mean= A + $\frac{\sum fidi}{\sum fi}$ x h = 55+($\frac{-5}{100}$x10) =55-0.5 =54.5

8)

CI	Frequency	Class mark (xi)	fixi
10-20	4	15	60
20-30	8	25	200
30-40	10	35	350
40-50	12	45	54
50-60	10	55	550
60-70	4	65	260
70-80	2	75	150
	$\sum f$=50		$\sum fixi$=2110

Mean= $\frac{\sum f\text{xi}}{\sum f\text{i}}$ =$\frac{2110}{50}$ =42.2

12 is the maximum frequency.so 40-50 is the modal class.

Mode = l+$\frac{f-f1}{2f-f1-f2} * h$ = l+$\frac{12-10}{2(12)-10-10} * 10$ =45

Empirical formula

3 Median= Mode + 2 Mean

Median=$\frac{Mode+2\ mean}{3}$ =$\frac{45+2*42.2}{3}$ =99.4

9)

CI	fi	xi	Ui=$\frac{xi-a}{h}$	fiui
20-30	8	25	-2	-16
30-40	6	35	-1	-6
40-50	X	45=a	0	0
50-60	11	55	1	11
60-70	y	65	2	2y
Total	$\sum fi = 25 + x + y$			$\sum fiui = 2y - 11$

Mean=a+$\frac{\sum fiui}{\sum fi}$ *h

Also

$\sum fi$=$25 + x + y = 50$

48=45+$\frac{2y-11}{50}$ *10 , y=13

x+y=25 , x=12

10)

CI	Frequency	Cu.freq
0-6	4	4
6-12	X	4+x
12-18	5	9+x
18-24	Y	9+x+y
24-30	1	10+x+y
Total	20	

$\sum f$=20

x+y=10, median is 14.4.So median class is 12-18

M= l+$\frac{\frac{n}{2}-c}{f} * h$ 14.4= 12+$\frac{\frac{20}{2}-(4+x)}{5} * 6$, x=4 and y=6

ANSWER KEY (4 MARKS)

1) mode = 55, modal class 45-60, l=45, h=15, fo=15, f1=x, f2=10

55 = 45+$\frac{15-x}{30-x-10} * 15$ we get x=5

2) median= 28.5 n=60, median class 20-30, l=20, h=10, f=20, cf=5+x

28.5= 20+$\frac{30-(5+x)}{20} * 10$ x=8, also x+y+45=60, y=7

3)

Daily wages	No.of workersfi)	xi	xi-A	ui=$\frac{xi-170}{20}$	fiui
100-120	10	110	-60	-3	-30
120-140	15	130	-40	-2	-30
140-160	20	150	-20	-1	-20
160-180	22	170	0	0	0
180-200	18	190	20	1	18
200-220	12	210	40	2	24
220-240	13	230	60	3	39
	$\sum f$=110				$\sum fiui$=1

Mean=a+$\frac{\sum fiui}{\sum fi}$ *h Mean=170+$\frac{1}{110}$ *20 =170.18, mean daily wages=170.18

Modal class 160-180, f=22, l=160, h=20, f1=20, f2=18

Mode = 160+$\frac{22-20}{44-20-18} * 20 =$ 166.67, modal daily wages= 166.67

4) 53+f1+f2= 100

f1+ f2 =47………….(A)

mean=53

$\frac{2730+30f1+70f2}{100}$ = 53…………..(B)

Solving A and B f1= 18, f2=29

Length (in mm)	Class Interval (inclusive)	Number of leaves	Cumulative Frequency
118 —126	117.5 - 126.5	3	3
127 —135	126.5 - 135.5	5	3 + 5=8
136 —144	135.5 - 144.5	9	8 + 9=17(F)
145 —153	144.5 - 153.5	12(f)	17 + 12=29
154 —162	153.5 - 162.5	5	29 + 5=34
163 —171	162.5 - 171.5	4	34 + 4=38
172 —180	171.5 - 180.5	2	38 + 2=40

5)

n = 40, n/2 = 20

median class is 144.5 – 153.5 , l= 144.5 ,cf= 17 , f= 12, h = 9

Using the formula, = 144.5 + 2.25 = 146.75

6) Mode =240

$240= 200+\left(\frac{270-230}{2X270-230-x} \right)x\ 100$

Solving x =210

7) Median $=125+\frac{34-22}{20}$ x20 =137 Mode $=125+\frac{20-13}{2x20-13-14}$ x20=135.7

Mean=137.7 (2 mean=3median-mode)

8)

the median of the following data is 31

Class	frequency	Cf
0-10	5	5
10-20	X	5 + x
20-30	6	11 +x
30-40	Y	11 +x +y
40-50	6	17 +x +y
50-60	5	22 + x+ y
Total	40	

Sum of frequencies = 40
5+x+6+y+6+5 =40
22+x+y = 40

median = 31

$l + \frac{\frac{N}{2}-cf}{f}$x h = 31

$30 + \frac{20-(11+x)}{y}$x10 =31

$\frac{20-(11+x)}{y}$x10 =31-30

$\frac{20-11-x}{y}$x10 =1

10(9-x) = y
90-10x =y
Sub in x+y =18
x + 90-10x =18
90-9x =18
90-18 =9x

x + y =40-22 x + y = 18	72 = 9x X = $\frac{72}{9}$ X = 8 Y = 90 -10 x = 90 –(10x8) = 90 – 80 = 10

9)

Class	frequency
0-20	8
20-40	11
40-60	X f_o
60-80	12 f_1
80-100	Y f_2
100-120	9
120-140	9
140-160	5

Sum of all frequencies =70 8+11+ x +12 +y +9 +9 +5 =70 54 +x+y = 70 x + y = 70-54 **x+ y = 16** Mode = l + ($\frac{f1-f0}{2f1-f0-f2}$xh) 65 = 60 + $\frac{12-x}{24-x-y}$x 20 65-60 = $\frac{12-x}{24-x-y}$x 20 5 =$\frac{12-x}{24-x-y}$x 20 $\frac{5}{20}$ = $\frac{12-x}{24-x-y}$ $\frac{1}{4}$ = $\frac{12-x}{24-x-y}$ 1(24-x-y) = 4(12-x) 24-x-y = 48 – 4x 24-48=-4x+ x+y 24-48 =y-3x -24 = y-3x	**Y = 3x-24** Sub in x+y =16 x + 3x-24 =16 4x = 16 +24 4x = 40 X = $\frac{40}{4}$ = 10 Y =(3x10)-24 = 30-24 =6

10)

Class	frequency
25-35	7
35-45	31 f_o
45-55	33 f_1
55-65	17 f_2
65-75	11
75-85	1

Mode = l + ($\frac{f1-f0}{2f1-f0-f2}$xh)

$= 45 + \frac{33-31}{66-31-17}x10$

$= 45 + \frac{2}{18}x10$

$= 45 + \frac{10}{9}$

= 45 + 1.11

= 46.11

CASE STUDY ANSWERS

1. (i) Modal class = 20-30 median class = 20-30

Required difference = 30-20=10

(ii) Use any formula to calculate Mean

Mean=19.82

2. (i) Average means Mean of the data. Use any formula to calculate mean.

Mean= 144.8

(ii) Average of the maximum number means Mode of the data. Use the formula to calculate

Mode= $l+ \frac{f1-f0}{2f1-f0-f2}h$

Mode= 150.9

3. (i) Median = 138.57

(ii) Mean = 145.2 km

4. (i) 10-12

(ii) mode=10.6yrs

5.

Class	Class mark xi	Frequency f1	xifi
15-19	17	10	170
20-24	22	15	330
25-29	27	12	324
30-34	32	8	256
35-39	37	5	185
Total		$\sum$fi=50	$\sum$xifi=1265

Mean, $\frac{\sum xifi}{\sum fi} = \frac{1265}{50}$=25.3

Thus mean time to complete the work for a worker =25.3 hours ≈ 3 days

(i) 25.3hrs

(ii) 3days

6.

Class interval	f	cf
199.5-209.5	4	4
209.5-219.5	14	18
219.5-229.5	26	44
229.5-239.5	10	54
239.5-249.5	6	60

Median= l + $\frac{\frac{N}{2}-cf}{f}$ **x h**

=219.5 +($\frac{30-18}{26}$)x10

=219.5+4.62

=224.12

i) 224.12

ii) 224.29km

7.

Age	Class interval	f	cf
Less than 10	0-10	3	3
Less than 20	10-20	10-3= 7	10
Less than 30	20-30	22-10=12	22
Less than 40	30-40	40-22=18	40
Less than 50	40-50	54-40=14	54
Less than 60	50-60	71-54=17	71

$\frac{N}{2}$=35.5

Median class 30-40

(i) Median= l + $\frac{\frac{N}{2}-cf}{f}$ x h

=30+$\frac{35.5-22}{18}$ x 10

= 30+7.5

=37.5

ii) Total amount=30x18=540 rupees

PROBABILITY

Probability is the study of the chances of events happening. By means of probability, the chance of the events is measured by a number lying from 0 to 1.

Experiment: An operation which produces some well-defined outcomes, is called an experiment. Eg. Tossing a coin, throwing a dice etc.

If an experiment is repeated under identical conditions and they do not produce the same outcomes every time, then it is said to be a **Random experiment**.

An event of an experiment is the collection of some outcomes of the experiment, generally denoted by E. Eg. Getting an odd number in a single throw of die is an event. This case there are three outcomes 1,3 and 5

Elementary event: An event having only one outcome of the random experiment is called an elementary event. Eg. Tossing a coin and getting H or T is an elementary event

Probability of an event : If E is an event associated with a random experiment , then probability of event E, denoted by P(E) represents the chance of occurrence of event E.

Eg. If E denotes the event of getting an odd number in a single throw of die, then P(E) represents the chance of occurrence of event E, i.e, the chance of getting 1,3 or 5

Compound event: The collection of two or more elementary events associated with an experiment is called a compound event. Eg. In the random experiment of tossing a die, if we define the event "getting a multiple of 3" .Then the event has two outcomes 3 and 6 and hence is a compound event.

Impossible event: An event which does not occur at all when an experiment is performed is called an impossible event. Eg. "Event of getting 7 on a die" when a die is tossed, is an impossible event.

Sure event: The event which always occurs when the experiment is carried out is called a sure event. Eg. If we toss a die, the total outcomes are 1,2,3,4,5,6. Let the event be "the number on the die is less than 7" then E is 1,2,3,4,5,6 and hence always occurs and is a sure event.

Equally likely outcomes: Two or more outcomes are said to be equally likely outcomes if each outcome has the same chance of appearing as any other. Eg. If we toss a coin, the two outcomes i.e, H or T are equally likely to appear, so they are equally likely outcomes.

Formula for finding probability: The probability of an event E is denoted by P(E) and is

defined as

$$P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ event\ E}{Total\ number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

The numerator in the definition of P(E) is always less than or equal to denominator of P(E)

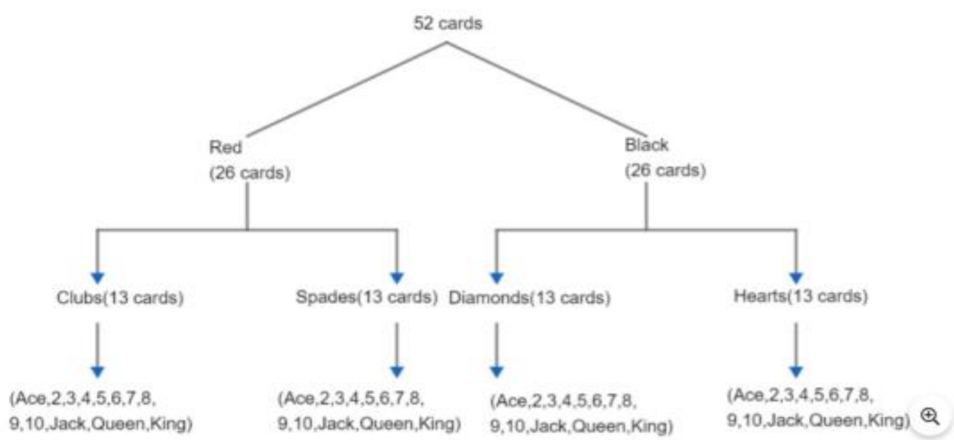
- $0 \leq P(E) \leq 1$
- For an impossible event P(E) = 0
- For sure event P(E) = 1
- Probability of an event cannot be negative
- The sum of probabilities of all the elementary events of an experiment = 1

Complement of an event / Negation of an event: If E is an event associated with a random experiment, then if we delete the outcomes in event E from total outcomes, then the event of collection of remaining outcomes is called complement event of 'event E' and is denoted by $\bar{E}$

• E and $\bar{E}$ are complementary events

• $P(E) + P(\bar{E}) = 1$

A pack of (or deck) of playing cards consists of four suits called Diamonds, Hearts, clubs and Spades. Each suit consists of 13 cards totaling 52 in all – 26 of red colour and 26 of black colour. Diamonds and Hearts are red cards whereas Clubs and Spades are black cards. Each suit contains an Ace, King, Queen and Jack ,2 3 , 4, 5, 6, 7 ,8, 9and,10. The Kings, Queens and Jacks are called face cards (there are 3 face cards in each suit). Thus, there are 12 face cards in all in a pack. Nine cards of each suit are numbered from 2 to 10.



