

CHAPTER-7

INTEGRALS

01 MARK TYPE QUESTIONS

12.	$\int \frac{dx}{1+\cos 2x}$ is equal to,	
	(a) $\tan x + c$	(b) $\frac{1}{2} \tan x + c$
	(c) $2 \tan x + c$	(d) none of these
13.	$\int_{-2}^2 x dx$ is equals to,	1
	(a) 0	(b) 2
	(c) 4	(d) 1
14.	$\frac{d}{dx} \int f(x) dx$ is equals to,	1
	(a) $f'(x)$	(b) $f(x)$
	(c) $f(x')$	(d) $f'(x')$
15.	What is the value of $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$	1
	(a) $\frac{\pi}{2}$	(b) $\frac{\pi}{4}$
	(c) $\frac{\pi}{8}$	(d) $\frac{\pi}{12}$
16.	What is the value of $\int_1^e \left(\frac{1+\log x}{x} \right) dx$	1
	(a) $\frac{3}{2}$	(b) $\frac{1}{2}$
	(c) e	(d) $\frac{1}{e}$
17.	What is the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x dx$	1
	(a) 0	(b) 1
	(c) -1	(d) 2
18.	Value of $\int_0^1 \left(\frac{x}{1+x} \right) dx$ is	1
	(a) $1 - \log 2$	(b) $\log 2 - 1$
	(c) $1 + \log 2$	(d) $\log 2$
19.	Assertion (A): $\int \frac{dx}{x^2+2x+3} = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x+1}{2} \right) + c$ Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$ (a) Both A and R are true and R is correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false and R is True	1
20.	Assertion (A): $\int e^x [\sin x + \cos x] dx = e^x \sin x + c$ Reason (R): $\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$	1

	(a) Both A and R are true and R is correct explanation of A (b) Both A and R are true but R is NOT the correct explanation of A (c) A is true but R is false (d) A is false and R is True	
21.	If $\frac{d}{dx}(f(x)) = 5x^4 - \frac{4}{x^5}$ such that $f(2) = 0$. Then $f(x)$ is (a) $x^5 + \frac{1}{x^4} - \frac{129}{8}$ (b) $x^5 + \frac{1}{x^4} + \frac{129}{8}$ (c) $x^5 + \frac{1}{x^4} - \frac{513}{16}$	1 $x^5 + \frac{1}{x^4} + \frac{513}{16}$
22.	$\int \frac{1}{\sin^2 x \cos^2 x} dx$ equals (a) $\tan x + \cot x + C$ (b) $\tan x - \cot x + C$ (c) $\tan x \cot x + C$ (d) $\tan x - \cot 2x + C$	1
23.	$\int \frac{1}{x(x^3+1)} dx$ equals (a) $\frac{1}{3} \log \log \left \frac{x^3}{x^3-1} \right + C$ (b) $\frac{1}{3} \log \log \left \frac{x^3+1}{x^3} \right + C$ (c) $\frac{1}{3} \log \log \left \frac{x^3}{x^3+1} \right + C$ (d) $\frac{1}{3} \log \log \left \frac{x^3-1}{x^3} \right + C$	1
24.	$\int \frac{5x^4 + 5^x 5^1}{x^5 + 5^x} dx$ equals (a) $5^x - x^5 + C$ (b) $5^x + x^5 + C$ (c) $(5^x - x^5)^{-1} + C$ (d) $\log(5^x + x^5) + C$	1
25.	$\int e^x \sec \sec x (1 + \tan x) dx$ equals (a) $e^x \cos x + C$ (b) $e^x \sec x + C$ (c) $e^x \sin x + C$ (d) $e^x \tan x + C$	1
26.	$\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to (a) $2(\sin x + x \cos \theta) + C$	1

	(b) $2(\sin x - x \cos \theta) + C$ (c) $2(\sin x + 2x \cos \theta) + C$ (d) $2(\sin x - 2x \cos \theta) + C$	
27.	$\int_0^{2/3} \frac{1}{4+9x^2} dx$ is equal to (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{24}$ (d) $\frac{\pi}{4}$	1
28.	$\int_{-1}^1 \frac{ x-2 }{x-2} dx, x \neq 2$ is equal to (a) 1 (b) -1 (c) 2 (d) -2	1
29.	The value of the integral $\int_0^{\frac{\pi}{2}} \log \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx$ is (a) 2 (b) $\frac{3}{4}$ (c) 0 (d) -2	1
30.	$\int_{a+c}^{b+c} f(x) dx$ is equal to (a) $\int_a^b f(x-c) dx$ (b) $\int_a^b f(x+c) dx$ (c) $\int_a^b f(x) dx$ (d) $\int_{a-c}^{b-c} f(x) dx$	1
31.	Anti derivative of $\sin \sin(ax+b)$ is (a) $\cos \cos(ax+b) + c$ (b) $a \cos \cos(ax+b) + c$ (c) $-\frac{\cos \cos(ax+b)}{a} + c$ (d) $-\frac{\cos \cos(ax+b)}{b} + c$	1
32.	$\int e^{2x} dx =$ (a) $e^x + c$ (b) $\frac{e^{2x}}{2} + c$ (c) $x^2 + c$ (d) $\frac{x^3}{3} + c$	1
33.	$\int \cos \cos \frac{7\pi}{6} dx =$ (a) $\frac{7\pi}{6} x + c$ (b) $\frac{5\pi}{6} x + c$ (c) $\frac{\pi}{6} x + c$ (d) $\frac{\pi}{3} x + c$	1
34.	$\int e^{(\sin x)^2} \sin 2x dx =$	1

	(a) $(\sin \sin x)^2 + c$ (b) $e^{(\cos x)^2} + c$ (c) $e^{(\sin x)^2} + c$ (d) none of these	
35.	$\int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx =$ (a) 0 (b) π (iii) 2π (iv) $\frac{\pi}{2}$	1
36.	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^3 (\sin x)^4 dx =$ (a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$	1
37.	$\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$ (a) $e^x + c$ (b) $\frac{e^x}{x} + c$ (c) $\frac{e^x}{x^2} + c$ (d) none of these	1
38.	$\int (e^x + 1)^2 e^x dx$ (a) $e^x + 1 + c$ (b) $(e^x + 1)^2 + c$ (c) $\frac{(e^x + 1)^3}{3} + c$ (d) $e^{2x} + c$	1
39.	$\int_0^3 [x] dx =$, where $[x]$ means the greatest integer less than or equal to x . (a) 0 (b) 3 (c) 2 (d) 1	1
40.	$\int x d(x^2 + 2)$ (a) $\frac{x^2}{2} + c$ (b) $x + c$ (c) $\frac{2x^3}{3} + c$ (d) $\frac{x^4}{4} + c$	1
41.	$\int 2^x 3^x dx$ is equal to (a) $\frac{2^x}{\ln 2} + C$ (b) $\frac{3^x}{\ln 3} + C$ (c) $\frac{2^x 3^x}{\ln 2 \ln 3} + C$ (d) $\frac{6^x}{\ln 6} + C$	1
42.	If $\int \frac{\sqrt{\cot x}}{\sin x \cos x} dx = A\sqrt{\cot x} + K$, then the value of A is __ (a) 2 (b) 1 (c) -2 (d) -1	1
43.	The anti-derivative of $\int \frac{\sin^2 x}{\cos^4 x} dx$ is (a) a polynomial of degree 5 in $\sin x$ (b) a polynomial of degree 4 in $\tan x$ (c) a polynomial of degree 5 in $\tan x$ (d) a polynomial of degree x in $\cos x$	1
44.	$\int \frac{x^9}{(4x^2+1)^6} dx$ is equal to (a) $\frac{1}{5x} \left(4 + \frac{1}{x^2} \right)^{-5} + C$ (c) $\frac{1}{5} \left(4 + \frac{1}{x^2} \right)^{-5} + C$ (b) $\frac{1}{10} \left(4 + \frac{1}{x} \right)^{-5} + C$ (d) $\frac{1}{10} \left(4 + \frac{1}{x^2} \right)^{-5} + C$	1
45.	$\int \frac{dx}{x(x^n-1)}$ is equal to	1

	(a) $\frac{1}{n} \log \left 1 - \frac{1}{x^n} \right + C$ (c) $\frac{1}{n} \log \left \frac{x^n}{x^{n-1}} \right + C$ (b) $\frac{1}{x^n} \log \left \frac{x^n}{x^{n-1}} \right + C$ (d) $\frac{1}{x^n} \log \left \frac{x^{n-1}}{x^n} \right + C$	
46.	$\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx = Px + Q \log 4e^x + 5e^{-x} + \text{Constant}$, then (a) $P = \frac{-1}{8}, Q = \frac{-7}{8}$ (c) $P = \frac{1}{8}, Q = \frac{7}{8}$ (b) $P = \frac{-1}{8}, Q = \frac{7}{8}$ (d) $P = \frac{1}{8}, Q = \frac{-7}{8}$	1
47.	The value of $\int_{-2}^3 1 - x^2 dx$ is (a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$	1
48.	If $\int_0^\pi xf(\sin x)dx = A \int_0^{\frac{\pi}{2}} f(\sin x)dx$, then A is (a) 2π (b) π (c) $\frac{\pi}{2}$ (d) 0	1
49.	$\int_0^\pi \sin x dx$ is (a) 2 (b) 2π (c) π (d) 0	1
50.	$\int_0^2 [x^2] dx$ is (a) $2 - \sqrt{2}$ (b) $2 - \sqrt{2}$ (c) $\sqrt{2} - 1$ (d) $-\sqrt{2} - \sqrt{3} + 5$	1

ANSWERS:

Q. NO	ANSWER	MARKS
1.	<p>a) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$</p> <p>let $I = \int \tan^{-1}\sqrt{x} dx$, put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2\sqrt{x} dt \Rightarrow 2t dt$</p> <p>so, $I = \int \tan^{-1}t 2t dt$</p> $= \tan^{-1}t 2\frac{t^2}{2} - \int \frac{1}{1+t^2} \cdot 2\frac{t^2}{2} dt \quad (\text{integrating by parts})$ $= t^2 \tan^{-1}t - \int \frac{1}{1+t^2} \cdot t^2 dt$ $= t^2 \tan^{-1}t - \int \frac{1+t^2-1}{1+t^2} dt$ $= t^2 \tan^{-1}t - \int \left(1 - \frac{1}{1+t^2}\right) dt$ $= t^2 \tan^{-1}t - [t - \tan^{-1}t] = t^2 \tan^{-1}t - t + \tan^{-1}t = \tan^{-1}t (t^2 + 1) - t = (x+1)$ $\tan^{-1}\sqrt{x} - \sqrt{x} + C$	1
2.	<p>(d) 2</p> <p>We have $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x dx = [\tan x]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \tan \frac{\pi}{4} - \tan(-\frac{\pi}{4}) = 1 + 1 = 2$</p>	1
3.	<p>a) $\tan(xe^x) + C$</p> <p>let $I = \int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$, put $xe^x = t \Rightarrow (xe^x + e^x) dx = dt$</p> $\Rightarrow e^x(x+1) dx = dt$ <p>So, $I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t dt = \tan t + C = \tan(xe^x) + C$</p>	1
4.	<p>(c) $\tan x - \cot x + C$</p> <p>$I = \int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \sec^2 x dx + \int \cosec^2 x dx = \tan x - \cot x + C$</p>	1
5.	<p>a) $a = \frac{-1}{8}$, b = $\frac{7}{8}$</p>	1
6.	<p>a) $\frac{4-\pi}{8}$</p> <p>$\int_0^{\frac{\pi}{8}} \tan^2(2x) dx = \int_0^{\frac{\pi}{8}} \sec^2 2x - 1 dx = [\frac{\tan 2x}{2} - x]_0^{\frac{\pi}{8}} = \frac{\tan \frac{\pi}{4}}{2} - \frac{\pi}{8} - 0 = \frac{1}{2} - \frac{\pi}{8} =$</p>	1
7.	<p>(b) $2 \log 2$</p> <p>$I = \int_{-1}^1 \frac{x^3 + x + 1}{x^2 + 2 x + 1} dx = \int_{-1}^1 \frac{x^3}{x^2 + 2 x + 1} dx + \int_{-1}^1 \frac{ x + 1}{x^2 + 2 x + 1} dx$</p> $= 0 + 2 \int_0^1 \frac{ x + 1}{(x+1)^2} dx \quad [\text{odd function} + \text{even function}]$ $= 2 \int_0^1 \frac{x+1}{(x+1)^2} dx$ $= 2 \int_0^1 \frac{1}{x+1} dx = 2 [\log x+1]_0^1 = 2 \log 2$	1
8.	<p>a) $\frac{8}{\pi}$</p> <p>since $I = \int_{-2}^2 x \cos \pi x dx = 2 \int_0^2 x \cos \pi x dx = 2 \left\{ \int_0^{\frac{1}{2}} x \cos \pi x dx + \right.$</p>	1

	$\int_{\frac{1}{2}}^{\frac{3}{2}} x \cos \pi x dx + \int_{\frac{3}{2}}^2 x \cos \pi x dx = \frac{8}{\pi}$	
9.	a) $\frac{1}{\sqrt{3}}$ $\int_0^{\frac{\pi}{6}} \sec^2(x - \frac{\pi}{6}) dx = [\tan \tan(x - \frac{\pi}{6})]_0^{\frac{\pi}{6}} = \tan(\frac{\pi}{6} - \frac{\pi}{6}) - \tan(0 - \frac{\pi}{6}) = \tan 0 - \tan(-\frac{\pi}{6}) = 0 + \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$	1
10.	(b) $\frac{ax^2}{2} + bx$ Given, $\frac{d}{dx} [f(x)] = ax + b$ and $f(0) = 0$ On integrating both sides, we have $f(x) = \int (ax + b) dx = \frac{ax^2}{2} + bx + C$ $\Rightarrow f(x) = \frac{ax^2}{2} + bx + C \dots\dots (i)$ Also, $f(0) = 0$, we have from (i) $f(0) = C$ $\Rightarrow 0 = C$ Putting in (i), we have $f(x) = \frac{ax^2}{2} + bx$	1
11.	(c)	1
12.	(b)	1
13.	(c)	1
14.	(b)	1
15.	(b)	1
16.	(a)	1
17.	(a)	1
18.	(a)	1
19.	(d)	1
20.	(a)	1
21.	(c)	1
22.	(b)	1
23.	(c)	1
24.	(d)	1
25.	(b)	1
26.	(a)	1
27.	(c)	1
28.	(b)	1
29.	(c)	1
30.	(b)	1
31.	(c)	1
32.	(d)	1
33.	(b)	1
34.	(c)	1
35.	(b)	1
36.	(a)	1
37.	(b)	1
38.	(c)	1

39.	(b)	1
40.	(c)	1
41.	(d) $\frac{6^x}{\ln 6} + C$	1
42.	(c) -2	1
43.	(c) a polynomial of degree 3 in $\tan x$	1
44.	(d) $\frac{1}{10} \left(4 + \frac{1}{x^2} \right)^{-5} + C$	1
45.	(a) $\frac{1}{n} \log \left 1 - \frac{1}{x^n} \right + C$	1
46.	(b) $P = \frac{-1}{8}, Q = \frac{7}{8}$	1
47.	(d) $\frac{28}{3}$	1
48.	(b) π	1
49.	(a) 2	1
50.	(d) $-\sqrt{2} - \sqrt{3} + 5$	1

CHAPTER-7
INTEGRALS
02 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Evaluate : $\int_0^1 x^2 e^x dx$	2
2.	Find : $\int \frac{\tan^3 x}{\cos^3 x} dx$	2
3.	Find: $\int \frac{x-1}{(x-2)(x-3)} dx$	2
4.	Find: $\int_{-\frac{\pi}{4}}^0 \frac{1+\tan x}{1-\tan x} dx$	2
5.	Evaluate: $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2} \right] e^x dx$	2
6.	Write the value of $\int \sec x (\sec x + \tan x) dx$	2
7.	Evaluate: $\int \frac{x^3 - x^2 + x - 1}{x-1} dx$	2
8.	Evaluate: $\int \frac{dx}{5-8x-x^2}$	2
9.	Evaluate: $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$	2
10.	Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5 x dx$	2
11.	If $f(x) = \int_0^x t \sin t dt$, then find the value of $f'(x)$	2
12.	Find $\int \frac{\sin^6 x}{\cos^8 x} dx$	2
13.	Evaluate : $\int_e^{e^2} \frac{dx}{x \log x}$	2
14.	Evaluate : $\int (\sin x) dx$	2
15.	Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x dx$	2

16.	Find the value of $\int \sin x \sqrt{1 - \cos 2x} dx$	2
17.	find the value of $\int \frac{1}{1+e^{-x}} dx$	2
18.	Find the value of $\int_0^{2\pi} \sin x dx$	2
19.	Find the value of $\int 5^{x+x} \left(\frac{x^2+2}{x^2+1}\right) dx$	2
20.	Find the value of $\int \sqrt{1 + \sin x} dx$	2
21.	Evaluate $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$	2
22.	Evaluate $\int \frac{(x+3)e^x}{(x+5)^3} dx$	2
23.	Evaluate $\int \operatorname{cosec}^3 x dx$	2
24.	Evaluate $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, \quad a > 0$	2
25.	Evaluate: $\int \tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right) dx, \quad x \in \left(-\frac{\pi}{2}, \frac{3\pi}{2} \right)$	2

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\int_0^1 x^2 e^x dx = [x^2 e^x]_0^1 - \int_0^1 2x e^x dx$ $= [x^2 e^x - 2x e^x + 2e^x]_0^1$ $= e - 2$	2
2.	<p>Given $I = \int \frac{\tan^2 x}{\cos^3 x} dx$ Let $\cos x = t$ So, $\sin x dx = -dt$</p> $I = \int \left[\frac{-1}{t^6} + \frac{1}{t^4} \right] dt = \frac{t^{-5}}{5} + \frac{t^{-3}}{-3} + C = \frac{1}{5(\cos x)^5} - \frac{1}{3(\cos x)^3} + C = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$	2
3.	$\int \frac{x-1}{(x-2)(x-3)} dx$ <p>Since, $\frac{x-1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$, on solving A = -1 and B = 2 $\Rightarrow \frac{x-1}{(x-2)(x-3)} = \frac{-1}{x-2} + \frac{2}{x-3}$ $\Rightarrow \int \frac{x-1}{(x-2)(x-3)} dx = - \int \frac{dx}{x-2} + 2 \int \frac{dx}{x-3}$ $= -\log(x-2) + 2\log(x-3) + C$ $= -\log(x-2) + \log((x-3)^2) + C = \log \frac{(x-3)^2}{(x-2)} + C$</p>	2
4.	$\int_{-\pi/4}^{0} \frac{1+tanx}{1-tanx} dx = \int_{-\pi/4}^{0} \tan \tan(\frac{\pi}{4} + x) dx =$ $= [\log \sec(\frac{\pi}{4} + x)]_{-\pi/4}^0$ $= \log \sec \frac{\pi}{4} - \log \sec(\frac{\pi}{4} - \frac{\pi}{4}) = \log(\sqrt{2}) - \log(\sec 0) = \log(\sqrt{2}) - \log 1 = \log(\sqrt{2}) = \frac{1}{2} \log 2$	2
5.	<p>Evaluate: $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2} \right] e^x dx$</p> <p>Put $2x = t$, $\therefore dx = \frac{1}{2} dt$</p> $\therefore \int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2} \right] e^x dx = \int_2^4 \left[\frac{1}{t} - \frac{1}{t^2} \right] e^t dt = \left[\frac{1}{t} e^t \right]_2^4 = \frac{e^4}{4} - \frac{e^2}{2}.$	2
6.	$I = \int \sec x (\sec x + \tan x) dx$ $= \int (\sec^2 x + \sec x \tan x) dx$ $= \int \sec^2 x dx + \int \sec x \tan x dx$ $= \tan x + \sec x + C$	2
7.	$\text{Let } I = \int \frac{x^3 - x^2 + x - 1}{x-1} dx = \int \frac{x^2(x-1) + 1(x-1)}{(x-1)} dx$ $= \int \frac{(x^2+1)(x-1)}{(x-1)} dx = \int (x^2 + 1) dx = \frac{x^3}{3} + x + C$	2
8.	$\int \frac{dx}{5 - 8x - x^2} = \int \frac{dx}{5 - 2.4x - x^2 + 4^2 - 4^2}$ $= \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$ $= \frac{1}{2\sqrt{21}} \log \left(\frac{\sqrt{21}+x+4}{\sqrt{21}-x-4} \right) + C$	2
9.	$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = [\tan^{-1} x]_1^{\sqrt{3}}$ $= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$	2
10.	<p>Use the property, $\int_{-a}^a f(x) dx = 0$, if $f(-x) = -f(x)$; $f(x)$ is an odd function</p> <p>$f(x) = \sin^5 x \Rightarrow f(-x) = \sin^5(-x) = -\sin^5 x = -f(x)$ so, $f(x)$ is an odd function</p> $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5 x dx = 0$	2
11.	<p>Differentiating both sides w.r.t to x we get</p> $f'(x) = [tsint]_0^x$ $= xsinx - 0 = xsinx$	1
		1