There are 4 honour cards of each suit (Ace, King, Queen and Jack). In total there are 16 honour cards

Possible outcomes

Tossing a coin--- Head , Tail

Tossing two coins ---(HH) (HT) (TT) (TH)

Tossing 3 coins ------(HHH) (HHT) (HTH) HTT) (TTH) (TTT) (THT) (THT)

Throwing a die-----1 ,2 , 3 , 4 , 5 , 6

Throwing two dices----(1,1) (1,2) (1,3) (1,4) (1 ,5) (1, 6)

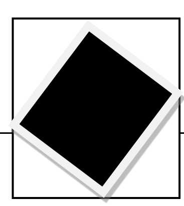
(2,1) (2,2) (2,3) (2,4) (2 ,5) (2, 6)

	(3,1) (3,2) (3,3) (3,4) (3 ,5) (3, 6) (4,1) (4,2) (4,3) (4,4) (4 ,5) (4, 6) (5,1) (5,2) (5,3) (5,4) (5 ,5) (5, 6) (6,1) (6,2) (6,3) (6,4) (6 ,5) (6, 6) Leap year 53 days… (Sunday, Monday) (Monday , Tuesday) (Tuesday, Wednesday) (Wednesday , Thursday) (Thursday , Friday) (Friday , Saturday) (Saturday , Sunday) Non leap year----Sunday , Monday , Tuesday , Wednesday , Thursday, Friday , Saturday
	PART A (1 MARK QUESTIONS)
1	The probability that a number selected from the numbers { 1,2,3,4,….20 }is a multiple of 5 is (A) $\frac{5}{20}$ (B) $\frac{1}{5}$ (C) $\frac{4}{5}$ (D) $\frac{2}{5}$
2	A letter of the English alphabet is chosen at random. The probability that the chosen letter is a consonant is (A) $\frac{21}{26}$ (B) $\frac{5}{26}$ (C) $\frac{2}{25}$ (D) None of these
3	Which of the following cannot be the probability of an event? (A) 1.5 (B) $\frac{3}{5}$ (C) 25% (D) 0.3
4	If P(E) = 0.05 , then the probability of P (not E) is (A) 0.85 (B) 0.75 (C) 0.25 (D) 0.95
5	In a single throw of a die , the probability of getting a multiple of 3 is (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{2}{3}$
6	A bag contains three green marbles, four blue marbles & two orange marbles. One marble is picked at random, then the probability that it is not an orange marble is (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{4}{9}$ (D) $\frac{7}{9}$

7	The probability of getting a bad egg in a lot of 400 eggs is 0.035. The number of bad eggs in the lot is (A) 7 (B) 14 (C) 21 (D) 28
8	Two coins are tossed simultaneously, then the probability of getting exactly one head is (A) $\frac{1}{3}$ (B) $\frac{2}{6}$ (C) $\frac{1}{2}$ (D) $\frac{1}{8}$
9	A card is drawn at random from a well shuffled pack of 52 cards. The probability that the drawn card is not an ace is (A) $\frac{1}{13}$ (B) $\frac{12}{13}$ (C) $\frac{9}{13}$ (D) $\frac{4}{13}$
10	The probability that a non – leap year has 53 Sundays is (A) $\frac{2}{7}$ (B) $\frac{5}{7}$ (C) $\frac{6}{7}$ (D) $\frac{1}{7}$
11	The Probability of guessing the right answer to a certain question in a test is $\frac{x}{12}$. If the probability of not guessing the correct answer to this question is $\frac{2}{3}$, then find value of x
12	Two coins are tossed simultaneously. Find the probability of getting at most one head
13	A die is thrown twice. Find the probability of getting a sum less than 8
14	A card is drawn from a pack of 52 cards. Find the probability that the card drawn is not a face card
15	A number is selected from first 50 natural numbers. What is the probability that it is a multiple of 3 or 5?
16	A card is drawn from a pack of 52 cards. Find the probability of getting a king of red colour
17	A box contains cards numbered 6 to 50. A card is drawn at random from the box. Find the probability that the card drawn has a number which is a perfect square
18	A bag contains 40 balls out of which some are red, some are blue and remaining are black. If the probability of drawing a red ball is $\frac{11}{20}$ and that of blue ball is $\frac{1}{5}$, then what is the no. of black balls?
19	A bag contains cards which are numbered from 2 to 90. A card is drawn at random from the bag. Find the probability that it bears a two digit number

20	A bag contains cards numbered from 1 to 49. After mixing the cards thoroughly a card is drawn from the bag at random, Find the probability that the number on the drawn card is an odd number
21	A card is drawn from a well shuffled deck of cards. What is the probability that the card drawn is neither a king nor a queen?
22	The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap.
23	A month is selected at random in a year. Find the probability that it is March or October.
	PART B (2 MARK QUESTIONS)
24	A coin & a die are tossed simultaneously. Find the probability that a tail & a prime number turns up
25	A letter is chosen at random from the letters of the word " ASSASSINATION", then the probability that the letter chosen is a vowel is in the form of $\frac{6}{2x+1}$,if so find the value of x
26	In a family of 3 children calculate the probability of having at least one boy.
27	A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a vowel
28	A coin is tossed two times. Find the probability of getting both heads or both tails
29	A box contains cards bearing numbers 6 to 70 . If one card is drawn at random from the box ,find the probability that it bears (i) a one digit number (ii) a number divisible by 5.
30	A box contains 20 cards numbered from 1 to 20 . A card drawn at random from the box. Find the probability that the card drawn at random is divisible by 2 or 3
31	From a bag containing 5 red , 8 black and 7 blue balls , a ball selected at random .Find the probability that (i) it is not a red ball (ii) it is a blue ball
32	Two different dice are tossed together. Find the probability (i)of getting a doublet

	(ii) of getting a sum 10 , on the two dice.
33	Two dice are rolled once. Find the probability of getting such numbers on the two dice, whose product is 12
34	All cards of ace , jack and queen are removed from a deck of playing cards. One card is drawn at random from the remaining cards. Find the probability that the card drawn is (i) a face card (ii) a black king
35	A number is selected at random from the numbers 3 ,5 , 5 , 7 , 7 ,7 , 9 , 9 , 9, 9, . Find the probability that the selected number is their average.
36	A number x is selected at random from the numbers 1 , 4 , 9 , 16 and another number y is selected at random from numbers 1 , 2 ,3 4 . Find the probability that the value of xy is more than 16
37	A group consists of 12 persons, out of which 4 are extremely patient, other 6 are extremely honest and the rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected , find the probability of selecting a person who is (i)extremely patient. (ii) extremely kind or honest.
38	A card is drawn from a well – shuffled pack of 52 cards. Find the probability that the card drawn is neither a red card nor a queen
39	A card is drawn from a well – shuffled pack of 52 cards. Find the probability that the card drawn is either a red card or a queen
40	A card is drawn from a well – shuffled pack of 52 cards. Find the probability of getting (i) a red king (ii) a queen or jack
41	A square of side 5 cm is drawn in the interior of another square of side 10 cm and shade as shown in the figure. A point is selected at random from the interior of the square. What is the probability that the point will be chosen from the shaded part?



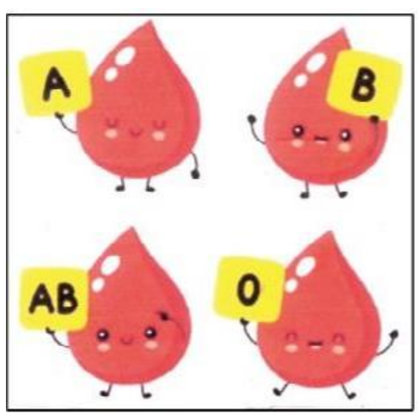
42	A bag contains 2 green, 3 red and 4 black balls. A ball is taken out of the bag at random. Find the probability that the selected ball is (i) not green (ii) not black
43	12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from the lot .Determine the probability that the pen taken out is a good one
44	Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?
	PART C - (3 MARK QUESTIONS)
45	A child has a die whose six faces show the letters as given below: ***A*** ***B*** ***C*** ***D*** ***E*** ***A*** The die is thrown once. What is the probability of getting (i) A (ii) D? (III) Vowels:
46	Out of 400 bulbs in a box, 15 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective.
47	A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.
48	A bag contains 12 balls out of which x are white. (i) If one ball is drawn at random, what is the probability that it will be a white ball? (ii) If 6 more white balls are put in the bag, the probability of drawing a white ball will be double that in case (i). Find x.
49	A pair of dice is thrown once. Find the probability of getting (i) doublet of prime numbers (ii) a doublet of odd numbers.

50	All the three face cards of spades are removed from a well- shuffled pack of 52 cards. A card is drawn at random from the remaining pack. Find the probability of getting (i) a black face cards (ii) a queen (iii) a black card
51	A box contains cards bearing numbers 6 to 70. If one card is drawn at random from the box ,find the probability that it bears (a) not a one digit number (b) a number not divisible by 5 (c) number is a perfect square
52	There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card I. is divisible by 9 and is a perfect square II. (ii) is a prime number greater than 80
53	A bag contains 24 balls of which x are red, 2x are white and 3x are blue. A ball is selected at random. What is the probability that (i) it is red (ii) it is blue (iii) neither red nor blue
54	A bag contains white, black and red balls only. A ball is drawn at random from the bag. The probability of getting a white ball is $\frac{3}{10}$ and that of black is $\frac{2}{5}$. Find the probability of getting a red ball. If the bag contains 20 black balls, then find the total number of balls in the bag.
55	A child's game has 6 triangles of which 3 are blue and the rest are red and 10 squares of which 6 are blue and the rest are red. One piece Is lost at random. Find the probability that that it is a (i) Triangle (ii) Square (iii) Square of blue colour
56	A lot consists of 48 mobile phones of which 42 are good, 3 have only minor defects and 3 have major defects. Varnika will buy a phone if it is good but the trader will

	only buy a mobile if it has no major defect. One phone is selected at random from the lot. What is the probability that it is: (i) acceptable to. Varnika? (ii) acceptable to the trader?
57	Five cards – the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random. (i) What is the probability that the card is the queen? (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?
58	A bag contains 5 black, 7 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is: (i) red. (ii) black or white. (iii) not black.
59	The probability of selecting a blue marble at random from a jar that contains only blue, black and green marbles is 1/5. The probability of selecting a black marble at random from the same jar is 1/4. If the jar contains 11 green marbles, find the total number of marbles in the jar.
	SECTION D (4 MARK QUESTIONS)
60	All the black Ace cards are removed from a pack of 52 playing cards. The remaining cards are well shuffled and then a card is drawn at random. Find the probability of getting i) a Ace card ii) a red card iii) a black card iv) a Jack
61	. Cards marked with the number 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from the box. Find the probability that the number on the card is: (i) An even number (ii) A number less than 14 (iii) A number is perfect square (iv) A prime number less than 20
62	Out of the families having three children, a family is chosen at random. Find the probability that the family has (i) Exactly one girl (ii) At least one girl (iii) At most one girl

63	The Ace, number 10 and jack of clubs are removed from a deck of 52 playing cards and remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (a) heart (b) Ace (c) clubs (d) either 10 or jack
64	Two dice are thrown simultaneously. What is the probability that: (a) 5 will not come up either of them? (b) 5 will come up at least one time? (c) 5 will come at both dice? (d) Sum of 5 comes on both the dice together
65	Cards bearing numbers 3, 5... 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing (a) a prime number less than 15 (b) a number divisible by 3 and 5
66	A bag contains 8 red balls & some blue balls. If the probability of drawing a blue ball is 3 times of a red ball, find the number of blue balls in the bag.
67	Three coins are tossed simultaneously. Find the probability of getting (i) Exactly 2 heads (ii) at least 1 head (iii) at most 2 tails (iv) exactly 3 heads
	Case study questions
68	

	Akshith & Dikshith are good friends. During vacation Dikshith went to Akshith's house to play Ludo.They played Ludo with 2 dice. (i)To win a game Dikshith needs a total of 7. What is the probability of winning a game by Dikshith ? (ii) Find the probability that 5 will come up at least in one die?
69	CASE STUDY 2  Two friends Neha and Sohan have some savings in their piggy bank. They decided to count the total coins they both had. After counting they find that they have fifty ₹ 1 coins, forty eight ₹ 2 coins, thirty six ₹ 5 coins, twenty eight ₹10 coins and eight ₹ 20 coins. Now, they said to Isha, their other friends, to choose a coin randomly (i) Find the probability of getting a denomination of ₹10 (ii) Find the probability of getting a denomination of₹ 2 or ₹ 5
70	CASE STUDY 3  Eric bought balls for decorating the Christmas tree. The bag contains 24 balls ,of which x number are red balls (i) If one ball is drawn at random, find the probability of getting a red ball (ii) If 6 more red balls are put in the bag ,the probability of drawing a red ball is double that in first case , find the number of red balls
	CASE STUDY 4

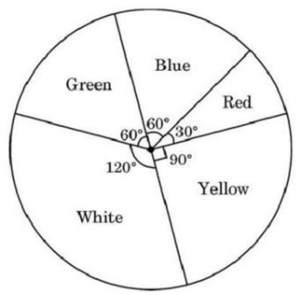
71	Gunjan is fond of playing cards. She tries to find out probability in different situations. One such situation is given below  **Five cards – ten, jack, queen, king, and an ace of diamonds are shuffled face downwards. One card is picked at random.** **(i) What is the probability that the card is a queen?** **(ii) If a king is drawn first and put aside, what is the probability that the second card picked up is the (a) ace? (b) king?**
72	**CASE STUDY 5** Blood Group describes the type of blood of a person has. It is a classification of blood based on the presence or absence of inherited antigenic substances on the surface of red blood cells. Blood type predicts whether a serious reaction will occur in a blood transfusion In a sample of 50 people, 21 had type O, 22 had A, 5 had Type B and rest had type AB blood group. (CBSE 2023 

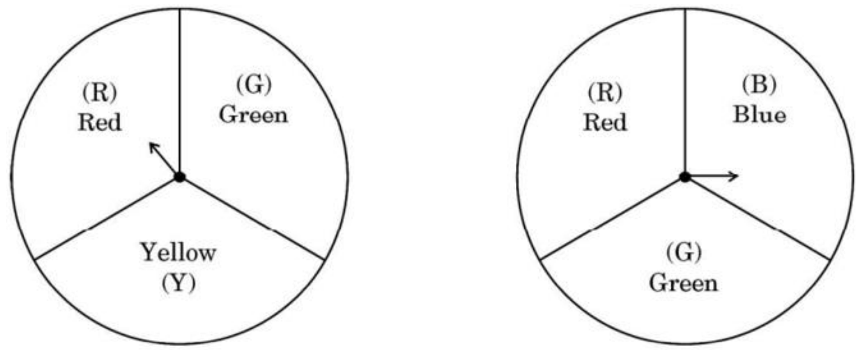
	Based on the above , answer the following questions i. What is the probability that a person chosen at random had type O blood? ii. What is the probability that a person chosen at random had Type AB blood group? iii. What is the probability that a person chosen at random had neither type A nor type B blood group? iv. What is the probability that person chosen at random had either type A, type B **or** type O blood group.
73	CASE STUDY 6 Computer – based learning (CBL)refers to any teaching methodology that makes use of computers for information transmission. At an elementary school level, computer application can be used to display multimedia lesson plans. A survey was done on 1000 elementary schools of Assam and they were classified by the number of computers they had. (CBSE – 2023  The following table shows the salaries (in percent) received by 50 employees during lockdown.

Number of computors	1 - 10	11 - 20	21 - 50	51 - 100	101 and more
Number of schools	250	200	290	180	80

Based on the above information, answer the following questions.

i. Find the probability that the school chosen at random has more than 100 computers.

	ii. (a) Find the probability that the school chosen at random has 50 or fewer computers (b) Find the probability that the school chosen at random has 10 or less more than 20 computers. iii. Find the probability that the school chosen at random has 10 or less than 10 computers
74	Some students were asked to list their favourite colour. The measure of each colour is shown by the central angle of a pie chart given below. (CBSE – 2023  Study the pie chart and answer the following questions. (i) If a student is chosen at random, then find the probability of his/her favourite colour being white. (ii) What is the probability of his/her favourite colour being blue or green? (iii) a) If 15 students liked the colour yellow, how many students participated in the survey? OR (iii) b) What is the probability of the favourite colourbeing red or blue?
75	A middle school decided to run the following spinner game as a fund raiser on Christmas carnival (CBSE – 2023



I II

Making purple : Spin each spinner once. Blue and red make purple. So, one spinner shows Red(R) and another Blue(B), then you 'win'. One such outcome is written as 'RB'.

Based on the above answer the following questions

(i) List all possible outcomes of the game.

(ii) Find the probability of 'Making Purple'.

(iii) a) For each win, a participant gets Rs 10, but if he /she loses, he/she has to pay Rs 5 to the school. If 99 participants played, calculate how much fund could the school have collected?

OR

(b) If the same amount of Rs 5 has been decided for winning or losing the game , then how much fund had been collected by school?

ANSWER KEY

Q NO	ANSWER	Q NO	ANSWER	Q NO	ANSWER
1	$\frac{1}{5}$	20	$\frac{25}{49}$	39	$\frac{7}{13}$
2	$\frac{21}{26}$	21	$\frac{11}{13}$	40	$\frac{1}{26}$, (ii) $\frac{2}{13}$
3	1.5	22	162	41	$\frac{1}{4}$
4	0.95	23	$\frac{1}{6}$	42	$\frac{7}{9}$, $\frac{5}{9}$
5	$\frac{1}{3}$	24	$\frac{1}{4}$	43	$\frac{11}{12}$
6	$\frac{7}{9}$	25	6	44	0.38

7	**14**	**26**	$\frac{7}{8}$	**45**	$\frac{1}{3}$ **,** $\frac{1}{6}$ **,** $\frac{1}{2}$
8	$\frac{1}{2}$	**27**	$\frac{5}{26}$	**46**	$\frac{77}{80}$
9	$\frac{12}{13}$	**28**	$\frac{1}{2}$	**47**	**45**
10	$\frac{1}{7}$	**29**	$\frac{4}{65}$ **, (ii)** $\frac{1}{5}$	**48**	$\frac{1}{4}$ **,3**
11	**4**	**30**	$\frac{13}{20}$	**49**	$\frac{1}{12}$ **,** $\frac{1}{12}$
12	$\frac{3}{4}$	**31**	$\frac{3}{4}$ **, (ii)** $\frac{7}{20}$	**50**	$\frac{3}{49}$ **,** $\frac{3}{49}$ **,** $\frac{23}{49}$
13	$\frac{7}{12}$	**32**	$\frac{1}{6}$ **, (ii)** $\frac{1}{12}$	**51**	$\frac{61}{65}$ **,** $\frac{52}{65}$ **,** $\frac{6}{65}$
14	$\frac{10}{13}$	**33**	$\frac{1}{9}$	**52**	$\frac{3}{100}$ **,** $\frac{3}{100}$
15	$\frac{23}{50}$	**34**	$\frac{1}{10}$ **(ii)** $\frac{1}{20}$	**53**	$\frac{1}{6}$ **,** $\frac{1}{2}$**,** $\frac{2}{3}$
16	$\frac{1}{26}$	**35**	$\frac{3}{10}$	**54**	$\frac{3}{10}$ **, 50**
17	$\frac{1}{9}$	**36**	$\frac{3}{8}$	**55**	$\frac{3}{8}$ **,** $\frac{5}{8}$ **,** $\frac{3}{8}$

18	**10**	**37**	$\frac{1}{3}$, **(ii)** $\frac{2}{3}$	**56**	$\frac{7}{8}$, $\frac{15}{16}$
19	$\frac{81}{89}$	**38**	$\frac{6}{13}$	**57**	$\frac{1}{5}$, $\frac{1}{4}$, **0**
58	$\frac{7}{15}$, $\frac{8}{15}$, $\frac{2}{3}$	**64**	$\frac{25}{36}$, $\frac{11}{36}$, $\frac{1}{36}$, $\frac{1}{9}$	**70**	$\frac{1}{6}$, **4**
59	**20**	**65**	$\frac{5}{17}$, $\frac{2}{17}$	**71**	$\frac{3}{47}$, $\frac{3}{46}$, $\frac{3}{46}$
60	$\frac{1}{25}$, $\frac{13}{25}$, $\frac{12}{25}$, $\frac{2}{25}$	**66**	**24**	**72**	i) $\frac{21}{50}$ ii) $\frac{1}{25}$ iii) $\frac{23}{50}$ iv) $\frac{24}{25}$
61	$\frac{1}{2}$, $\frac{3}{25}$, $\frac{9}{100}$, $\frac{2}{25}$	**67**	$\frac{3}{8}$, $\frac{7}{8}$, $\frac{7}{8}$, $\frac{1}{8}$	**73**	(i) $\frac{80}{1000}$ or 0.08 (ii) (a) $\frac{740}{1000}$ or 0.74 (b) $\frac{450}{1000}$ or 0.45 (iii) $\frac{250}{1000}$ or 0.25
62	$\frac{3}{8}$, $\frac{7}{8}$, $\frac{1}{2}$	**68**	$\frac{1}{6}$, $\frac{11}{36}$		
63	$\frac{13}{49}$, $\frac{3}{49}$, $\frac{10}{49}$, $\frac{6}{49}$	**69**	$\frac{14}{85}$, $\frac{42}{85}$		
74	i. P(favourite colour being white) $=\frac{120}{360} = \frac{1}{3}$ (ii) P(favourite colour being blue or green) = $\frac{60+60}{360}$ or $\frac{1}{3}$ (a) Let the total number of students be $x \Rightarrow \frac{15}{x} = \frac{1}{4}$ $\Rightarrow x = 60$ Total number of participants = 60 (b) P(favourite colour being Red or Blue) = $\frac{60+30}{360} = \frac{1}{4}$				
75	(i) All possible out come: **RR, RG,RB,GR,GB,GG,YR,YB,YG** (ii) Number of favourable outcome (RB) = 1				

	(iii) P(Making Purple) = $\frac{1}{9}$ P(winning)= $99 \times \frac{1}{9} = 11$ Game lost by 88 persons Fund collected = $88 \times ₹5 - 11 \times ₹10 = ₹330$

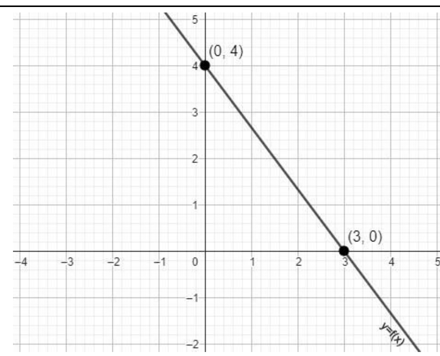
SAMPLE QUESTION PAPER
Class X Session 2023-24
MATHEMATICS STANDARD (Code No.041)

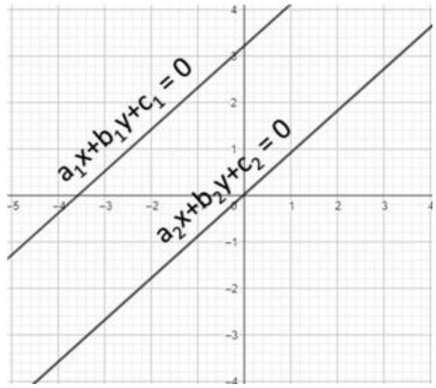
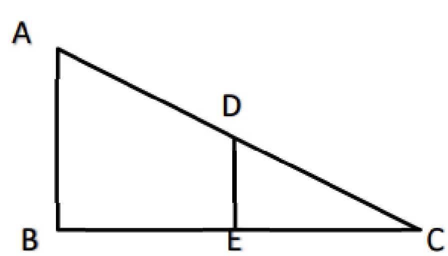
TIME: 3 HOURS **MAX.MARKS: 80**

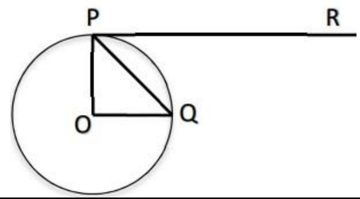
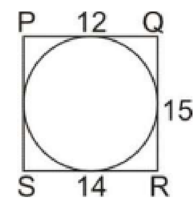
General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E

Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

	SECTION A	
	Section A consists of 20 questions of 1 mark each.	
1	If two positive integers a and b are written as a = x^3y^2 and b = xy^3, where x, y are prime numbers, then the result obtained by dividing the product of the positive integers by the LCM (a, b) is (a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2	1
2	The given linear polynomial y = f(x) has (a) 2 zeros (b) 1 zero and the zero is '3' (c) 1 zero and the zero is '4' (d) No zero 	1