12.	$\int \frac{\sin^6 x}{\cos^8 x} dx = \int \tan^6 x \sec^2 x dx$ $= \int t^6 dt, \text{ where } \tan x = t \Rightarrow \sec^2 x dx = dt$ $= \frac{t^7}{7} + C = \frac{\tan^7 x}{7} + C$	1 1
13.	<p>Let $I = \int_e^{e^2} \frac{dx}{x \log x}$, Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$</p> <p>When $x = e, t = \log e = 1$ and when $x = e^2, t = 2\log e = 2$</p> $Let I = \int_e^{e^2} \frac{dx}{x \log x} = \int_1^2 \frac{dt}{t} = [\log t]_1^2 = \log 2 - \log 1 = \log 2$	1 1
14.	$\int (\sin x) dx = \int \left(\cos \left(\frac{\pi}{2} - x \right) \right) dx$ $= \int \left(\frac{\pi}{2} - x \right) dx = \frac{\pi}{2} x - \frac{x^2}{2} + C$	1 1
15.	<p>Let $f(x) = x \cos^2 x$</p> $\Rightarrow f(-x) = (-x) \cos^2(-x) = -x \cos^2 x = -f(x)$ <p>$\Rightarrow f(x)$ is an odd function</p> $\Rightarrow \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x dx = 0$	1 1
16.	$\int \sin x \sqrt{1 - \cos 2x} dx$ $= \int (\sin x)(\sqrt{2} \sin x) dx$ $= \frac{\sqrt{2}}{2} \int 2(\sin x)^2 dx$ $= \frac{1}{\sqrt{2}} \int (1 - \cos 2x) dx$ $= \frac{x}{\sqrt{2}} - \frac{\sin 2x}{2\sqrt{2}} + c \text{ (Answer)}$	2
17.	$\int \frac{1}{1 + e^{-x}} dx$ $= \int \frac{e^x}{e^x + 1} dx$ $= \int \frac{d(e^x + 1)}{e^x + 1}$ $= \log \log e^x + 1 + c \text{ (Answer)}$	2
18.	$\int_0^{2\pi} \sin x dx$ $= \int_0^\pi \sin x dx + \int_\pi^{2\pi} -\sin x dx$ $= [-\cos x]_0^\pi + [\cos x]_\pi^{2\pi}$ $= -\cos \pi + \cos 0 + \cos 2\pi - \cos \pi$	2

	= 4(answer)	
19.	$\int 5^{x+x} \left(\frac{x^2 + 2}{x^2 + 1} \right) dx$ $= \int 5^{x+x} \left(1 + \frac{1}{x^2 + 1} \right) dx$ $= \int 5^u du \quad [Taking, u = x + x, hence du = (1 + \frac{1}{1+x^2})dx]$ $= \frac{5^u}{\log 5} + c$ $= \frac{5^{x+x}}{\log \log 5} + c \text{ (Answer)}$	2
20.	$\int \sqrt{1 + \sin x} dx$ $= \int \sqrt{(\sin \frac{x}{2})^2 + (\cos \frac{x}{2})^2 + 2\cos \frac{x}{2} \sin \frac{x}{2}} dx$ $= \int (\sin \frac{x}{2} + \cos \frac{x}{2}) dx$ $= -2\cos \frac{x}{2} + 2\sin \frac{x}{2} + c \text{ (Answer)}$	2
21.	$\int \frac{dx}{\cos x + \sqrt{3}\sin x} = \frac{1}{2} \int \frac{dx}{\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x} = \frac{1}{2} \int \frac{dx}{\sin \frac{\pi}{6}\cos x + \cos \frac{\pi}{6}\sin x} = \frac{1}{2} \int \frac{dx}{\sin(x + \frac{\pi}{6})}$ $= \frac{1}{2} \int \operatorname{cosec}(x + \frac{\pi}{6}) dx = \frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + C$	2
22.	$\int \frac{(x+3)e^x}{(x+5)^3} dx = \int \frac{(x+5-2)e^x}{(x+5)^3} dx = \int e^x \left\{ \frac{1}{(x+5)^2} - \frac{2}{(x+5)^3} \right\} dx = e^x \frac{1}{(x+5)^2} + C$	2
23.	$I = \int \operatorname{cosec}^3 x dx = \int \operatorname{cosec} x \operatorname{cosec}^2 x dx =$ $= \operatorname{cosec} x \int \operatorname{cosec}^2 x dx - \int \frac{d}{dx} (\operatorname{cosec} x) \int \operatorname{cosec}^2 x dx$ $= -\operatorname{cosec} x \operatorname{cot} x - \int \operatorname{cosec} x \operatorname{cot}^2 x dx$ $= -\operatorname{cosec} x \operatorname{cot} x - \int \operatorname{cosec} x (\operatorname{cosec}^2 x - 1) dx$ $= -\operatorname{cosec} x \operatorname{cot} x - \int \operatorname{cosec}^3 x dx + \int \operatorname{cosec} x dx$ $= -\operatorname{cosec} x \operatorname{cot} x - I + \int \operatorname{cosec} x dx$ $2I = -\operatorname{cosec} x \operatorname{cot} x + \log \left \tan \frac{x}{2} \right $ $\therefore I = -\frac{1}{2} \operatorname{cosec} x \operatorname{cot} x + \frac{1}{2} \log \left \tan \frac{x}{2} \right + C$	2
24.	$I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx ----- (i)$ $\text{Also } I = \int_{-\pi}^{\pi} \frac{\cos^2(-x)}{1+a^{-x}} dx = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x} dx ----- (ii)$ $\text{Adding } 2I = \int_{-\pi}^{\pi} \cos^2 x dx = 2 \int_0^{\pi} \cos^2 x dx = 2.2 \int_0^{\frac{\pi}{2}} \cos^2 x dx$ $I = 2 \int_0^{\frac{\pi}{2}} \cos^2 x dx = 2 \int_0^{\frac{\pi}{2}} \sin^2 x dx = 2 \int_0^{\frac{\pi}{2}} dx - 2 \int_0^{\frac{\pi}{2}} \cos^2 x dx$ $I + I = 2 \int_0^{\frac{\pi}{2}} dx = \pi \therefore I = \frac{\pi}{2}$	2
25.	$\int \tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right) dx, x \in \left(-\frac{\pi}{4}, \frac{\pi}{4} \right)$	2

$$\begin{aligned}&= \int \tan^{-1} \left(\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right)^2} \right) dx \\&= \int \tan^{-1} \left(\frac{\cos \frac{x}{2} + \sin \frac{x}{2}}{\cos \frac{x}{2} - \sin \frac{x}{2}} \right) dx = \int \tan^{-1} \left(\frac{1 + \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} \right) dx = \int \tan^{-1} \left(\tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \right) dx \\&= \int \left(\frac{x}{2} + \frac{\pi}{4} \right) dx = \frac{x^2}{4} + \frac{\pi x}{4} + C\end{aligned}$$

DRAFT

CHAPTER-7
INTEGRALS
03 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find: $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)} dx$	3
2.	Evaluate : $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$	3
3.	Evaluate : $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$	3
4.	Find the value of $\int \sin x \cdot \log \cos x dx$.	3
5.	Evaluate: $\int \frac{x^2+x+1}{(x^2+1)(x+2)} dx$	3
6.	Evaluate: $\int \frac{(x-3)}{(x-1)^3} e^x dx$	3
7.	Evaluate : $\int_0^\pi \frac{x \tan x}{\sec x \cosec x} dx$	3
8.	Evaluate : $\int_{-1}^2 f(x) dx$, where $f(x) = x+1 + x + x-1 $	3
9.	Evaluate : $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9+16 \sin 2x} dx$	3
10.	$Find the value of \int \frac{x^7}{x+1} dx$	3
11.	Find the value of $\int \tan \tan x \tan \tan 2x \tan \tan 3x dx$	3
12.	find the value $\int \frac{1}{\sqrt{(1-e^{2x})}} dx$	3
13.	Evaluate $\int \frac{6e^{2x}+7e^x}{\sqrt{(e^x-5)(e^x-4)}} dx$	3
14.	Find the value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$	3
15.	Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$	3