3	The lines representing the given pair of linear equations are non-intersecting. Which of the following statements is true? (a) $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$ (b) $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$ (c) $\frac{a1}{a2} \neq \frac{b1}{b2} = \frac{c1}{c2}$ (d) $\frac{a1}{a2} \neq \frac{b1}{b2} \neq \frac{c1}{c2}$ 	1
4	The nature of roots of the quadratic equation $9x^2 – 6x – 2 = 0$ is: (a) No real roots (b) 2 equal real roots (c) 2 distinct real roots (d) (d) More than 2 real roots	1
5	Two APs have the same common difference. The first term of one of these is –1 and that ofthe other is – 8. The difference between their 4th terms is (a) 1 (b) -7 (c) 7 (d) 9	1
6	What is the ratio in which the line segment joining (2,-3) and (5, 6) is divided by x-axis? (a) 1:2 (b) 2:1 (c) 2:5 (d) 5:2	1
7	A point (x,y) is at a distance of 5 units from the origin. How many such points lie in the thirdquadrant? (a) 0 (b) 1 (c) 2 (d) infinitely many	1
8	In Δ ABC, DE || AB. If AB = a, DE = x, BE = b and EC = c. Then x expressed in terms of a, b and c is: (a) $\frac{ac}{b}$ (b) $\frac{ac}{b+c}$ (c) $\frac{ab}{c}$ (d) $\frac{ab}{b+c}$ 	1

9	If O is centre of a circle and Chord PQ makes an angle 50° with the tangent PR at the point of contactP, then the angle subtended by the chord at the centre (a) 130° (b) 100° (c) 50° (d) 30° 		
10	A quadrilateral PQRS is drawn to circumscribe a circle. If PQ = 12 cm, QR = 15 cm and RS = 14 cm, then find the length of SP is (a) 15 cm (b) 14 cm (c) 12 cm (d) 11 cm 		
11	Given that $sin\theta = \frac{a}{b}, then\ cos\theta$ is (a) $\frac{b}{\sqrt{b^2-a^2}}$ (b) $\frac{b}{a}$ (c) $\frac{\sqrt{b^2-a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2-a^2}}$		
12	(sec A + tan A) (1 – sin A) equals: (a) sec A (b) sin A (c) cosec A (d) cos A		(c) cosec A
13	If a pole 6 m high casts a shadow 2 $\sqrt{3}$m long on the ground, then the Sun's elevation is (a) 60° (b) 45° (c) 30° (d) 90°		
14	If the perimeter and the area of a circle are numerically equal, then the radius of the		

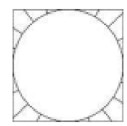
circleis

(a) 2 units
(b) π units
(c) 4 units
(d) 7 units

15 It is proposed to build a new circular park equal in area to the sum of areas of two circularparks of diameters 16 m and 12 m in a locality. The radius of the new park is

(a) 10m (b) 15m (c) 20m (d) 24m

16 There is a square board of side '2a' units circumscribing a red circle. Jayadev is asked to keep a dot on the above said board. The probability that he keeps the dot on the shaded region is.



a) $\frac{\pi}{4}$

b) $\frac{4-\pi}{4}$

c) $\frac{\pi-4}{4}$

d) $\frac{4}{\pi}$

17 2 cards of hearts and 4 cards of spades are missing from a pack of 52 cards. A card is drawn atrandom from the remaining pack. What is the probability of getting a black card?

a) $\frac{22}{52}$

b) $\frac{22}{46}$

c) $\frac{24}{52}$

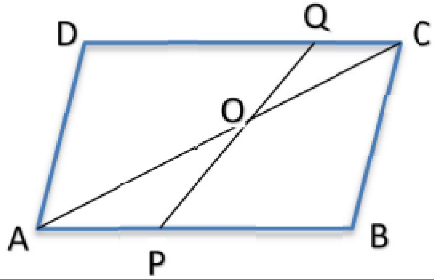
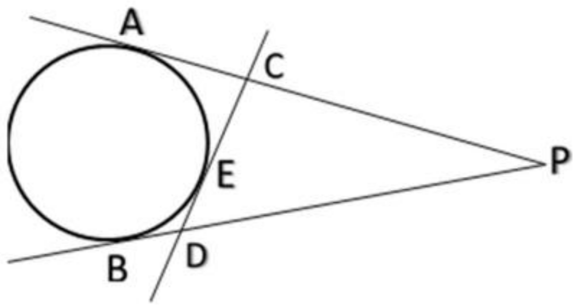
d) $\frac{24}{46}$

18 The upper limit of the modal class of the given distribution is:

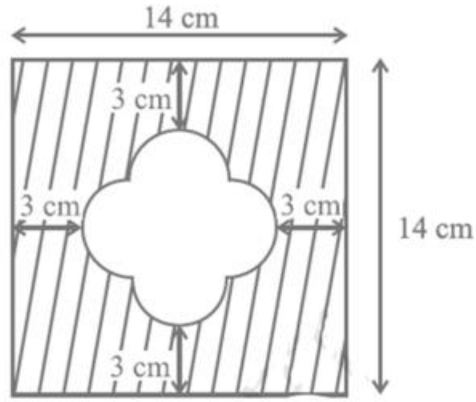
Height [in cm]	Below 140	Below 145	Below 150	Below 155	Below 160	Below 165
Number of girls	4	11	29	40	46	51

(a) 165 (b) 160 (c) 155 (d) 150

19	DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed bya statement of Reason (R). Choose the correct option Statement A (Assertion): Total Surface area of the top is the sum of the curved surface area of the hemisphere and the curved surface area of the cone. Statement R(Reason): Top is obtained by joining the plane surfaces of th hemisphere and cone together. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanationof assertion (A) (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) (c) Assertion (A) is true but reason (R) is false. Assertion (A) is false but reason (R) is true.	
20	Statement A (Assertion): -5, -5/2, 0, 5/2 … is in Arithmetic Progression. Statement R (Reason) : The terms of an Arithmetic Progression cannot have both positiveand negative rational numbers. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanationof assertion (A) (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) (c) Assertion (A) is true but reason (R) is false. (d) Assertion (A) is false but reason (R) is true.	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21	Prove that $\sqrt{2}$ is an irrational number.	2

22	ABCD is a parallelogram. Point P divides AB in theratio 2:3 and point Q divides DC in the ratio 4:1. Prove that OC is half of OA.  	2
23	From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At a point E on the circle, a tangent is drawnto intersect PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of ΔPCD.  	2
24	If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0° < A + B < 90°$; $A > B$, find A and B.	2
	OR	
	Find the value of x if $2cosec^2 30° + x sin^2 60° - \frac{3}{4}\tan^2 30° = 10$	
25	With vertices A, B and C of ΔABC as centres, arcs are drawn with radii 14 cm and the threeportions of the triangle so obtained are removed. Find the total area removed from the triangle.	2
	OR	

Find the area of the unshaded region shown in thegiven figure.



SECTION C

Section C consists of 6 questions of 3 marks each

26 National Art convention got registrations from students from all parts of the country, of which 60 are interested in music, 84 are interested in dance and 108 students are interested in handicrafts. For optimum cultural exchange, organisers wish to keep them in minimum number of groups such that each group consists of students interested in the same artformand the number of students in each group is the same. Find the number of students in eachgroup. Find the number of groups in each art form. How many rooms are required if each group will be allotted a room? 3

27 If α, β are zeroes of quadratic polynomial $5x^2 + 5x + 1$, find the value of 3

1. $\alpha^2 + \beta^2$
2. $\alpha^{-1} + \beta^{-1}$

28 The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there? 3

OR

Solve:- $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$; $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1, x, y > 0$

29 PA and PB are tangents drawn to a circle of centre O from an external point P. Chord ABmakes an angle of 30° with the radius at the point of contact.If length of the chord is 6 cm, find the length of the tangent PA and the length of the radiusOA. 3

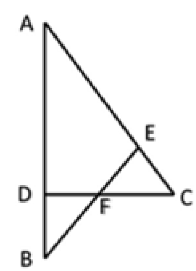
OR

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠ PTQ = 2 ∠ OPQ.

30 If $1 + \sin^2\theta = 3 sin\theta cos\,\theta$, then prove that$tan\theta = 1\ or\ \frac{1}{2}$ 3

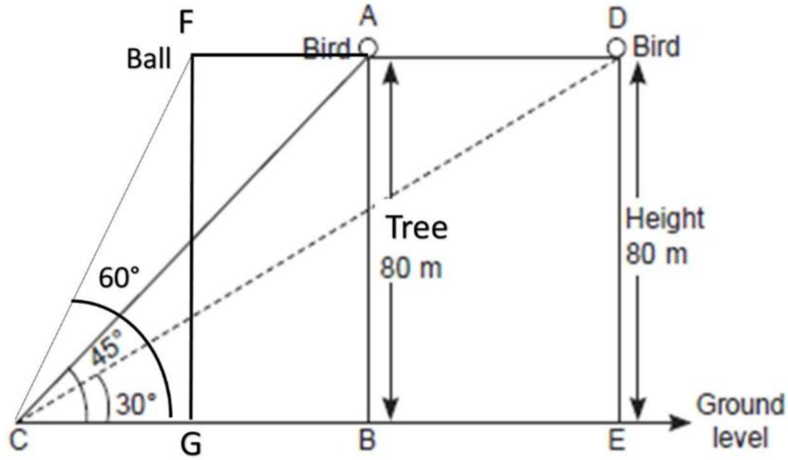
31 The length of 40 leaves of a plants are measured correct to nearest millimeter, and the data obtained is represented in the following table. Find the mean length of the leaves. 3

Length [in mm]	Number of leaves

	118 - 126	3	
	127 – 135	5	
	136 – 144	9	
	145 – 153	12	
	154 – 162	5	
	163 – 171	4	
	172 – 180	2	
	SECTION D		
	Section D consists of 4 questions of 5 marks each		
32	A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream.		5
	OR		
	Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.		
33	(a) State and prove Basic Proportionality theorem. (b) In the given figure ∠CEF = ∠CFE. F is the midpoint of DC. Prove that $\frac{AB}{BD} = \frac{AE}{FD}$ 		5
34	Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by21 cm? What should be the speed of water if the rise in water level is to be attained in 1 hour?		5
	OR		
	A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius		

	of the cylindrical part are 3 m and 14 m respectively, and the total height of the tent is 13.5 m, find the area of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of ₹ 500 per m^2.	
35	The median of the following data is 50. Find the values of 'p' and 'q', if the sum of all frequencies is 90. Also find the mode of the data. <table><tr><td>Marks obtained</td><td>Number of students</td></tr><tr><td>20 – 30</td><td>p</td></tr><tr><td>30 – 40</td><td>15</td></tr><tr><td>40 – 50</td><td>25</td></tr><tr><td>50 – 60</td><td>20</td></tr><tr><td>60 – 70</td><td>q</td></tr><tr><td>70 – 80</td><td>8</td></tr><tr><td>80 - 90</td><td>10</td></tr></table>	5
	SECTION E	
	CASE BASED QUESTIONS	
36	Manpreet Kaur is the national record holder for women in the shot-put discipline. Her throw of18.86m at the Asian Grand Prix in 2017 is the maximum distance for an Indian female athlete. Keeping her as a role model, Sanjitha is determinedto earn gold in Olympics one day. Initially her throw reached 7.56m only. Being an athlete in school, she regularly practiced both in themornings and in the evenings and was able to improve the distance by 9cm every week. During the special camp for 15 days, she started with40 throws and every day kept increasing the numberof throws by 12 to achieve this remarkable progress. 	