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$\int \frac{1}{\sqrt{x}(\sqrt{x+1})(\sqrt{x+2})} dx$ <p>Let $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow \frac{1}{\sqrt{x}} dx = 2 dt$</p> $= 2 \int \frac{1}{(t+1)(t+2)} dt = 2 \int \frac{(t+2)-(t+1)}{(t+1)(t+2)} dt = 2 \left(\int \frac{1}{t+1} dt - \int \frac{1}{t+2} dt \right)$ $= 2 [\log t+1 - \log t+2] + C$ $= 2 \log \left \frac{t+1}{t+2} \right + C$ $= 2 \log \left \frac{\sqrt{x}+1}{\sqrt{x}+2} \right + C$	3
2.	<p>Let $I = \int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$</p> <p>Put $e^x = t \Rightarrow e^x dx = dt$</p> $\therefore I = \int \frac{dt}{\sqrt{5-4t-t^2}} = \int \frac{dt}{\sqrt{-t^2+4t-5}} = \int \frac{dt}{\sqrt{-(t^2+2t+2^2-9)}} = \int \frac{dt}{\sqrt{3^2-(t+2)^2}} = \sin^{-1} \left(\frac{t+2}{3} \right) + C = \sin^{-1} \left(\frac{e^x+2}{3} \right) + C$	3
3.	$\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$ <p>Let $I = \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx = \int \frac{(\sin^2 x)^3 + (\cos^2 x)^3}{\sin^2 x \cos^2 x} dx$</p> $= \int \frac{(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x)}{\sin^2 x \cos^2 x} dx$ $= \int \frac{\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x}{\sin^2 x \cos^2 x} dx$ $= \int \tan^2 x dx - \int dx + \int \cot^2 x dx$ $= \int \sec^2 x - 1 dx - x + \int \cosec^2 x - 1 dx$ $= \tan x - \cot x - 3x + C$	3
4.	<p>Put $\cos x = t \Rightarrow -\sin x dx = dt$</p> $\therefore - \int \log t dt \Rightarrow - \int (\log t) \cdot 1 dt$ $\Rightarrow [\log t \int 1 dt - \int \left\{ \frac{d}{dx} (\log t) \int 1 dt \right\} dt]$ $\Rightarrow [(\log t) \cdot t - \int \frac{1}{t} \cdot t dt]$ $\Rightarrow -[t \cdot \log t - \int 1 dt]$ $\Rightarrow -[t \log t - t] + C$ $\Rightarrow -t \cdot \log t + t + C$ $\Rightarrow -\cos x \log \cos x + \cos x + C$	3
5.	$\Rightarrow \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} = \frac{A}{x + 2} + \frac{Bx + C}{x^2 + 1}$ $\Rightarrow x^2 + x + 1 = A(x^2 + 1) + (Bx + C)(x + 2)$ $\Rightarrow x^2 + x + 1 = x^2(A + B) + x(2B + C) + (A + 2C)$ <p>On comparing the coefficients of x^2, x and constant terms both sides, we get</p> $A + B = 1 \dots\dots (ii)$ $2B + C = 1 \dots\dots (iii)$ <p>and $A + 2C = 1 \dots\dots (iv)$</p> <p>On substituting the value of B from q. (ii) in Eq. (iii), we get</p> $2(1 - A) + C = 1$ $\Rightarrow 2 - 2A + C = 1$ $\Rightarrow 2A - C = 1 \dots\dots (v)$ <p>From above equations we get</p> $\Rightarrow A = \frac{3}{5}, B = \frac{2}{5} \text{ and } C = \frac{1}{5}$ $\Rightarrow \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} = \frac{A}{x + 2} + \frac{Bx + C}{x^2 + 1}$	3

	$\Rightarrow \frac{3}{5} \int \frac{dx}{x+2} + \frac{1}{5} \int \frac{2x+1}{x^2+1} dx$ $\Rightarrow \frac{3}{5} \int \frac{dx}{x+2} + \frac{1}{5} \int \frac{2x}{x^2+1} dx + \frac{1}{5} \int \frac{1}{x^2+1} dx$ $\Rightarrow \frac{3}{5} \log(x+2) + \frac{1}{5} \log(x^2+1) + \frac{1}{5} \tan^{-1}(x) + c$	
6.	$\int \frac{(x-3)}{(x-1)^3} e^x dx = \int \frac{(x-1-2)}{(x-1)^3} e^x dx$ $\Rightarrow \int \left[\frac{x-1}{(x-1)^3} - \frac{2}{(x-1)^3} \right] e^x dx$ $\Rightarrow \int \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right] e^x dx$ <p style="text-align: center;">we know that $\Rightarrow \int e^x [f(x) + f'(x)] dx = e^x \cdot f(x) + c$</p> <p style="text-align: center;">where $f(x) = \frac{1}{(x-1)^2} \Rightarrow f'(x) = -\frac{2}{(x-1)^3}$</p> <p style="text-align: center;">hence $\int \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right] e^x dx = \frac{e^x}{(x-1)^2} + c$</p>	3
7.	$Let I = \int_0^\pi \frac{x \tan x}{\sec x \cosec x} dx = \int_0^\pi x \sin^2 x dx$ $\Rightarrow I = \int_0^\pi (\pi - x) \sin^2(\pi - x) dx$ $\Rightarrow I = \int_0^\pi (\pi - x) \sin^2 x dx \Rightarrow 2I = \pi \int_0^\pi \sin^2 x dx$ $\Rightarrow 2I = \frac{\pi}{2} \int_0^\pi (1 - \cos 2x) dx = \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right]_0^\pi = \frac{\pi^2}{2}$ $\Rightarrow I = \frac{\pi^2}{4}$	1 1 1
8.	<p>We can redefine f as</p> $f(x) = \begin{cases} 2-x, & \text{if } -1 \leq x < 0 \\ x+2, & \text{if } 0 \leq x < 1 \\ 3x, & \text{if } 1 \leq x < 2 \end{cases}$ $\Rightarrow \int_{-1}^2 f(x) dx = \int_{-1}^0 (2-x) dx + \int_0^1 (x+2) dx + \int_1^2 3x dx$ $= \left[2x - \frac{x^2}{2} \right]_{-1}^0 + \left[\frac{x^2}{2} + 2x \right]_0^1 + \left[\frac{3x^2}{2} \right]_1^2$ $= \frac{5}{2} + \frac{5}{2} + \frac{9}{2} = \frac{19}{2}$	1 1 1 1
9.	$Let I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 [1 - (\sin x - \cos x)^2]} dx$ <p>Put $\sin x - \cos x = t \Rightarrow (\sin x + \cos x) dx = dt$</p> <p>When $x = 0, t = -1$, when $x = \frac{\pi}{4}, t = 0$</p> $\Rightarrow I \int_{-1}^0 \frac{1}{9 + 16(1 - t^2)} dt = \int_{-1}^0 \frac{1}{25 + 16t^2} dt = \frac{1}{16} \int_{-1}^0 \frac{1}{\left(\frac{5}{4}\right)^2 + t^2} dt$ <p>It is of the form $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left \frac{a+x}{a-x} \right + C$</p> <p>After evaluating, we get $I = \frac{1}{20} \log 3$</p>	1 1 1

10.	<p>We know that, $x^7 + 1 = (x+1)(x^6 - x^5 + x^4 - x^3 + x^2 - x + 1)$</p> $\begin{aligned} & \int \frac{x^7}{x+1} dx \\ &= \int \frac{x^7 + 1 - 1}{x+1} dx \\ &= \int \frac{x^7 + 1}{x+1} dx - \int \frac{dx}{x+1} \\ &= \int (x+1) \frac{x^6 - x^5 + x^4 - x^3 + x^2 - x + 1}{(x+1)} dx - \log \log x+1 \\ &= \frac{x^7}{7} - \frac{x^6}{6} + \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x - \log \log x+1 + c \text{ (Answer)} \end{aligned}$	3
11.	<p>We know that, $\tan \tan 3x = \tan \tan (2x+x) = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$</p> <p>So, we get $\tan \tan x \tan \tan 2x \tan \tan 3x = \tan \tan 3x - \tan \tan 2x - \tan \tan x$</p> $\begin{aligned} & \int \tan \tan x \tan \tan 2x \tan \tan 3x dx \\ &= \int (\tan \tan 3x - \tan \tan 2x - \tan \tan x) dx \\ &= \int \tan \tan 3x dx - \int \tan \tan 2x dx - \int \tan \tan x dx \\ &= \frac{1}{3} \log \cos \cos 3x - \frac{1}{2} \log \log \cos \cos 2x - \log \log \cos \cos x + c \\ & \text{(Answer)} \end{aligned}$	3
12.	$\begin{aligned} & \int \frac{1}{\sqrt{(1-e^{2x})}} dx \\ &= \int \frac{e^{-x}}{\sqrt{e^{-2x}-1}} dx \dots (i) \\ & \text{taking, } e^{-x} = u \\ \therefore -e^{-x} dx &= du \\ (i) \text{ becomes } \int \frac{-du}{\sqrt{u^2-1}} \\ &= -\log \log u + \sqrt{u^2-1} + c \\ &= -\log \log e^{-x} + \sqrt{e^{-2x}-1} + c \text{ (Answer)} \end{aligned}$	3
13.	$\int \frac{(6e^x+7)e^x}{\sqrt{(e^x-5)(e^x-4)}} dx$ <p>Let $e^x = t$, then $e^x dx = dt$</p> $\therefore I = \int \frac{(6t+7)}{\sqrt{(t-5)(t-4)}} dt$ <p>Using the expression $6t+7 = A \frac{d}{dt}(t^2 - 9t + 20) + B$</p> <p>Solving we get $A = 3$ and $B = 34$</p> $\begin{aligned} \therefore I &= \int \frac{(6t+7)}{\sqrt{(t-5)(t-4)}} dt = 3 \int \frac{(2t-9)}{\sqrt{t^2 - 9t + 20}} dt + 34 \int \frac{1}{\sqrt{t^2 - 9t + 20}} dt \\ &= 6\sqrt{t^2 - 9t + 20} + 34 \log \left \left(t - \frac{9}{2} \right) + \sqrt{t^2 - 9t + 20} \right + C \text{ where } t = e^x \end{aligned}$	3
14.	$\begin{aligned} I &= \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x} dx + 2 \int_{-\pi}^{\pi} \frac{x \sin x}{1+\cos^2 x} dx \\ &= 0 + 2 \int_{-\pi}^{\pi} \frac{x \sin x}{1+\cos^2 x} dx \quad (f(x) = \frac{2x}{1+\cos^2 x} \text{ is an odd fn.}) \end{aligned}$	3

$= 4 \int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$ ($g(x) = \frac{x \sin x}{1 + \cos^2 x}$ is an even fn.)
 Also $I = 4 \int_0^\pi \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx$
 Adding we get, $2I = 4\pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx$
 $I = 2\pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx$
 Putting $t = \cos x$, $dt = -\sin x dx$, Also as $x = 0, t = 1$ & $x = \pi, t = -1$
 The integral reduces to $I = 2\pi \int_{-1}^1 \frac{dt}{1+t^2} = \pi^2$

15. $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ Putting $x = \tan \theta$, then the integral reduces to 3

$$I = \int_0^{\frac{\pi}{2}} \log(1 + \tan \theta) d\theta$$

Using the property $\int_0^a f(x) dx = \int_0^a f(a - x) dx$

$$I = \frac{\pi}{8} \log 2$$

CHAPTER-7
INTEGRALS
04 MARK TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	Find : $\int \frac{dx}{\sin x + \sin \sin 2x}$	4
2.	Find: $\int_0^{\pi} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$	4
3.	<p>The given integral $\int f(x) dx$ can be transformed into another form by changing the independent variable x to t by substitution $x=g(t)$</p> <p>Consider $I = \int f(x) dx$, put $x = g(t) \Rightarrow \frac{dx}{dt} = g'(t)$</p> $\Rightarrow dx = g'(t) dt \Rightarrow I = \int f(g(t)) g'(t) dt$ <p>This change of variable formula is one of the important tools available to us in the name of integration by substitution.</p> <p>Based on the above information, answer the following questions:</p> <ol style="list-style-type: none"> 1. Find the value of $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$ 2. . Find the value of $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ 3. Find the value of $\int \frac{\sin x}{(1+\cos x)^2} dx$ 4. Find the value of $\int \frac{\log x}{x} dx$ 	4
4.	<p>There are many practical applications of Definite Integration. Definite integrals can be used to determine the mass of an object if its density function is known. We can also find work by integrating a force function, and the force exerted on an object submerged in a liquid. The most important application of Definite Integration is finding the area under the curve.</p> <p>Let f be a continuous function defined on the closed interval $[a,b]$ and F be an antiderivative of f then</p> $\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$ <p>It is very useful because it gives us a method of calculating the definite integral more easily. There is no need to keep integration constant C because if we consider $F(x) + C$ instead of $F(x)$.</p> $\begin{aligned} \int_a^b f(x) dx &= [F(x) + C]_a^b = F(b) + C - F(a) - C \\ &= F(b) - F(a) \end{aligned}$ <p>Based on the above information, answer the following questions:</p> <ol style="list-style-type: none"> 1. Find the value of $\int_2^3 x^2 dx$ 	4

	<p>2. Find the value of $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx$</p> <p>3. Find the value of $\int_{-1}^1 (x+1)dx$</p> <p>4. Find the value of $\int_2^3 \frac{1}{x} dx$</p>	
5.	<p>Three students in a group, studying the concept of the partial fraction, but they were confused while solving the question they did not have any idea about how to start the solution. One of the students tells them for the integration by partial fraction, first we must check we are dealing with polynomial and degree of numerator is less than the degree of denominator and proceed for partial fraction, if not, divide numerator by denominator and write it as $\frac{\text{Numerator}}{\text{Denominator}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Denominator}}$</p> <p>Based on the above information, answer the following:</p> <ul style="list-style-type: none"> (i) If the function is $f(x) = \frac{2}{(1-x)(1+x^2)}$, write the partial fraction of the given function in constant term A, B and C. (ii) Find the value of the constant taken in the numerator of the factor $(1-x)$, while reducing $f(x)$ into partial fractions. (iii) Find the value of both the constants taken in the numerator of the factor $(1+x^2)$, while reducing $f(x)$ into partial fractions. <p>Or</p> <p>Find the integration of the function $f(x)$</p>	4
6.	<p>If u and v are two functions of x, then</p> $\int uv dx = u \int v dx - \int \left\{ \frac{du}{dx} \int v dx \right\} dx$ <p>i.e, the integral of the product of two functions = first function x Integral of second - Integral of (derivative of the first x integral of second). Here the choice of the first function is important. We can use the order ILATE, where I= Inverse trigonometric functions L = Logarithmic functions, A= Algebraic functions T = trigonometric functions, E = exponential functions If the integrand contains only one function, we take that function as the first function and 1 as the second function.</p> <p>Based on the above information, answer the following:</p> <ul style="list-style-type: none"> (i) I = $\int x dx$, which functions should be taken as first and second functions (ii) How to evaluate the integral $\int \frac{x}{1+x^2} dx$ <p>Write the integral as given in (i)</p>	4
7.	<p>Find the value of $\int \frac{x}{x - \sqrt{x^2 - 1}} dx$</p>	4
8.	<p>find the value of $\int (\sqrt{3}\sin x + \cos x)^{-1} dx$</p>	4
9.	<p>Let the definite integral be defined by the formula $\int_a^b f(x) dx = \frac{b-a}{2} (f(a) + f(b))$. For more accurate result for $c \in (a, b)$, we can use</p> $\int_a^b f(x) dx = \frac{b-a}{2} (f(a) + f(b) + 2f(c))$ <p>where $c = \frac{a+b}{2}$. Then</p> <p>(i) Evaluate with more accuracy as stated: $\int_0^{\pi/2} \sin x dx$</p>	4

	(ii) Evaluate with min accuracy as stated: $\int_0^{\frac{\pi}{2}} \cos x dx$	
10.	For the integral $\int_0^{\frac{3}{2}} x \cos \pi x dx$ (i) Find possible c if $a = -1$ and $b = \frac{3}{2}$ provided $a \leq c \leq b$ (ii) Evaluate the integral.	4

DRAFT

ANSWERS:

Q. NO	ANSWER	MARKS
1.	$I = \int \frac{dx}{\sin x + \sin 2x} = \int \frac{1}{\sin x + 2\sin x \cos x} dx = \int \frac{1}{\sin x(1+2\cos x)} dx = \int \frac{\sin x}{\sin^2 x(1+2\cos x)} dx =$ $\int \frac{\sin x}{(1-\cos^2 x)(1+2\cos x)} dx$ <p>Put $\cos x = t$, so $-\sin x dx = dt$</p> $\therefore I = \int \frac{-dt}{(1-t^2)(1+2t)} = - \int \frac{dt}{(1+t)(1-t)(1+2t)}$ <p>Then write $\frac{1}{(1+t)(1-t)(1+2t)} = \frac{A}{1+t} + \frac{B}{1-t} + \frac{C}{1+2t}$, on solving $A = \frac{-1}{2}$, $B = \frac{1}{6}$, $C = \frac{4}{3}$</p> $I = \frac{1}{2} \log 1+t + \frac{1}{6} \log 1-t - \frac{4}{3x^2} \log 1+2t + C$ $I = \frac{1}{2} \log 1+\cos x + \frac{1}{6} \log 1-\cos x - \frac{4}{3x^2} \log 1+2\cos x + C$	4
2.	$\text{Let } I = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{2\sin 2x}} = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{2.2 \sin x \cos x}} = \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{\frac{\sin x}{\cos x} \cos^2 x}} = \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^4 x \sqrt{\tan x}} = \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{\sec^2 x \sec^2 x dx}{\sqrt{\tan x}}$ <p>Put $\tan x = t \Rightarrow \sec^2 x dx = dt$, $x=0 \Rightarrow t=0$ and $x=\frac{\pi}{4} \Rightarrow t=1$</p> $\therefore I = \frac{1}{2} \int_0^1 \frac{(1+t^2)dt}{\sqrt{t}} = \frac{1}{2} \left[\frac{t^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} \right]_0^1 + \frac{1}{2} \left[\frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right]_0^1 = \frac{6}{5}$	4
3.	<p>1. $I = \int \frac{e^{\tan^{-1} x}}{1+x^2} dx$</p> <p>Let $\tan^{-1} x = t \Rightarrow \frac{dx}{1+x^2} = dt$</p> $\Rightarrow I = \int e^t dt = e^t + c = e^{\tan^{-1} x} + c$ <p>2. $I = \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$</p> <p>Let $\sin^{-1} x = t \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$</p> $\Rightarrow I = \int t dt = \frac{t^2}{2} + c = \frac{(\sin^{-1} x)^2}{2} + c$ <p>3. $I = \int \frac{\sin x}{(1+\cos x)^2} dx$</p> <p>Let $1+\cos x = t \Rightarrow -\sin x dx = dt$</p> $\Rightarrow I = \int \frac{1}{t^2} dt = \frac{-1}{t} + c = \frac{-1}{1+\cos x} + c$ <p>4. $I = \int \frac{\log x}{x} dx$</p> <p>Let $\log x = t \Rightarrow \frac{dx}{x} = dt$</p> $\Rightarrow I = \int t dt = \frac{t^2}{2} + c = \frac{(\log x)^2}{2} + c$	4
4.	<p>(i) $I = \int_2^3 x^2 dx = \left[\frac{x^3}{3} \right]_2^3 = \frac{27}{3} - \frac{8}{3} = \frac{19}{3}$</p> <p>(ii) $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx = [\tan^{-1} x]_1^{\sqrt{3}} = \tan^{-1}(\sqrt{3}) - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$</p> <p>(iii) $I = \int_{-1}^1 (x+1) dx = \left[\frac{x^2}{2} + x \right]_{-1}^1 = \left(\frac{1}{2} + 1 \right) - \left(\frac{1}{2} - 1 \right) = 2$</p> <p>(iv) $I = \int_2^3 \frac{1}{x} dx = [\log x]_2^3 = \log 3 - \log 2 = \log \left(\frac{3}{2} \right)$</p>	4
5.	<p>(i) $f(x) = \frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$</p>	1