	(i) How many throws Sanjitha practiced on 11^{th} day of the camp?	1
	(ii) What would be Sanjitha's throw distance at the end of 6 weeks? (or) When will she be able to achieve a throw of 11.16 m?	2
	(iii) How many throws did she do during the entire camp of 15 days ?	1
37	Tharunya was thrilled to know that the football tournament is fixed with a monthly timeframe from 20th July to 20th August 2023 and for the first time in the FIFA Women's World Cup's history, two nations host in 10 venues. Her father felt that the game can be better understood if the position of players is represented as points on a coordinate plane.  	
	(i) At an instance, the midfielders and forward formed a parallelogram. Find the position of the central midfielder (D) if the position of other players who formedthe parallelogram are :- A(1,2), B(4,3) and C(6,6)	1
	(ii) Check if the Goal keeper G(-3,5), Sweeper H(3,1) and Wing-back K(0,3) fall on asame straight line. [or] Check if the Full-back J(5,-3) and centre-back I(-4,6) are equidistant from forward C(0,1) and if C is the mid-point of IJ.	2
	(iii) If Defensive midfielder A(1,4), Attacking midfielder B(2,-3) and Striker E(a,b) lie on the same straight line and B is equidistant from A and E, find the position of E.	1

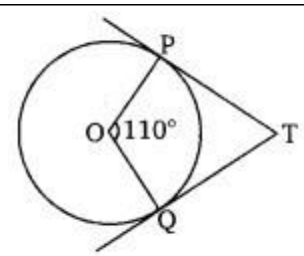
38	One evening, Kaushik was in a park. Children were playing cricket. Birds were singing on anearby tree of height 80m. He observed a bird on the tree at an angle of elevation of 45°. When a sixer was hit, a ball flew through the tree frightening the bird to fly away. In 2 seconds, he observed the bird flying at the same height at an angle of elevation of 30° and the ball flying towards him at the same height at an angle of elevation of 60°.  	
	(i) At what distance from the foot of the tree was he observing the bird sitting on the tree?	1
	(ii) How far did the bird fly in the mentioned time? (or) After hitting the tree, how far did the ball travel in the sky when Kaushik saw theball?	2
	(iii) What is the speed of the bird in m/min if it had flown $20(\sqrt{3} + 1)$ m?	1

SAMPLE QUESTION PAPER Class X Session 2023-24 MATHEMATICS BASIC (241) TIME: 3 HOURS MAX.MARKS: 80

General Instructions:

1. **This Question Paper has 5 Sections A, B, C, D, and E.**
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. **Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.**
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. **Section D has 4 Long Answer (LA) type questions carrying 5 marks each.**
6. Section E has 3 sourced based/Case Based/passage based/integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. **All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.**
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

	SECTION A	
	Section A consists of 20 questions of 1 mark each.	
1	If two positive integers a and b are written as a = x^3y^2 and b = xy^3, where x, y are prime numbers, then HCF (a, b) is (a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2	1
2	The LCM of smallest two digit composite number and smallest composite number is: a) 12 b) 4 c) 20 d) 44	1
3	1. If x = 3 is one of the roots of the quadratic equation $x^2 - 2kx - 6 = 0$, then the value ofk is	1

	a) $-\frac{1}{2}$ b) $\frac{1}{2}$ c) 3 d) 2	
4	The pair of equations y = 0 and y = -7 has: a) one solution b) two solutions c) infinitely many solutions d) no solution	1
5	Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is : a) 0 only b) 4 c) 8 only d) 0,8	1
6	The distance of the point (3, 5) from x-axis is k units, then k equals: a) 3 b) 4 c) 5 d) 8	1
7	If in Δ ABC and Δ PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ a) ΔPQR ~ΔCAB b) ΔPQR ~ΔABC c) ΔCBA ~ΔPQR d) ΔBCA ~ΔPQR	1
8	Which of the following is NOT a similarity criterion of triangles? a) AA b) SAS c) AAA d) RHS	1
9	In figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to (a) 60° (b) 70° (c) 80° (d) 90° 	
10	If $cosA = \frac{4}{5}, then\ tanA$ is (e) $\frac{3}{5}$	

(f) $\frac{3}{4}$

(g) $\frac{4}{3}$

(h) $\frac{1}{8}$

11 In figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to

(a) 60° (b) 70° (c) 80° (d) 90°

12 $(1 - \cos^2 A)$ is equal to

a) $\sin^2 A$ b) $\tan^2 A$ c) $1 - \sin^2 A$ d) $\sec^2 A$

(c) cos

13 The radius of a circle is same as the side of a square. Their perimeters are in the ratio

a) 1 : 1 b) $2 : \pi$ c) $\pi : 2$ d) $\sqrt{\pi} : 2$

14 The area of the circle is $154 cm^2$. The radius of the circle is

a) 7cm b) 14cm c) 3.5cm d) 17.5cm

15 When a dice is thrown once, the probability of getting an even number less than 4 isa) 1/4 b) 0 c) 1/2 d) 1/6

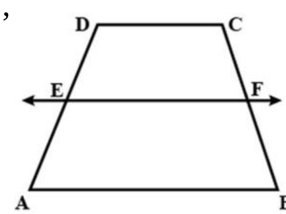
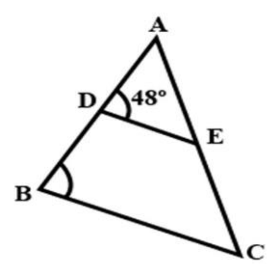
16 For the following distribution:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

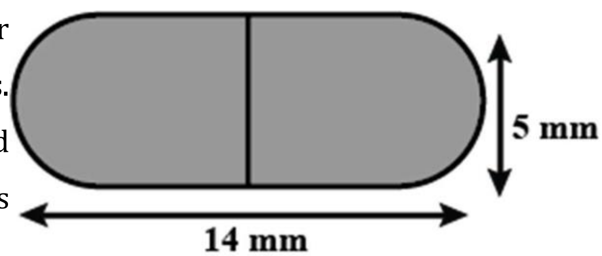
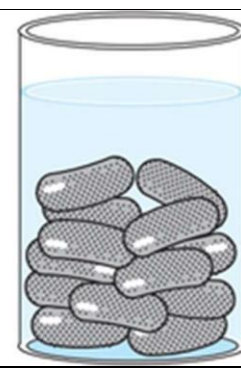
The lower limit of modal class is:

a) 15 b) 20 c) 10 d) 5

17	A rectangular sheet of paper 40cm x 22cm, is rolled to form a hollow cylinder ofheight 40cm. The radius of the cylinder(in cm) is : e) 3.5 f) 7 g) $\frac{80}{7}$ h) 5	
18	Consider the following frequency distribution: <table><tr><td>Class</td><td>0-6</td><td>6-12</td><td>12-18</td><td>18-24</td><td>24-30</td></tr><tr><td>Frequency</td><td>12</td><td>10</td><td>15</td><td>8</td><td>11</td></tr></table>The median class is: a) 6-12 b) 12-18 c) 18-24 d) 24-30	
19	DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option Assertion (A): The point (0, 4) lies on y-axis. Reason(R): The x-coordinate of a point on y-axis is zero (a) Both assertion (A) and reason (R) are true and reason (R) is the correctexplanation of assertion (A). (b) Both assertion (A) and reason (R) are true but reason (R) is not the correctexplanation of assertion (A). (c) Assertions (A) is true but reason (R) is false. (d) Assertions (A) is false but reason (R) is true.	

20	2. Assertion (A): The HCF of two numbers is 5 and their product is 150. Then their LCMis 40. **Reason(R): For any two positive integers a and b, HCF (a, b) x LCM (a, b) = a x b.** (a) Both assertion (A) and reason (R) are true and reason (R) is the correctexplanation of assertion (A). (b) Both assertion (A) and reason (R) are true but reason (R) is not the correctexplanation of assertion (A). (c) Assertions (A) is true but reason (R) is false. (d) Assertions (A) is false but reason (R) is true.	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21	Find whether the following pair of linear equations is consistent or inconsistent: 3x + 2y = 8 **6x – 4y = 9**	2
22	In the given figure, if ABCD is a trapezium in which AB ∥ CD ∥ EF, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$ 	2
	OR	
	In figure, if AD = 6cm, DB = 9cm, AE = 8cm and EC = 12cm and ∠ADE = 48˚. Find ∠ABC. . 	2
	OR	
23	The length of a tangent from a point A at distance 5cm from thecentre of the circle is 4cm. Find the radius of the circle.	
24	Evaluate: $\sin^2 60^\circ + 2\tan 45^\circ - \cos^2 30^\circ$.	2

25	Find the diameter of a circle whose area is equal to the sum of the areas of two circles ofradii 40cm and 9cm.	
	OR	
	A chord of a circle of radius 10cm subtends a right angle at the centre. Find the area of minor segment. (Use π = 3.14)	
	SECTION C	
	Section C consists of 6 questions of 3 marks each	
26	Prove that $\sqrt{3}$ is an irrational number.	3
27	Find the zeroes of the quadratic polynomial $4s^2 - 4s + 1$ and verify the relationshipbetween the zeroes and the coefficients.	3
28	The coach of a cricket team buys 4 bats and 1 ball for Rs. 2050. Later, she buys 3 batsand 2 balls for ₹1600. Find the cost of each bat and each ball.	3
	OR	
	A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.	
29	A circle touches all the four sides of quadrilateral ABCD. Prove that AB + CD = AD + BC.	3
30	Prove that $$(cosec\theta - cot\theta)^2 = \frac{1 - cos\theta}{1 + cos\theta}$$	
	OR	
30	Prove that sec A (1 – sin A) (sec A + tan A) = 1.	3
31	A bag contains 6 red, 4 black and some white balls. I. Find the number of white balls in the bag if the probability of drawing a white ball is $\frac{1}{2}$. II. How many red balls should be removed from the bag for the probability of drawing a white ball to be $\frac{1}{2}$?	3
	SECTION D	
	Section D consists of 4 questions of 5 marks each	

32	A train travels 360km at a uniform speed. If the speed had been 5km/h more, it wouldhave taken 1 hour less for the same journey. Find the speed of the train.	5
	OR	
	A motor boat whose speed is 18km/h in still water takes 1 hour more to go 24km upstreamthan to return downstream to the same spot. Find the speed of the stream	
33	Prove that If a line is drawn parallel to one side of a triangle to intersect the other twosides in distinct points, the other two sides are divided in the same ratio. In ΔPQR, S and T are points on PQ and PR respectively.$\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ. Prove that PQR is an isosceles triangle	5
34	A medicine capsule is in the shape of a cylinder with two hemispheres stuck at each of its ends. The length of the entire capsule is 14mm and the diameter of the capsule is 5mm. Find its surface area. 	5
	OR	
	A gulab jamun, contains sugar syrup up to about 30% of itsvolume. Find approximately how much syrup would be found in 45 gulab jamuns, each shape like cylinder with two hemispherical ends with length 5cm and diameter 2.8cm. 	
35	The following table gives the distribution of the life time of 400 neon lamps:	5

Life time (in hours)	Number of lamps
1500-2000	14
2000-2500	56

2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the average life time of a lamp.

SECTION E

CASE BASED QUESTIONS

36 India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher qualityproduction runs. The production of TV sets in a factory increases uniformly by a fixed number every year. Itproduced 16000 sets in 6th year and 22600 in 9th yeardistance by 9cm every week. During the special camp for 15 days, she started with40 throws and every day kept increasing the numberof throws by 12 to achieve this remarkable progress.

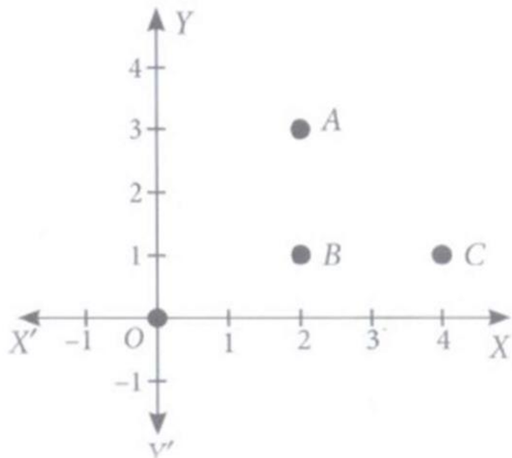
1) In which year, the production is 29,200 sets? 1

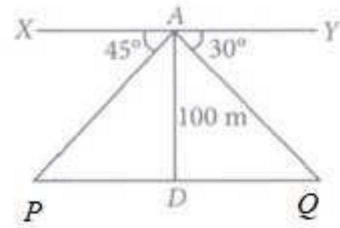
2) Find the production in the 8^{th} year. 2

OR

Find the production in first 3 years.

3) Find the difference of the production in 7th year and 4th year. 1

37	Alia and Shagun are friends living on the same street in Patel Nagar. Shagun's house is at theintersection of one street with another street on which there is a library. They both study in the same school and that is not far from Shagun's house. Suppose the school is situated at the point O, i.e., the origin, Alia's house is at A. Shagun's house is at B and library is at C. Based on the above information, answer the following questions. 	
	(i) How far is Alia's house from Shagun's house?	1
	(i) How far is the library from Shagun's house?	2
	(i) Show that for Shagun, school is farther compared to Alia's house and library. **OR** Show that Alia's house, shagun's house and library for an isosceles right triangle	1
38	A boy is standing on the top of light house. He observed that boat P and boat Q are approaching the light house from opposite directions. He finds that angle of depression of boat P is 45° and angle of depression of boat Q is 30°. He also knows that height of thelight house is 100 m.	