	<p>(ii) $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$ $\Rightarrow 2 = A(1+x^2) + (1-x)(Bx+C)$ On solving, we get, $A = B = C = 1$</p> <p>(iii) $B = 1, C = 1$ Or</p> $\int f(x)dx = \int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x+1}{1+x^2} dx$ $= -\log 1-x + \frac{1}{2} \log \log 1+x^2 + x + C$	1 2
6.	<p>(i) x as first function and 1 as second function. (ii) $\int \frac{x}{1+x^2} dx = \frac{1}{2} \int \frac{2x}{1+x^2} dx = \frac{1}{2} \log 1+x^2$ (iii) $I = \int x dx = x \int 1 dx - \int \left\{ \frac{d(x)}{dx} \int 1 dx \right\} dx$ $= x - \int \frac{x}{1+x^2} dx = xx - \frac{1}{2} \log 1+x^2 + C$</p>	1 1 1 1
7.	$\begin{aligned} & \int \frac{x}{x-\sqrt{x^2-1}} dx \\ &= \int \frac{x(x+\sqrt{x^2-1})}{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})} dx \\ &= \int \frac{x^2+x\sqrt{x^2-1}}{x^2-(x^2-1)} dx \\ &= \int x^2 dx + \int x\sqrt{x^2-1} dx \\ &= \frac{x^3}{3} + \int x\sqrt{x^2-1} dx \dots (i) \end{aligned}$ <p>taking, $x^2 - 1 = u$ in 2nd part of (i)</p> <p>We get, $2xdx = du$</p> <p>(i) becomes $\int \frac{x}{x-\sqrt{x^2-1}} dx$</p> $\begin{aligned} &= \frac{x^3}{3} + \frac{1}{2} \int u^{\frac{1}{2}} du \\ &= \frac{x^3}{3} + \frac{1}{2} \times \frac{2}{3} u^{\frac{3}{2}} + c \\ &= \frac{x^3}{3} + \frac{1}{3} (x^2 - 1)^{\frac{3}{2}} + c \text{ (Answer)} \end{aligned}$	4
8.	$\begin{aligned} & \int (\sqrt{3}\sin x + \cos x)^{-1} dx \\ &= \int \frac{1}{\sqrt{3} \sin x \sin x + \cos x \cos x} dx \\ &= \int \frac{1}{2(\frac{\sqrt{3}}{2} \sin x \sin x + \frac{1}{2} \cos x \cos x)} dx \\ &= \int \frac{1}{2(\sin x \cos \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6})} dx \end{aligned}$	4

$$\begin{aligned}
 &= \frac{1}{2} \int \frac{1}{\sin(x + \frac{\pi}{6})} dx \\
 &= \frac{1}{2} \int \csc(x + \frac{\pi}{6}) dx \\
 &= \frac{1}{2} \log \log |\tan \tan \left(\frac{x}{2} + \frac{\pi}{12} \right)| + c \quad (\text{Answer})
 \end{aligned}$$

9. (i) $\frac{\pi}{8}(1 + \sqrt{2})$ (ii) $\frac{\pi}{8}$

10. (i) The value of c is $\frac{1}{2}$. (ii) $\frac{5}{2\pi} - \frac{1}{\pi^2}$

4

4

CHAPTER-7
INTEGRALS
05 MARKS TYPE QUESTIONS

Q. NO	QUESTION	MARK
1.	<p>For a function $f(x)$, if $f(-x) = f(x)$, then f is an even function and if $f(-x) = -f(x)$, then f is an odd function .Again ,we have</p> $\int_{-a}^a f(x)dx = \begin{cases} 2 \int_0^a f(x)dx, & \text{if } f(x) \text{ is even} \\ 0, & \text{if } f(x) \text{ is odd} \end{cases}$ <p>On the above information answer the following questions ,</p> <p>i) $f(x) = x^2 \sin x$ is an a)even (ii) odd (iii) neither even nor odd (iv) none of these</p> <p>ii) $\int_{-\pi}^{\pi} f(x)dx$ is equal to a)$\frac{\pi}{4}$ (b) 2π (c)$\frac{\pi}{2}$ (d) 0</p> <p>iii) $f(x) = x \sin x$, then $\int_{-\pi}^{\pi} f(x)dx$ is a) π (b) 2π (c) 3π (d) 4π</p> <p>(iv) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x dx$ is equal to a) 0 (b) 1 (c) 2 (d) 3</p>	5
2.	<p>For any function we have ,$\int_a^b f(x)dx = \int_a^{c_1} f(x)dx + \int_{c_1}^{c_2} f(x)dx + \dots + \int_{c_n}^b f(x)dx$, where $a < c_1 < c_2 < \dots < c_n < b$,</p> <p>Based on the above information , answer the following questions</p> <p>i) $\int_0^1 3x - 2 dx$ a)$\frac{15}{18}$ (b) $\frac{1}{2}$ (iii) $\frac{7}{3}$ (d) $\frac{11}{2}$</p> <p>ii) $\int_0^{\pi} \cos x dx$ a) 1 (b) 0 (c) 2 (d) 3</p> <p>(iii) $\int_0^2 [x] dx$ a) 0 (b) 1 (c) 2 (d) 3</p> <p>(iv) $\int_{-1}^1 e^{ x } dx$ a) e (b) 3(e-1) (c) 2(e-1) (d) 4</p>	5
3.	Evaluate: $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$	5
4.	Evaluate $\int \frac{2x}{(x^2 + 1)(x^2 + 2)^2} dx$	5
5.	Evaluate : $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$	5
6.	Find the value of $\int \frac{x + \sin x}{1 + \cos x} dx$	5
7.	Find the value of $\int \frac{1}{(2 - 3 \cos 2x)} dx$	5
8.	Find the value of $\int e^x \frac{x^2 + 1}{(x+1)^2} dx$	5
9.	Prove that $\int_0^a \sin^{-1} \sqrt{\frac{x}{a+x}} dx = \frac{a}{2}(\pi - 2)$	5
10.	Evaluate $\int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$	5

ANSWERS:

$$\Rightarrow 28 - C = -1$$

Now, from Eqs. (iv) and (y), we get

$$-B = 1 \Rightarrow B = -1$$

$$\therefore A = 1 \text{ and } C = -1$$

$$\Rightarrow I = \int \frac{1}{t+1} dt - \int \frac{1}{t+2} dt - \int \frac{1}{(t+2)^2} dt$$

$$\Rightarrow I = \log(t+1) - \log(t+2) + \frac{1}{t+2} + c$$

$$\Rightarrow I = \log(x^2+1) - \log(x^2+2) + \frac{1}{x^2+2} + c$$

5.

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx = \int_0^{\frac{\pi}{2}} \frac{(\frac{\pi}{2} - x) \sin(\frac{\pi}{2} - x) \cos(\frac{\pi}{2} - x)}{\sin^4(\frac{\pi}{2} - x) + \cos^4(\frac{\pi}{2} - x)} dx \\ &= \int_0^{\frac{\pi}{2}} \frac{(\frac{\pi}{2} - x) \sin x \cos x}{\sin^4 x + \cos^4 x} dx \Rightarrow 2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx \\ \Rightarrow I &= \frac{\pi}{2 \times 4} \int_0^{\frac{\pi}{2}} \frac{2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx \Rightarrow I = \frac{\pi}{8} \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx \\ \Rightarrow I &= \frac{\pi}{8} \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{(\sin^2 x)^2 + (1 - \sin^2 x)^2} dx \end{aligned}$$

Put $\sin^2 x = t \Rightarrow \sin 2x dx = dt$, when $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = 1$

$$I = \frac{\pi}{8} \int_0^1 \frac{dt}{t^2 + (1-t)^2} = \frac{\pi}{8} \int_0^1 \frac{dt}{2t^2 - 2t + 1}$$

$$I = \frac{\pi}{16} \int_0^1 \frac{dt}{(t-1/2)^2 + (\frac{1}{2})^2} = \frac{\pi}{8} [(2t-1)]_0^1 = \frac{\pi^2}{16}$$

6.

$$\begin{aligned} I &= \int \frac{x + \sin x}{1 + \cos x} dx = \int \frac{x + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} dx \\ &= \int \left(\frac{x}{2 \cos^2 \frac{x}{2}} + \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right) dx = \int \left(\frac{1}{2} x \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right) dx \\ &= \frac{1}{2} [x \int \sec^2 \frac{x}{2} dx - \int (\frac{d}{dx}(x) \int \sec^2 \frac{x}{2} dx) dx] + \int \tan \frac{x}{2} dx \\ \text{By doing integration by parts we get } I &= x \tan \frac{x}{2} + C \end{aligned}$$

7.

$$\begin{aligned} &\int \frac{1}{(2 - 3 \cos 2x)} dx \\ &= \int \frac{1}{2(\cos \cos x)^2 + 2(\sin \sin x)^2 - 3\{(\cos x)^2 - (\sin \sin x)^2\}} dx \\ &= \int \frac{1}{5(\sin \sin x)^2 - (\cos \cos x)^2} dx \\ &= \int \frac{(\sec \sec x)^2}{5(\tan \tan x)^2 - 1} dx \dots (i) \end{aligned}$$

Taking, $\sqrt{5} \tan \tan x = u$

$$\text{We get, } \sqrt{5}(\sec \sec x)^2 dx = du$$

$$(i) \text{ becomes } \int \frac{1}{(2 - 3 \cos 2x)} dx$$

$$= \int \frac{1}{\sqrt{5}(u^2 - 1)} du$$

1

1

1

1

1

1

2

5

$$= \frac{1}{2\sqrt{5}} \log \log \left| \frac{u-1}{u+1} \right| + c$$

$$= \frac{1}{2\sqrt{5}} \log \log \left| \frac{\sqrt{5} \tan \tan x - 1}{\sqrt{5} \tan \tan x + 1} \right| + c, [\text{ put the value of } u]$$

is the required answer.

8.

$$\begin{aligned} & \int e^x \frac{x^2 + 1}{(x+1)^2} dx \\ &= \int e^x \frac{x^2 + 2x + 1 - 2x}{(x+1)^2} dx \\ &= \int e^x \frac{(x+1)^2 - 2x}{(x+1)^2} dx \\ &= \int e^x dx - 2 \int \frac{xe^x}{(x+1)^2} dx \\ &= e^x - 2 \int \frac{(x+1-1)e^x}{(x+1)^2} dx \\ &= e^x - 2 \int \left(\frac{e^x}{x+1} - \frac{e^x}{(x+1)^2} \right) dx \dots (i) \\ \text{Taking, } & \frac{e^x}{x+1} = u \\ \text{We get, } & \left(\frac{e^x}{x+1} - \frac{e^x}{(x+1)^2} \right) dx = du \\ \text{Now, (i) becomes, } & \int e^x \frac{x^2+1}{(x+1)^2} dx = e^x - 2 \int du \\ &= e^x - 2u + c \\ &= e^x - \frac{2e^x}{x+1} + c \text{ (Answer)} \end{aligned}$$

5

9.

$$\int_0^a \sin^{-1} \sqrt{\frac{x}{a+x}} dx = \frac{a}{2}(\pi - 2)$$

5

(Put $x = a \tan^2 \theta$ and apply Integration by parts)

10.

$$\int_0^\pi \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x} (= \frac{\pi^2}{2ab})$$

Apply the following properties in series:

$$(i) \int_0^a f(x) dx = \int_0^a f(a-x) dx \quad (ii) \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$$

5



ARTHAM
RESOURCE MATERIAL
NO. 1 EDUCATIONAL RESOURCES

As Per Revised
CBSE Curriculum
2023-24

Classroom Teaching & Animated Videos Playlists



We take immense pleasure in serving you. Now, revel in our seamless online services completely free of charge. view our animated and classroom teaching Playlists customized for students from grade 1 to 12,Covering a wide range of subjects to enhance your comprehension and knowledge. Simply click on the provided playlist links to access **Playlists based on the latest NCERT Syllabus for 2023-24.**

Our content includes Competency-Based Questions, Assertion-Reason Questions, Previous Year Questions (PYQ), and Case Study-Based Questions to enhance your learning experience. For the most up-to-date videos, consider subscribing to our YouTube channel at <https://www.youtube.com/@PrincipalsHandbookandDiary> additionally, you're encouraged to join our expanding WhatsApp community group to stay updated with the latest curriculum-related content and updates.