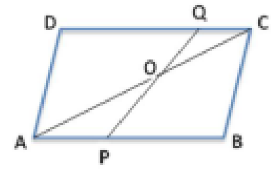
	 	
	(i) What is the measure of ∠APD?	1
	(ii) If ∠YAQ = 30°, then ∠AQD is also 30°, Why?	1
	(iii) Find length of PD OR Find length of DQ	2

Marking Scheme
Class X Session 2023-24
MATHEMATICS STANDARD (Code No.041)

TIME: 3 hours **MAX.MARKS: 80**

	SECTION A	
	Section A consists of 20 questions of 1 mark each.	
1.	(b) xy^2	1
2.	(b) 1 zero and the zero is '3'	1
3.	(b) $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$	1
4.	(c) 2 distinct real roots	1
5.	(c) 7	1
6.	(a) 1:2	1
7.	(d) infinitely many	1
8.	(b) $\frac{ac}{b+c}$	1
9.	(b) 100°	1
10.	(d) 11 cm	1
11.	(c) $\frac{\sqrt{b^2-a^2}}{b}$	1
12.	(d) cos A	1
13.	(a) 60°	1
14.	(a) 2 units	1
15.	(a) 10m	1
16.	(b) $\frac{4-\pi}{4}$	1
17.	(b) $\frac{22}{46}$	1
18.	(d) 150	1
19.	(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)	1
20.	(c) Assertion (A) is true but reason (R) is false.	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21.	Let us assume, to the contrary, that $\sqrt{2}$ is rational. So, we can find integers *a* and *b* such that $\sqrt{2} = \frac{a}{b}$ where *a* and *b* are coprime. So, b $\sqrt{2}$ = a. Squaring both sides, we get $2b^2 = a^2$. Therefore, 2 divides a^2 and so 2 divides a. So, we can write a = 2c for some integer c. Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$. This means that 2 divides b^2, and so 2 divides b Therefore, a and b have at least 2 as a common factor. But this contradicts the fact that a and b have no common factors other than 1. This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational. So, we conclude that $\sqrt{2}$ is irrational.	½ ½ ½ ½

22. ABCD is a parallelogram. ½



AB = DC = a

Point P divides AB in the ratio 2:3

$AP = \frac{2}{5}a$, $BP = \frac{3}{5}a$

point Q divides DC in the ratio 4:1. ½

$DQ = \frac{4}{5}a$, $CQ = \frac{1}{5}a$

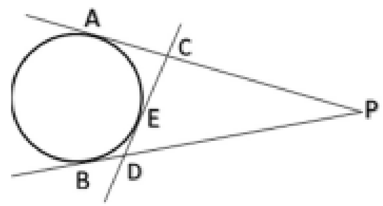
Δ APO ~ Δ CQO [AA similarity] ½

$$\frac{AP}{CQ} = \frac{PO}{QO} = \frac{AO}{CO}$$ ½

$$\frac{AO}{CO} = \frac{\frac{2}{5}a}{\frac{1}{5}a} = \frac{2}{1} \Rightarrow OC = \tfrac{1}{2}\,OA$$

23.

PA = PB; CA = CE; DE = DB [Tangents to a circle] ½



Perimeter of ΔPCD = PC + CD + PD

= PC + CE + ED + PD

= PC + CA + BD + PD

= PA + PB 1

Perimeter of ΔPCD = PA + PA = 2PA = 2(10) = 20 cm ½

24. $\because \tan(A+B) = \sqrt{3} \quad \therefore A+B = 60^0$ ...(1) ½

$\because \tan(A-B) = \frac{1}{\sqrt{3}} \quad \therefore A-B = 30^0$ ...(2) ½

Adding (1) & (2), we get $2A = 90^0 \Rightarrow A = 45^0$ ½

Also (1) –(2), we get $2B = 30^0 \Rightarrow B = 45^0$ ½

[or]

$2\,\mathrm{cosec}^2 30 + x\sin^2 60 - \frac{3}{4}\tan^2 30 = 10$

$\Rightarrow \quad 2(2)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$ 1

$\Rightarrow \quad 2(4) + x\left(\frac{3}{4}\right) - \frac{3}{4}\left(\frac{1}{3}\right) = 10$ ½

$\Rightarrow \quad 8 + x\left(\frac{3}{4}\right) - \frac{1}{4} = 10$

$\Rightarrow \quad 32 + x(3) - 1 = 40$ ½

$\Rightarrow \quad 3x = 9 \quad \Rightarrow \quad x = 3$

25. Total area removed $= \frac{\angle A}{360}\pi r^2 + \frac{\angle B}{360}\pi r^2 + \frac{\angle C}{360}\pi r^2$ ½

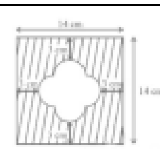
$= \frac{\angle A + \angle B + \angle C}{360}\pi r^2$

$= \frac{180}{360}\pi r^2$ ½

$= \frac{180}{360} \times \frac{22}{7} \times (14)^2$ ½ ½

$= 308\ cm^2$

[or]



The side of a square = Diameter of the semi-circle = a ½

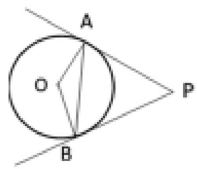
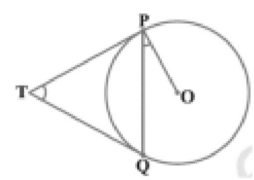
Area of the unshaded region

= Area of a square of side 'a' + 4(Area of a semi-circle of diameter 'a')

The horizontal/vertical extent of the white region = 14-3-3 = 8 cm ½

Radius of the semi-circle + side of a square + Radius of the semi-circle = 8 cm

	2 (radius of the semi-circle) + side of a square = 8 cm $2a = 8$ cm $\Rightarrow a = 4$ cm Area of the unshaded region = Area of a square of side 4 cm + 4 (Area of a semi-circle of diameter 4 cm) $= (4)^2 + 4 \times \frac{1}{2}\pi(2)^2 = (16 + 8\pi)$ cm^2	½ ½
	SECTION C	
	Section C consists of 6 questions of 3 marks each	
26.	Number of students in each group subject to the given condition = HCF (60,84,108) HCF (60,84,108) = 12 Number of groups in Music $= \frac{60}{12} = 5$ Number of groups in Dance $= \frac{84}{12} = 7$ Number of groups in Handicrafts $= \frac{108}{12} = 9$ Total number of rooms required = 21	½ ½ ½ ½ ½ ½
27.	$P(x) = 5x^2 + 5x + 1$ $\alpha+\beta = \frac{-b}{a} = \frac{-5}{5} = -1$ $\alpha\beta = \frac{c}{a} = \frac{1}{5}$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $= (-1)^2 - 2\left(\frac{1}{5}\right)$ $= 1 - \frac{2}{5} = \frac{3}{5}$ $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$ $= \frac{(\alpha+\beta)}{\alpha\beta} = \frac{(-1)}{\frac{1}{5}} = -5$	½ ½ ½ ½ ½ ½
28.	Let the ten's and the unit's digits in the first number be x and y, respectively. So, the original number = 10x + y When the digits are reversed, x becomes the unit's digit and y becomes the ten's Digit. So the obtain by reversing the digits= 10y + x According to the given condition. (10x + y) + (10y + x) = 66 i.e., 11(x + y) = 66 i.e., x + y = 6 ---- (1) We are also given that the digits differ by 2, therefore, either x – y = 2 ---- (2) or y – x = 2 ---- (3) If x – y = 2, then solving (1) and (2) by elimination, we get x = 4 and y = 2. In this case, we get the number 42. If y – x = 2, then solving (1) and (3) by elimination, we get x = 2 and y = 4. In this case, we get the number 24. Thus, there are two such numbers 42 and 24.	½ ½ ½ ½ ½ ½
	[or]	
	Let $\frac{1}{\sqrt{x}}$ be 'm' and $\frac{1}{\sqrt{y}}$ be 'n', Then the given equations become $2m + 3n = 2$ $4m - 9n = -1$	½ ½

	$(2m + 3n = 2)$ X- 2 $\Rightarrow -4m - 6n = -4$ ...(1)	
	4m - 9n = -1 $\quad 4m - 9n = -1$ ...(2)	
	Adding (1) and (2)	
	We get $-15n = -5 \Rightarrow n = \frac{1}{3}$	½
	Substituting $n = \frac{1}{3}$ in $2m + 3n = 2$, we get	½
	$2m + 1 = 2$	
	$2m = 1$	
	$m = \frac{1}{2}$	1
	$m = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$ and $n = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9$	
29.		
	$\angle OAB = 30°$	
	$\angle OAP = 90°$ [Angle between the tangent and the radius at the point of contact]	
	$\angle PAB = 90° - 30° = 60°$	½
	AP = BP [Tangents to a circle from an external point]	
	$\angle PAB = \angle PBA$ [Angles opposite to equal sides of a triangle]	½
	In ΔABP, $\angle PAB + \angle PBA + \angle APB = 180°$ [Angle Sum Property]	
	$60° + 60° + \angle APB = 180°$	
	$\angle APB = 60°$	½
	$\therefore$ ΔABP is an equilateral triangle, where AP = BP = AB.	
	PA = 6 cm	½
	In Right ΔOAP, $\angle OPA = 30°$	
	$\tan 30° = \frac{OA}{PA}$	
	$\frac{1}{\sqrt{3}} = \frac{OA}{6}$	½
	$OA = \frac{6}{\sqrt{3}} = 2\sqrt{3}cm$	½
	[or]	
		
	Let $\angle TPQ = \theta$	
	$\angle TPO = 90°$ [Angle between the tangent and the radius at the point of contact]	½
	$\angle OPQ = 90° - \theta$	
	TP = TQ [Tangents to a circle from an external point]	
	$\angle TPQ = \angle TQP = \theta$ [Angles opposite to equal sides of a triangle]	½
	In ΔPQT, $\angle PQT + \angle QPT + \angle PTQ = 180°$ [Angle Sum Property]	½
	$\theta + \theta + \angle PTQ = 180°$	½
	$\angle PTQ = 180° - 2\theta$	½
	$\angle PTQ = 2(90° - \theta)$	½
	$\angle PTQ = 2\angle OPQ$ [using (1)]	
30.	Given, $1 + \sin^2\theta = 3\sin\theta\cos\theta$	
	Dividing both sides by $\cos^2\theta$,	
	$\frac{1}{\cos^2\theta} + \tan^2\theta = 3\tan\theta$	
	$\sec^2\theta + \tan^2\theta = 3\tan\theta$	½
	$1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$	½
	$1 + 2\tan^2\theta = 3\tan\theta$	½
	$2\tan^2\theta - 3\tan\theta + 1 = 0$	½
	If $\tan\theta = x$, then the equation becomes $2x^2 - 3x + 1 = 0$	

$\Rightarrow (x-1)(2x-1) = 0$ x = 1 or $\frac{1}{2}$

tan θ = 1 or $\frac{1}{2}$ 1

31.

Length [in mm]	Number of leaves (f)	CI	Mid x	d	fd
118 – 126	3	117.5- 126.5	122	-27	-81
127 – 135	5	126.5– 135.5	131	-18	-90
136 – 144	9	135.5– 144.5	140	-9	-81
145 – 153	12	144.5 – 153.5	a = 149	0	0
154 – 162	5	153.5 – 162.5	158	9	45
163 – 171	4	162.5 – 171.5	167	18	72
172 – 180	2	171.5 – 180.5	176	27	54

2

½

½

Mean = a + $\frac{\sum fd}{\sum f}$ = 149 + $\frac{-8}{40}$

= 149 – 2.025 = 146.975

Average length of the leaves = 146.975

SECTION D

Section D consists of 4 questions of 5 marks each

32. Let the speed of the stream be x km/h.

The speed of the boat upstream = (18 – x) km/h and

the speed of the boat downstream = (18 + x) km/h. 1

The time taken to go upstream = $\frac{distance}{speed} = \frac{24}{18-x}$ hours

the time taken to go downstream = $\frac{distance}{spe} = \frac{24}{18+x}$ hours 1

According to the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$ 1

24(18 + x) – 24(18 – x) = (18 – x) (18 + x)

x^2 + 48x – 324 = 0

x = 6 or – 54 1

Since x is the speed of the stream, it cannot be negative.

Therefore, x = 6 gives the speed of the stream = 6 km/h. 1

[or]

Let the time taken by the smaller pipe to fill the tank = x hr.