We are committed to enriching your educational journey!!!



Nageen Group of Schools

ANIMATED & CLASSROOM TEACHING VIDEOS PLAYLISTS

(As per revised CBSE Curriculum– 2023-24)

ANIMATED VIDEOSPLAYLISTS (CLASS 1)

Class 1 EVS(EnglishLanguage)(CBSE)	Click here for playlist
Class 1 Mathematics (EnglishLanguage)(CBSE)	Click here for Playlist
Class 1 EVS (HindiLanguage)(CBSE)	Click here for Playlist
Class 1 Mathematics(Hindi Language)(CBSE)	Click here for Playlist

ANIMATED VIDEOS PLAYLISTS (CLASS 2)

Class 2 EVS (EnglishLanguage)(CBSE)	Click here for Playlist
Class 2 Mathematics (EnglishLanguage)(CBSE)	Click here for Playlist
Class 2 EVS(HindiLanguage)(CBSE)	Click here for Playlist
Class 2 Mathematics (Hindi Language)(CBSE)	Click here for Playlist

ANIMATED VIDEOS PLAYLISTS (CLASS 3)

Class 3 Mathematics (EnglishLanguage)(CBSE)	Click here for Playlist
Class 3 EVS (EnglishLanguage)(CBSE)	Click here for Playlist
Class 3 EVS (HindiLanguage)(CBSE)	Click here for Playlist
Class 3 Mathematics (HindiLanguage)(CBSE)	Click here for Playlist

ANIMATED VIDEOS PLAYLISTS (CLASS 4)

Class 4 Mathematics (EnglishLanguage)(CBSE)	Click here for Playlist
Class 4 EVS(EnglishLanguage)(CBSE)	Click here for Playlist
Class 4 Mathematics (HindiLanguage)(CBSE)	Click here for Playlist
Class 4 EVS (HindiLanguage)(CBSE)	Click here for Playlist

CLASSROOM TEACHING VIDEOS PLAYLISTS (CLASS 4)

Class 4 General Science (CBSE)	Click here for Playlist
--------------------------------	---

ANIMATED VIDEOS PLAYLISTS (CLASS 5)

Class 5 Mathematics (EnglishLanguage)(CBSE)	Click here for Playlist
Class 5 Science (EnglishLanguage)(CBSE)	Click here for Playlist
Class 5 Mathematics(HindiLanguage)(CBSE)	Click here for Playlist
Class 5 Science (HindiLanguage)(CBSE)	Click here for Playlist

CLASSROOM TEACHING VIDEOS PLAYLISTS (CLASS 5)

Class 5 General Science (CBSE)	Click here for Playlist
Class 5 EVS (CBSE)	Click here for Playlist

ANIMATED VIDEOS PLAYLISTS(CLASS 6)

Class 6 Mathematics (EnglishLanguage)(CBSE)	Click here for Playlist
Class 6 Social Science (EnglishLanguage)(CBSE)	Click here for Playlist
Class 6 Science (EnglishLanguage) (CBSE)	Click here for Playlist
Class 6 Mathematics (Hindi Language)(CBSE)	Click here for Playlist
Class 6 Science All Chapters (CBSE)	Click here for Playlist

CLASSROOM TEACHING VIDEOS PLAYLISTS (CLASS 6)

Class 6 Mathematics (CBSE)	Click here for Playlist
Class 6 Social Science (CBSE)	Click here for Playlist
Class 6 Sanskrit (CBSE)	Click here for Playlist
Class 6 Hindi (CBSE)	Click here for Playlist
Class 6 Science (CBSE)	Click here for Playlist

ANIMATED VIDEOS PLAYLISTS (CLASS 7)

Class 7 Science(CBSE)	Click here for Playlist
Class 7 Mathematics(CBSE)	Click here for Playlist
Class 7 Social Science(CBSE)	Click here for Playlist
Class 7 Mathematics(CBSE)	Click here for Playlist
Class 7 Science (CBSE)	Click here for Playlist

CLASSROOM TEACHING VIDEOS PLAYLISTS (CLASS 7)

Class 7 Science (CBSE)	Click here for Playlist
Class 7 Hindi (CBSE)	Click here for Playlist
Class 7 Sanskrit (CBSE)	Click here for Playlist
Class 7 Social Science (CBSE)	Click here for Playlist
Class 7 Mathematics (CBSE)	Click here for Playlist

ANIMATED VIDEOS PLAYLISTS (CLASS 8)

Class 8 Science(CBSE)	Click here for Playlist
Class 8 Mathematics(CBSE)	Click here for Playlist
Class 8 Social Science(CBSE)	Click here for Playlist
Class 8 Mathematics(CBSE)	Click here for Playlist
Class 8 Science(CBSE)	Click here for Playlist

CLASSROOM TEACHING VIDEOS PLAYLISTS (CLASS 8)

Class 8 Hindi (CBSE)	Click here for Playlist
Class 8 Sanskrit (CBSE)	Click here for Playlist

ANIMATED VIDEOS PLAYLISTS (CLASS 9)

Class 9 Biology(CBSE)	Click here for Playlist
Class 9 Physics(CBSE)	Click here for Playlist
Class 9 Chemistry(CBSE)	Click here for Playlist
Class 9 Social Science (CBSE)	Click here for Playlist

Class 9 Mathematics (CBSE)	Click here for Playlist
Class 9 Science (CBSE)	Click here for Playlist

CLASSROOM TEACHING VIDEOS PLAYLISTS (CLASS 9)

Class 9 Social Science (CBSE)	Click here for Playlist
Class 9 Mathematics(CBSE)	Click here for Playlist
Class 9 English (CBSE)	Click here for Playlist
Class 9 Hindi (CBSE)	Click here for Playlist

ANIMATED VIDEOS PLAYLISTS (CLASS 10)

Class 10 Biology (CBSE)	Click here for Playlist
Class 10 Physics (CBSE)	Click here for Playlist
Class 10 Chemistry (CBSE)	Click here for Playlist
Class 10 Social Science (CBSE)	Click here for Playlist
Class 10 Mathematics(CBSE) (English Language)	Click here for Playlist
Class 10 Mathematics(CBSE) (Hindi Language)	Click here for Playlist
Class 10 Science(CBSE) (Hindi Language)	Click here for Playlist

CLASSROOM TEACHING VIDEOS PLAYLISTS (CLASS 10)

Class 10 English (CBSE)	Click here for Playlist
Class 10 Hindi (CBSE)	Click here for Playlist
Class 10 Mathematics (CBSE)	Click here for Playlist
Class 10 Social Science (CBSE)	Click here for Playlist
Class 10 Magical Science Board Exam Preparation in 1 min (CBSE)	Click here for Playlist
Class 10: Science (CBSE)	Click here for Playlist

ANIMATED VIDEOS PLAYLISTS (CLASS 11)

Class 11 Physics (CBSE) (English Language)	Click here for Playlist
Class 11 Chemistry (CBSE) (English Language)	Click here for Playlist
Class 11 Biology (CBSE) (English Language)	Click here for Playlist
Class 11 Mathematics(CBSE) (English Language)	Click here for Playlist
Class 11 Accountancy (CBSE) (English Language)	Click here for Playlist
Class 11 Business Studies (CBSE) (English Language)	Click here for Playlist
Class 11 Statistics (CBSE) (English Language)	Click here for Playlist
Class 11 Biology (CBSE) (Hindi Language)	Click here for Playlist
Class 11 Mathematics (CBSE) (Hindi Language)	Click here for Playlist
Class 11 Physics (CBSE) (Hindi Language)	Click here for Playlist
Class 11 Chemistry (CBSE) (Hindi Language)	Click here for Playlist
Class 11Micro Economy (CBSE) (English Language)	Click here for Playlist

CLASSROOM TEACHING VIDEOS PLAYLISTS (CLASS 11)

Class 11Mathematics (CBSE)	Click here for Playlist
Class 11 Accounts (CBSE)	Click here for Playlist
Class 11 Business Studies (CBSE)	Click here for Playlist

Class 11 Hindi (CBSE)	Click here for Playlist
Class 11 Psychology (CBSE)	Click here for Playlist
Class 11 Economics (CBSE)	Click here for Playlist
Class 11 Physics (CBSE)	Click here for Playlist
Class 11 Chemistry (CBSE)	Click here for Playlist
Class 11 English (CBSE)	Click here for Playlist
Class 11 Biology (CBSE)	Click here for Playlist
Class 11 Biology Shorts (CBSE)	Click here for Playlist

ANIMATED VIDEOS PLAYLISTS (CLASS 12)

Class 12 Physics (CBSE)	Click here for Playlist
Class 12 Chemistry (CBSE)	Click here for Playlist
Class 12 Biology(CBSE)	Click here for Playlist
Class 12 Macro Economy (CBSE)	Click here for Playlist
Class 12 Economic (CBSE)	Click here for Playlist
Class 12 Mathematics (CBSE)	Click here for Playlist
Class 12 Accountancy (CBSE)	Click here for Playlist
Class 12 Business Studies (CBSE)	Click here for Playlist
Class 12 Physics (CBSE)	Click here for Playlist
Class 12 Mathematics (CBSE)	Click here for Playlist
Class 12 Biology (CBSE)	Click here for Playlist
Class 12 Chemistry (CBSE)	Click here for Playlist

CLASSROOM TEACHING VIDEOS PLAYLISTS (CLASS 12)

Class 12 CHEMISTRY (CBSE)	Click here for Playlist
Class 12 Business Studies (CBSE)	Click here for Playlist
Class 12 Hindi (CBSE)	Click here for Playlist
NEET Biology in 1 min	Click here for Playlist
Class 12 History (CBSE)	Click here for Playlist
Class 12 Political Science (CBSE)	Click here for Playlist
Class 12 Physics (CBSE)	Click here for Playlist
Class 12 Biology (CBSE)	Click here for Playlist
Class 12 : Accounts (CBSE)	Click here for Playlist



SCHOOL OF EDUCATORS

You will get Pre- Board Papers PDF, Word file, PPT, Lesson Plan, Worksheet, practical tips and Viva questions , reference books , smart content , curriculum , syllabus , marking scheme , toppers answer scripts , revised exam pattern , revised syllabus , Blue Print etc. here **Join Your Subject WhatsApp Group.**

Kindergarten

Class 1 Class 2 Class 3 Class 4

Class 5 Class 6 Class 7 Class 8

Class 9 Class 10 Class 11 (Science) Class 11 (Commerce)

Class 11 (Humanities) Class 12 (Science) Class 12 (Commerce) Class 12 (Humanities)

Subject Wise Groups Secondary and Senior Secondary

Secondary Groups (IX & X)

SST



[Click to Join](#)

Mathematics



[Click to Join](#)

Science



[Click to Join](#)

English



[Click to Join](#)

Hindi



[Click to Join](#)

Information Technology (402)



[Click to Join](#)

Senior Secondary Groups (XI & XII)

Physics



[Click to Join](#)

Chemistry



[Click to Join](#)

English



[Click to Join](#)

Mathematics



[Click to Join](#)

Biology



[Click to Join](#)

Accountancy



[Click to Join](#)

Economics



[Click to Join](#)

History



[Click to Join](#)

Geography



[Click to Join](#)

Sociology



[Click to Join](#)

Mathematics



[Click to Join](#)

Hindi Core



[Click to Join](#)

Home Science



[Click to Join](#)

Sanskrit



[Click to Join](#)

Hindi Elective



[Click to Join](#)

Political Science



[Click to Join](#)

Painting



[Click to Join](#)

Vocal Music



[Click to Join](#)

Comp. Science



[Click to Join](#)

IP



[Click to Join](#)

Physical Education



[Click to Join](#)

App. Mathematics



[Click to Join](#)

IIT /NEET



[Click to Join](#)

Legal Studies



[Click to Join](#)

Entrepreneurship



[Click to Join](#)

French



[Click to Join](#)

Teachers Jobs



[Click to Join](#)

SOE CBSE Principals (Group for Principals Only)



[Click to Join](#)

Rules & Regulations of the Group

1. No introduction
2. No Good Morning/Any wish type message
3. No personal Chats & Messages
4. No Spam
5. You can also ask your difficulties here.

Just get learning resources & post learning resources.

Helpline number only WhatsApp: +91-95208-77777



Why Artham Resource Material?

Resource materials for teachers and students are essential tools for effective teaching and learning. They provide valuable information, guidance, and support to both teachers and students, making the teaching and learning process more efficient and productive.

For teachers, Artham resource materials include lesson plans, instructional guides, assessment tools, professional development materials, and teaching aids. These materials are well researched and created according to 2023-24 NEP and NCERT guidelines.

For students, resource materials can include textbooks, study guides, homework assignments, reference books, online learning platforms, and educational videos. These materials can be obtained from school libraries, educational publishers, online resources, and teachers.

Both teachers and students can also benefit from Artham educational resources which are free and openly licensed educational materials that can be used and shared for teaching and learning. Artham resource material include textbooks, courses, lesson plans, and multimedia resources that are available online.

In summary, resource materials are critical components of effective teaching and learning. They provide a wealth of information and support that can enhance the quality of education and help students achieve academic success.

Teachers and students can also purchase these resources from the links provided with every resource.

JOIN TELEGRAM GROUP/CHANNELS FOR CLASS WISE HIGH QUALITY RESOURCE MATERIAL

SOE CBSE Groups

- [Click to Join CBSE Group...All classes](#)
- [Click to Join SOE CBSE Kindergarten Group](#)
- [Click to Join SOE CBSE Class 1 Group](#)
- [Click to Join SOE CBSE Class 2 Group](#)
- [Click to Join SOE CBSE Class 3 Group](#)
- [Click to Join SOE CBSE Class 4 Group](#)
- [Click to Join SOE CBSE Class 5 Group](#)
- [Click to Join SOE CBSE Class 6 Group](#)
- [Click to Join SOE CBSE Class 7 Group](#)
- [Click to Join SOE CBSE Class 8 Group](#)
- [Click to Join SOE CBSE Class 9 Group](#)
- [Click to Join SOE CBSE Class 10 Group](#)
- [Click to Join SOE CBSE Class 11 \(Science\) Group](#)
- [Click to Join SOE CBSE Class 11 \(Commerce\) Group](#)
- [Click to Join SOE CBSE Class 11 \(Humanities\) Group](#)
- [Click to Join SOE CBSE Class 12 \(Science\) Group](#)
- [Click to Join SOE CBSE Class 12\(Commerce\) Group](#)

- [Click to Join SOE CBSE Class 12 \(Humanities\) Group](#)
- [Click to Join SOE JEE/NEET Group](#)
- [Click to Join SOE CUET Group](#)
- [Click to Join SOE NDA, OLYMPIAD, NTSE Group](#)
- [Click to Join SOE School Principal Professional Development Group](#)
- [Click to Join SOE School Teacher Professional Development Group](#)
- [Click to Join SOE CBSE Project File Group for Class 9th to 12th All Subjects](#)

SOE ICSE Groups

- [Click to Join SOE ICSE Kindergarten Group](#)
- [Click to Join SOE ICSE Class 1 Group](#)
- [Click to Join SOE ICSE Class 2 Group](#)
- [Click to Join SOE ICSE Class 3 Group](#)
- [Click to Join SOE ICSE Class 4 Group](#)
- [Click to Join SOE ICSE Class 5 Group](#)
- [Click to Join SOE ICSE Class 6 Group](#)
- [Click to Join SOE ICSE Class 7 Group](#)
- [Click to Join SOE ICSE Class 8 Group](#)
- [Click to Join SOE ICSE Class 9 Group](#)
- [Click to Join SOE ICSE Class 10 Group](#)
- [Click to Join SOE ICSE Class 11 \(Science\) Group](#)
- [Click to Join SOE ICSE Class 11 \(Commerce\) Group](#)
- [Click to Join SOE ICSE Class 11 \(Humanities\) Group](#)
- [Click to Join SOE ICSE Class 12 \(Science\) Group](#)
- [Click to Join SOE ICSE Class 12\(Commerce\) Group](#)
- [Click to Join SOE ICSE Class 12 \(Humanities\) Group](#)
- [Click to Join SOE JEE/NEET Group](#)
- [Click to Join SOE CUET Group](#)
- [Click to Join SOE NDA, OLYMPIAD, NTSE Group](#)
- [Click to Join SOE School Principal Professional Development Group](#)
- [Click to Join SOE School Teacher Professional Development Group](#)

Nageen CBSE Channels

- [Click to Join Nageen CBSE Kindergarten Channel](#)
- [Click to Join Nageen CBSE Class 1 Channel](#)
- [Click to Join Nageen CBSE Class 2 Channel](#)
- [Click to Join Nageen CBSE Class 3 Channel](#)
- [Click to Join Nageen CBSE Class 4 Channel](#)
- [Click to Join Nageen CBSE Class 5 Channel](#)
- [Click to Join Nageen CBSE Class 6 Channel](#)
- [Click to Join Nageen CBSE Class 7 Channel](#)
- [Click to Join Nageen CBSE Class 8 Channel](#)
- [Click to Join Nageen CBSE Class 9 Channel](#)
- [Click to Join Nageen CBSE Class 10 Channel](#)
- [Click to Join Nageen CBSE Class 11 \(Science\) Channel](#)
- [Click to Join Nageen CBSE Class 11 \(Humanities\) Channel](#)
- [Click to Join Nageen CBSE Class 11 \(Commerce\) Channel](#)
- [Click to Join Nageen CBSE Class 12 \(Science\) Channel](#)
- [Click to Join Nageen CBSE Class 12 \(Commerce\) Channel](#)
- [Click to Join Nageen CBSE Class 12 \(Humanities\) Channel](#)

- [Click to Join JEE/NEET Channel](#)
- [Click to Join CUET Channel](#)
- [Click to Join NDA, OLYMPIAD, NTSE Channel](#)

Nageen ICSE Channels

- [Click to Join Nageen ICSE Kindergarten Channel](#)
- [Click to Join Nageen ICSE Class 1 Channel](#)
- [Click to Join Nageen ICSE Class 2 Channel](#)
- [Click to Join Nageen ICSE Class 3 Channel](#)
- [Click to Join Nageen ICSE Class 4 Channel](#)
- [Click to Join Nageen ICSE Class 5 Channel](#)
- [Click to Join Nageen ICSE Class 6 Channel](#)
- [Click to Join Nageen ICSE Class 7 Channel](#)
- [Click to Join Nageen ICSE Class 8 Channel](#)
- [Click to Join Nageen ICSE Class 9 Channel](#)
- [Click to Join Nageen ICSE Class 10 Channel](#)
- [Click to Join Nageen ICSE Class 11 \(Science\) Channel](#)
- [Click to Join Nageen ICSE Class 11 \(Commerce\) Channel](#)
- [Click to Join Nageen ICSE Class 11 \(Humanities\) Channel](#)
- [Click to Join Nageen ICSE Class 12 \(Science\) Channel](#)
- [Click to Join Nageen ICSE Class 12 \(Commerce\) Channel](#)
- [Click to Join Nageen ICSE Class 12 \(Humanities\) Channel](#)
- [Click to Join JEE/NEET Channel](#)
- [Click to Join CUET Channel](#)
- [Click to Join NDA, OLYMPIAD, NTSE Channel](#)