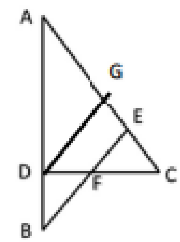
Time taken by the larger pipe = (x – 10) hr ½

Part of the tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of the tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$ 1

The tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together. ½

Part of the tank filled by both the pipes in 1 hour = $\frac{8}{75}$ ½

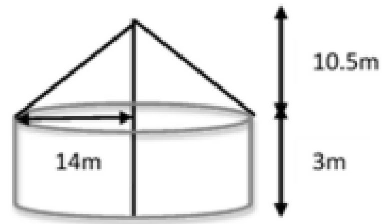
	Therefore, $\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$ $8x^2 - 230x + 750 = 0$ $x = 25, \frac{30}{8}$ Time taken by the smaller pipe cannot be 30/8 = 3.75 hours, as the time taken by the larger pipe will become negative, which is logically not possible. Therefore, the time taken individually by the smaller pipe is 25 hours and the larger pipe will be 25 – 10 =15 hours.	½ 1 ½ ½
33.	(a) Statement – ½ Given and To Prove – ½ Figure and Construction ½ Proof – 1 ½  [b] Draw DG ‖ BE In Δ ABE, $\frac{AB}{BD}=\frac{AE}{GE}$ [BPT] CF = FD [F is the midpoint of DC] ---(i) In Δ CDG, $\frac{DF}{CF}=\frac{GE}{CE}=1$ [Mid point theorem] GE = CE ---(ii) ∠CEF = ∠CFE [Given] CF = CE [Sides opposite to equal angles] ---(iii) From (ii) & (iii) CF = GE ---(iv) From (i) & (iv) GE = FD $\therefore \frac{AB}{BD}=\frac{AE}{GE} \Rightarrow \frac{AB}{BD}=\frac{AE}{FD}$	3 ½ ½ ½ ½
34.	Length of the pond, l= 50m, width of the pond, b = 44m Water level is to rise by, h = 21 cm = $\frac{21}{100}$ m Volume of water in the pond = lbh = 50 x 44 x $\frac{21}{100}$ m^3 =462 m^3 Diameter of the pipe = 14 cm Radius of the pipe, r = 7cm = $\frac{7}{100}$ m Area of cross-section of pipe = πr^2 $= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} = \frac{154}{10000}$ m^2 Rate at which the water is flowing through the pipe, h = 15km/h = 15000 m/h Volume of water flowing in 1 hour = Area of cross-section of pipe x height of water coming out of pipe $= \left(\frac{154}{10000} \times 15000\right) m^3$ Time required to fill the pond = $\frac{Volume\ of\ the\ pond}{Volume\ of\ water\ flowing\ in\ 1\ hour}$ $= \frac{462 \times 10000}{154 \times 15000}$ = 2 hours Speed of water if the rise in water level is to be attained in 1 hour = 30km/h	1 1 ½ ½ 1 1
	[or]	

Radius of the cylindrical tent (r) = 14 m

Total height of the tent = 13.5 m

Height of the cylinder = 3 m

Height of the Conical part = 10.5 m ½



Slant height of the cone (l) = $\sqrt{h^2 + r^2}$

$= \sqrt{(10.5)^2 + (14)^2}$

$= \sqrt{110.25 + 196}$

$= \sqrt{306.25} = 17.5$ m 1

Curved surface area of cylindrical portion

= 2πrh

$= 2\text{x} \frac{22}{7} \times 14 \times 3$ 1

= 264 m²

Curved surface area of conical portion

=πrl

$= \frac{22}{7} \times 14 \times 17.5$ 1

=770 m² ½

Total curved surface area = 264 m² + 770 m² = 1034 m²

Provision for stitching and wastage = 26 m² ½

Area of canvas to be purchased = 1060 m²

Cost of canvas = Rate × Surface area ½

= 500 x 1060 = ₹ 5,30,000/-

35.

Marks obtained	Number of students	Cumulative frequency
20 – 30	p	p
30 – 40	15	p + 15
40 – 50	25	p + 40
50 – 60	20	p + 60
60 – 70	q	p + q + 60
70 – 80	8	p + q + 68
80 - 90	10	p + q + 78
	90	

1

½

½

p + q + 78 = 90

p + q = 12

Median $= (l) + \frac{\frac{n}{2} - c}{f} . h$

$50 = 50 + \frac{45-(p+40)}{20} . 10$ ½

$\frac{45-(p+40)}{20} . 10 = 0$ ½

$45 - (p + 40) = 0$ ½

P = 5 ½

5 + q = 12

q = 7 1

Mode $= l + \frac{f1-f0}{2f1-f0-f2} . h$

	$= 40 + \frac{25-15}{2(25)-15-20} . 10$ $= 40 + \frac{100}{15} = 40 + 6.67 = 46.67$	
	SECTION E	
36.	(i) Number of throws during camp. a = 40; d = 12 $t_{11} = a + 10d$ $= 40 + 10 \times 12$ $= 160\ throws$	1
	(ii) a = 7.56 m; d = 9cm = 0.09 m n = 6 weeks t_n = a + (n-1) d = 7.56 + 6(0.09) = 7.56 + 0.54 Sanjitha's throw distance at the end of 6 weeks = 8.1 m (or) a = 7.56 m; d = 9cm = 0.09 m t_n =11.16 m t_n = a + (n-1) d 11.16 = 7.56 + (n-1) (0.09) 3.6 = (n-1) (0.09) n-1 = $\frac{3.6}{0.09}$ = 40 n = 41 Sanjitha's will be able to throw 11.16 m in 41 weeks.	½ ½ ½ ½ ½ ½ ½ ½
	(iii) a = 40; d = 12; n = 15 $S_n = \frac{n}{2}$ [2a + (n-1) d] $S_n = \frac{15}{2}$ [2(40) + (15-1) (12)] $= \frac{15}{2}$ [80 + 168] $= \frac{15}{2}$ [248] =1860 throws	 ½ ½
37.	(i) Let D be (a,b), then Mid point of AC = Midpoint of BD $\left(\frac{1+6}{2}, \frac{2+6}{2}\right) = \left(\frac{4+a}{2}, \frac{3+b}{2}\right)$ 4 + a = 7 3+ b = 8 a = 3 b = 5 Central midfielder is at (3,5)	 ½ ½

Basic Mathematics (241)
Marking Scheme
2023-24

Section A

1) (b) xy^2	1
2) (c) 20	1
3) (b) ½	1
4) (d) No Solution	1
5) (d) 0,8	1
6) (c) 5 Unit	1
7) (a) $\Delta PQR \sim \Delta\, CAB$	1
8) (d) RHS	1
9) (b) 70°	1
10) (b) ¾	1
11) (b) 45°	1
12) (a) $\sin^2 A$	1
13) (c) $\pi : 2$	1
14) (a) 7 cm	1
15) (d) $\frac{1}{6}$	1
16) (a) 15	1
17) (a) 3.5 cm	1
18) (b) 12-18	1
19) (a) Both assertion and reason are true and reason is the correct explanation of assertion.	1
20) (d) Assertion (A) is false but reason(R) is true.	1

SECTION B

21) $3x+2y = 8$

$6x - 4y = 9$

$a_1=3, \quad b_1=2, \quad c_1 = 8$

$a_2=6, \quad b_2=-4, \quad c_2 = 9$ 1

$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2} \qquad \frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2} \qquad \frac{c_1}{c_2} = \frac{8}{9}$ 1/2

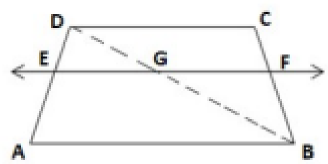
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given pair of linear equations are consistent. 1/2

---

22) Given:-AB II CD II EF

To prove:- $\frac{AE}{ED} = \frac{BF}{FC}$



Construction:- Join BD to intersect EF at G. 1/2

Proof:- in Δ ABD

EG II AB (EF II AB)

$\frac{AE}{ED} = \frac{BG}{GD}$ (by BPT)___________(1) 1/2

In ΔDBC

GF II CD (EF II CD)

$\frac{BF}{FC} = \frac{BG}{GD}$ (by BPT)___________(2) 1/2

$from$ (1) & (2)

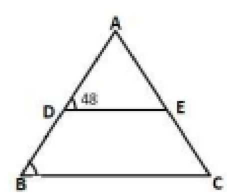
$\frac{AE}{ED} = \frac{BF}{FC}$ 1/2

OR

Given AD=6cm, DB=9cm

AE=8cm, EC=12cm, ∠ADE=48

To find:- ∠ABC=?



Proof:

In ΔABC

$\frac{AD}{DB} = \frac{6}{9} = \frac{2}{3}$ ......(1)

$\frac{AE}{EC} = \frac{8}{12} = \frac{2}{3}$ ....(2)

From (1) & (2) 1

$\frac{AD}{DB} = \frac{AE}{EC}$

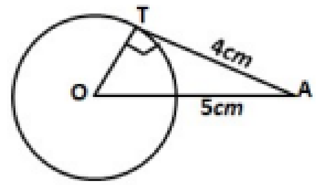
DE II BC (Converse of BPT)

∠ADE=∠ABC (Corresponding angles)

⇒ ∠ABC=48° 1

23) In Δ OTA, $\angle$OTA = 90°

By Pythagoras theorem



$OA^2 = OT^2 + AT^2$ 1/2

$(5)^2 = OT^2 + (4)^2$

$25 - 16 = OT^2$

$9 = OT^2$ 1/2

OT=3cm

radius of circle = 3cm. 1

24) $\text{Sin}^2 60^\circ + 2 \tan 45^\circ - \cos^2 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$ 1

$= \frac{3}{4} + 2 - \frac{3}{4}$

$= 2$ 1

25) Area of the circle= sum of areas of 2 circles

$\pi R^2 = \pi(40)^2 + \pi(9)^2$ 1/2

$\pi R^2 = \pi \times (40^2 + 9^2)$ 1/2

$R^2 = 1600 + 81$

$R^2 = 1681$

$R = 41\ cm.$ 1/2

$Diameter\ of\ given\ circle = 41 \times 2 = 82cm$ 1/2

OR

radius of circle = 10cm, $\theta = 90^\circ$

Area of minor segment = $\frac{\theta}{360^\circ} \pi r^2$ - Area of Δ

$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} \times b \times h$ 1/2

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 - \frac{1}{2} \times 10 \times 10$ 1/2

$= \frac{314}{4} - 50$

$= 78.5 - 50 = 28.5\ cm^2$ 1/2

Area of minor segment = $28.5\ cm^2$ 1/2

Section C

26) Let us assume that $\sqrt{3}$ be a rational number

$\sqrt{3}=\frac{a}{b}$ where a and b are co-prime. 1

squaring both the sides

$\left(\sqrt{3}\right)^2=\left(\frac{a}{b}\right)^2$ 1/2

$3=\frac{a^2}{b^2} \Rightarrow a^2=3b^2$

a^2 is divisible by 3 so a is also divisible by 3________(1)

let a=3c for any integer *c*.

$(3c)^2=3b^2$ 1/2

$9c^2=3b^2$

$b^2=3c^2$

since b^2 is divisible by 3 so, b is also divisible by 3 _____(2)

From (1) & (2) we can say that 3 in a factor of a and b 1/2

which is contradicting the fact that a and b are co- prime.

Thus, our assumption that $\sqrt{3}$ is a rational number is wrong.

Hence, $\sqrt{3}$ is an irrational number. 1/2

27) $P(S)=4S^2-4S+1$

$4S^2-2S-2S+1=0$

$2S(2S-1)-1(2S-1)=0$

$(2S-1)(2S-1)=0$

$S=½$ $S=½$ 1

$a=4$ $b=-4$ $c=1$ $\propto=½$ $\beta=½$

$\propto+\beta=\frac{-b}{a}$, $\propto\beta=\frac{c}{a}$

$\frac{1}{2}+\frac{1}{2}=\frac{-4}{4}$, $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$ 1

$\frac{1+1}{2}=\frac{+4}{4}$, $\frac{1}{4}=\frac{1}{4}$

$\frac{2}{2}=1$

$1=1$ 1

28) Let cost of one bat be Rs x

Let cost of one ball be Rs y 1/2

ATQ

$4x+1y=2050$__________(1)

$3x+2y=1600$__________(2) 1/2

from (1) $4x+1y=2050$

$y=2050-4x$ 1/2

$Substite\ \ value\ of\ y\ in\ (2)$

$3x + 2(2050 - 4x) = 1600$

$3x + 4100 - 8x = 1600$

$-5x = -2500$

$x = 500$ 1/2

$Substiture\ value\ of\ x\ in\ (1)$

$4x + 1y = 2050$

$4(500) + y = 2050$

$2000 + y = 2050$

$y = 50$ 1/2

Hence

Cost of one bat = Rs. 500 1/2

Cost of one ball = Rs. 50

OR

Let the fixed charge for first 3 days= Rs. x

And additional charge after 3 days= Rs. y 1/2

ATQ

$x + 4y = 27$---------------(1)

$x + 2y = 21$ ---------------(2) 1/2

Subtract eq^n (2) from (1)

$2y = 6$

$y = 3$ 1

Substitute value of y in (2)

$x + 2(3) = 21$

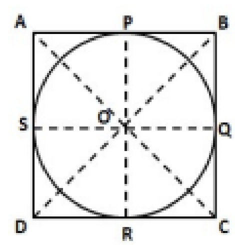
$x = 21 - 6$

$x = 15$ 1

Fixed charge= Rs. 15

Additional charge per day = Rs. 3

29) Given circle touching sides of ABCD at P,Q,R and S



To prove- AB+CD=AD+BC

Proof- 1

AP=AS--------(1) tangents from an external point

PB=BQ--------(2) to a circle are equal in length

DR=DS--------(3)

CR=CQ--------(4) 1

Adding eq^n (1),(2),(3) & (4)

AP+BP+DR+CR=AS+DS+BQ+CQ

AB+DC=AD+BC 1

30) $(cosec\theta - \cot\theta)^2 = \frac{1-cos\theta}{1+cos\theta}$

LHS=$(cosec\theta - cot\theta)^2$

$= \left(\frac{1}{sin\theta} - \frac{\cos\theta}{sin\theta}\right)^2$ 1/2

$= \left(\frac{1-\cos\theta}{sin\theta}\right)^2$ 1/2

$= \dfrac{(1-\cos\theta)^2}{sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{1-cos^2\theta}$ 1

$= \dfrac{(1-\cos\theta)^2}{(1-cos\theta)(1+cos\theta)}$

$= \frac{1-cos\theta}{1+cos\theta}$ =RHS 1

LHS = RHS, Hence Proved

OR

$secA\ (1-sinA)(\sec A + tanA)$=1

LHS=$\frac{1}{cosA}(1-sinA)\left(\frac{1}{cosA}+\frac{sinA}{cosA}\right)$ 1

= $\frac{(1-\sin A)}{\cos A}\frac{(1+\sin A)}{\cos A}$

= $\frac{(1-\sin A)\ (1+\sin A)}{cos^2 A}$

= $\frac{1-sin^2 A}{cos^2 A}$ (1-$sin^2 A = cos^2 A$) 1

= $\frac{cos^2 A}{cos^2 A}$

= 1 = RHS 1

LHS=RHS. Hence Proved

31) (i) Red balls= 6 , Black balls = 4 , White balls = x

P(white ball) = $\frac{x}{10+x} = \frac{1}{3}$ 1

⇒ 3x = 10 + x ⇒ x= 5 white balls 1/2

(ii) Let y red balls be removed, black balls = 4, white balls = 5

P(white balls)= $\frac{5}{(6-y)+4+5} = \frac{1}{2}$ 1

$\Rightarrow \frac{5}{15-y} = \frac{1}{2} \Rightarrow 10 = 15 - y \Rightarrow y = 5$ ½

So 5 balls should be removed.

Section D

32) Let the speed of train be $x\ km/hr$ 1/2

distance= 360 km

Speed = $\frac{distance}{time}$

Time = $\frac{360}{x}$ 1/2

New speed = $(x+5)km/hr$

Time = $\frac{D}{S}$

$x + 5 = \frac{360}{\left(\frac{360}{x}-1\right)}$ 1

$(x+5)\left(\dfrac{360}{x}-1\right) = 360$

$(x+5)(360-x) = 360x$

$-x^2 - 5x + 1800 = 0$

$x^2 + 5x - 1800 = 0$ 1

$x^2 + 45x - 40x - 1800 = 0$

$x(x+45) - 40(x+45) = 0$

$(x+45)(x-40) = 0$ 1

$x + 45 = 0$, $x - 40 = 0$

$x = -45$, $x = 40$

Speed cannot be negative

Speed of train =40km/hr 1

OR

Let the speed of the stream=xkm/hr 1/2

Speed of boat= 18 km/hr

Upstream speed= $(18-x)km/hr$

Downstream speed=$(18+x)km/hr$ 1/2

Time taken (upstream)=$\frac{24}{(18-x)}$

Time taken (downstream)=$\frac{24}{(18+x)}$

ATQ

$$\frac{24}{(18-x)} = \frac{24}{(18+x)}+1$$ 1

$$\frac{24}{(18-x)} - \frac{24}{(18+x)} = 1$$

$24(18+x) - 24(18-x) = (18-x)(18+x)$

$24(18+x-18+x) = (18)^2 - x^2$

$24(2x) = 324 - x^2$

$48x - 324 + x^2 = 0$

$x^2 + 48x - 324 = 0$ 1

$x^2 - 6x + 54x - 324 = 0$

$x(x-6) + 54(x-6) = 0$

$(x-6)(x+54) = 0$ 1

$x - 6 = 0$, $x + 54 = 0$

$x = 6$, $x = -54$

Speed cannot be negative 1

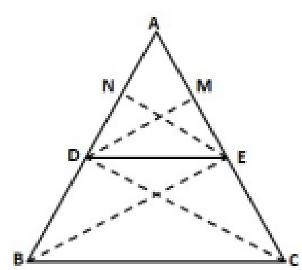
Speed of stream=$6km/hr$

33) Given ΔABC , DE || BC

To prove $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: join BE and CD 1/2

Draw DM $\perp$ AC and EN $\perp$ AB



Proof: $Area\ of\ \ \Delta ADE = \frac{1}{2}$ x b x h

$= \frac{1}{2}$x AD x EN-----------------------(1)

Area $(\Delta DBE) = \frac{1}{2}$x DB x EN--------(2)

Divide eq^n (1) by (2)

$\frac{\text{ar } \Delta ADE}{\text{ar } \Delta DBE} = \frac{\frac{1}{2} \text{x } AD \text{ x } EN}{\frac{1}{2} \text{x } DB \text{ x } EN} = \frac{AD}{DB}$------------(3) 1 area $\Delta ADE = \frac{1}{2}$ x AE x DM --------(4)

area $\Delta DEC = \frac{1}{2}$ x EC x DM --------(5)

Divide eq^n (4) by (5)

$\frac{\text{ar } \Delta ADE}{\text{ar } \Delta DEC} = \frac{\frac{1}{2} X\ AE\ X\ DM}{\frac{1}{2} X\ EC\ X\ DM} = \frac{AE}{EC}$-------------(6) 1

ΔBDE and ΔDEC are on the same base DE and between same parallel lines BC and DE

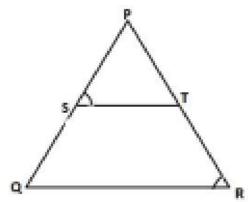
$\therefore\ area\ (\Delta DBE) = ar\ (\ DEC)$

$hence$

$\frac{ar(\ \Delta ADE)}{ar\ (\Delta DBE} = \frac{ar(\ \Delta ADE)}{ar\ (\Delta DEC)}$ [LHS of (3) =RHS of (6)]

$\frac{AD}{DB} = \frac{AE}{EC}$ [RHS of (3) = RHS of (6) 1/2

Since $\frac{PS}{SQ} = \frac{PT}{TR}$ $\therefore ST\ \parallel QR\ \ (by\ converse\ of\ BPT)$



∠PST = ∠PQR (Corresponding angles) 1

But ∠PST = ∠PRQ (given)

∠PQR = ∠PRQ

PR = PQ (sides opposite to equal angles are equal

Hence ΔPQR is isosceles. 1

34) Diameter of cylinder and hemisphere = 5mm radius, (r) = $\frac{5}{2}$

Total length = 14mm

Height of cylinder = 14 - 5 = 9mm 1

CSA of cylinder = $2\pi rh$

$= 2 \text{ x } \frac{22}{7} \text{ x } \frac{5}{2} \text{ x } 9$

$= \frac{990}{7}\ mm^2$ 1

CSA of hemispheres = $2\pi r^2$

$= 2\text{x} \frac{22}{7} \text{ x } \left(\frac{5}{2}\right)^2$

$= \frac{275}{7}\ mm^2$ 1

CSA of 2 hemispheres = 2 x $\frac{275}{7}$

$= \frac{550}{7}\ mm^2$ 1

Total area of capsule = $\frac{990}{7} + \frac{550}{7}$

$= \frac{1540}{7}$

$= 220\ mm^2$ 1

OR

Diameter of cylinder = 2.8 cm

radius of cylinder = $\frac{2.8}{2}$ = 1.4 cm

radius of cylinder = *radius* of hemisphere = 1.4 cm

Height of cylinder = 5-2.8 1

= 2.2 cm

Volume of 1 Gulab jamun = vol. of cylinder + 2 x vol. of hemisphere

$= \pi r^2 h + 2 \times \frac{2}{3} \pi r^3$ 1

$\frac{22}{7} \times (1.4)^2 \times 2.2 + 2 \times \frac{2}{3} \times \frac{22}{7} \times (1.4)^3$

$= 13.55 + 11.50$

$= 25.05\ cm^3$ 1

$volume\ of\ 45\ Gulab\ jamun = 45 \times 25.05$

$syrup\ in\ 45\ Gulab\ jamun = 30\% \times 45 \times 25.05$

$= \frac{30}{100} \times 45 \times 25.05$ 1

$= 338.175\ cm^3$

$\approx 338\ cm^3$

35)

Life time (in hours)	Number of lamps(f)	Mid x	d	fd
1500-2000	14	1750	-1500	-21000
2000-2500	56	2250	-1000	-56000
2500-3000	60	2750	-500	-30000
3000-3500	86	3250	0	0
3500-4000	74	3750	500	37000
4000-4500	62	4250	1000	62000
4500-5000	48	4750	1500	72000
	400			64000

2

Mean = a + $\frac{\Sigma fd}{\Sigma f}$ 1/2

a = 3250 1/2

Mean $= 3250 + \frac{64000}{400}$ 1

$= 3250 + 160$

$= 3410$

Average life of lamp is 3410 hr 1

Section E

36) $a_6 = 16000$ $a_9 = 22600$

$a+5d=16000$-------(1)

$a+8d=22600$ --------(2)

substitute $a = 1600 - 5d$ from (1)

$16000-5d + 8d = 22600$

$3d = 22600-16000$

$3d=6600$

$d = \frac{6600}{3} = 2200$

$a = 16000-5(2200)$

$a = 16000-11000$

$a = 5000$

(i) $a_n = 29200$, $a = 5000$, $d = 2200$

$a_n = a + (n-1)d$

$29200 = 5000 + (n - 1)2200$ 1/2

$29200-5000 = 2200n-2200$

$24200+2200=2200n$

$26400=2200n$

$n= \frac{264}{22}$

$n=12$ 1/2

in 12th year the production was Rs 29200

(ii) $n=8$, $a=5000$, $d=2200$

$a_n = a + (n-1)d$ 1/2

$= 5000+(8-1)2200$ 1/2

$= 5000+7 \times 2200$

$= 5000+15400$ 1/2

$= 20400$

The production during 8th year is $= 20400$ 1/2

OR

$n = 3$, $a = 5000$, $d = 2200$

$s_n = \frac{n}{2}[\,2a + (n-1)d]$ 1/2

$= \frac{3}{2}$ [2(5000) + (3-1) 2200]

$S_3 = \frac{3}{2}$ (10000 + 2 x 2200) 1/2

$= \frac{3}{2}$ (10000 + 4400) 1/2

= 3 x 7200

= 21600 1/2

The production during first 3 year is 21600

(iii) a_4 = a+3d

= 5000 + 3 (2200)

= 5000 + 6600

= 11600 1/2

a_7 = a+6d

= 5000 + 6 x 2200

=5000 + 13200

= 18200

$a_7 - a_4$ = 18200-11600 = 6600 1/2

37) coordinates of A (2, 3) Alia's house

coordinates of B (2, 1) Shagun's house

coordinates of C (4,1) Library

(i) AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2-2)^2 + (1-3)^2}$ 1/2

$= \sqrt{(0^2 + (-2)^2}$

$AB = \sqrt{0+4} = \sqrt{4}$ = 2 units 1/2

Alia's house from shagun's house is 2 units

(ii) C(4,1), B (2,1)

CB = $\sqrt{(2-4)^2 + (1-1)^2}$ 1/2

$= \sqrt{(-2)^2 + 0^2}$

$= \sqrt{4+0} = \sqrt{4}$ = 2 unit 1/2

(iii) 0(0,0), B(2,1)

OB = $\sqrt{(2-0)^2 + (1-0)^2}$

$= \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$ units 1

Distance between Alia's house and Shagun's house, AB = 2 units

Distance between Library and Shagun's house, CB = 2 units 1/2

OB is greater than AB and CB, 1/2

For shagun, school [O] is farther than Alia's house [A] and Library [C]

OR

C (4, 1), A(2, 3)

CA = $\sqrt{(2-4)^2 + (3-1)^2}$

= $\sqrt{(-2)^2 + 2^2}$+ = $\sqrt{4+4}$ = $\sqrt{8}$

= $2\sqrt{2}$ units AC^2= 8 1

Distance between Alia's house and Shagun's house, AB = 2 units

Distance between Library and Shagun's house, CB = 2 units 1/2

$AB^2 + BC^2 = 2^2 + 2^2 = 4 + 4 = 8 = AC^2$ ½

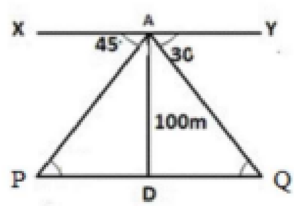
Therefore A, B and C form an isosceles right triangle.

38)

(i) XY ∥PQ and AP is transversal.

∠APD = ∠PAX (alternative interior angles) 1/2

∠APD=45° 1/2



(ii) Since XY || PQ and AQ is a transversal

so alternate interior angles are equal

hence ∠YAQ = ∠AQD=30° 1/2

(iii) In ΔADP, $\theta = 45^\circ$

$\tan\theta = \frac{P}{B}$

$\tan 45^\circ = \frac{100}{PD}$ 1/2

PD=100 m

1/2

Boat P is 100 m from the light house 1

OR

In ΔADQ, $\theta = 30^\circ$

$\tan\theta = \frac{P}{B}$ 1/2

$\tan 30 = \frac{100}{DQ}$

$\frac{1}{\sqrt{3}} = \frac{100}{DQ}$ 1/2

DQ = $100\sqrt{3}$ m 1


Best Wishes