ASSIGNMENT

Ch 4 Determinants

Based on NCERT Exercise 4.6

(Prepared by Amit Bajaj)

Q1. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations:

$x + y + z = 6$, $x + 2z = 7$, $3x + y + z = 12$.

Q2. If $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations :

$x + 2y + z = 4$, $-x + y + z = 0$, $x - 3y + z = 4$

Q3. If $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$, find A^{-1}. Hence solve the following system of equations:

$x + 2y + 5z = 10$, $x - y - z = -2$, $2x + 3y - z = -11$.

Q4. If $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations :

$3x - 4y + 2z = -1$, $2x + 3y + 5z = 7$ and $x + z = 2$.

Q5. If $A = \begin{bmatrix} 8 & -4 & 1 \\ 10 & 0 & 6 \\ 8 & 1 & 6 \end{bmatrix}$, find A^{-1}, Hence solve the following system of equations :

$8x - 4y + z = 5$, $10x + 6z = 4$, $8x + y + 6z = \frac{5}{2}$

Q6. Using matrix method, solve the following system of linear equations:

(i) $\begin{aligned} 4x+2y+3z&=2 \\ x+y+z&=1 \\ 3x+y-2z&=5 \end{aligned}$

(ii) $\begin{aligned} x+2y+z&=7 \\ x+3z&=11 \\ 2x-3y&=1 \end{aligned}$

(iii) $\begin{aligned} x-y+z&=2 \\ 2x-y&=0 \\ 2y-z&=1 \end{aligned}$

(iv) $\begin{aligned} x+y-z&=1 \\ 3x+y-2z&=3 \\ x-y-z&=-1 \end{aligned}$

(v) $\begin{aligned} x-y&=3 \\ 2x+3y+4z&=17 \\ y+2z&=7 \end{aligned}$

(vi) $\begin{aligned} 2x+y+z&=7 \\ x-y-z&=-4 \\ 3x+2y+z&=10 \end{aligned}$

Q7. Solve the given system of equations:

(i) $\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10$; $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10$; $\frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13$

(ii) $\frac{2}{x}+\frac{3}{y}-\frac{4}{z}=1$; $\frac{3}{x}+\frac{3}{y}+\frac{8}{z}=\frac{31}{6}$; $\frac{6}{x}+\frac{2}{y}+\frac{1}{z}=\frac{47}{12}$

(iii) $\frac{2}{x}+\frac{3}{y}+\frac{4}{z}=-3$, $\frac{5}{x}-\frac{4}{y}-\frac{6}{z}=12$, $\frac{3}{x}-\frac{2}{y}-\frac{2}{z}=6$

(iv) If $A=\begin{pmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{pmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations:

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=2, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=5, \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=-4.$

Q8. Let $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}, B=\begin{bmatrix}4 & -6\\ -2 & 4\end{bmatrix}$, then compute AB. Hence, solve the following system of equations: $2x+y=4$, $3x+2y=1$

Q9. Use product $\begin{bmatrix}\mathbf{1} & \mathbf{-1} & \mathbf{2}\\ \mathbf{0} & \mathbf{2} & \mathbf{-3}\\ \mathbf{3} & \mathbf{-2} & \mathbf{4}\end{bmatrix}\begin{bmatrix}\mathbf{-2} & \mathbf{0} & \mathbf{1}\\ \mathbf{9} & \mathbf{2} & \mathbf{-3}\\ \mathbf{6} & \mathbf{1} & \mathbf{-2}\end{bmatrix}$ to solve the system of equations:

$x + 3z = 9$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$

Q10. Find the product of matrices $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$ and use it for solving the equations $x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2.$

Q11. If $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$, find AB and use this result to solve the following equations: $2x-y+z=-1$; $-x+2y-z=4$; $x-y+2z=-3$.

Q12. Use the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ to solve the equations $\begin{aligned} x-y+z&=4 \\ x-2y-2z&=9 \\ 2x+y+3z&=1 \end{aligned}$

Q13. Given $A=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations: $\begin{aligned} y+2z&=7 \\ x-y&=3 \\ 2x+3y+4z&=17 \end{aligned}$.

Q14. If A = $\begin{bmatrix} \mathbf{1} & \mathbf{-2} & \mathbf{0} \\ \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{0} & \mathbf{-2} & \mathbf{1} \end{bmatrix}$ and B = $\begin{bmatrix} \mathbf{7} & \mathbf{2} & \mathbf{-6} \\ \mathbf{-2} & \mathbf{1} & \mathbf{-3} \\ \mathbf{-4} & \mathbf{2} & \mathbf{5} \end{bmatrix}$, find AB. Hence solve the system of equations :

$x - 2y = 10$, $2x + y + 3z = 8$ and $-2y + z = 7$.

Q15. If $A=\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, find AB and use this result to solve the equations $x+3y+3z=9, x+4y+3z=10, x+3y+4z=10$.

Q16. If $A=\begin{bmatrix} 1 & 2 & 2 \\ 1 & -1 & 1 \\ 1 & 1 & -3 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & 8 & 4 \\ 4 & -5 & 1 \\ 2 & 1 & -3 \end{bmatrix}$, find AB and use this result to solve the equations $x+y+z=4, 2x-y+z=-1, 2x+y-3z=-9$.

Q17. Find inverse of $\begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 2 \\ 4 & 2 & -3 \end{bmatrix}$.

Hence solve $2x+3y+4z=17, 3x-2y+2z=11, 4x+2y-3z=8$.

Q18. If $A=\begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the following system of equations: $2x+3y+2z=9, -x+6z=4, x-y=1$.

Q19. Using elementary transformation, find the inverse of the matrix $A=\begin{bmatrix}8 & 4 & 3\\ 2 & 1 & 1\\ 1 & 2 & 2\end{bmatrix}$ and use it to solve the equations: $8x+4y+3z=19, 2x+y+z=5, x+2y+2z=7$

Q20. If $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 5 & 3\\ 0 & 2 & 1\end{bmatrix}$, find A^{-1} using elementary transformation. Hence solve the equations: $x-y=3, 2x+5y+3z=5, 2y+z=0$.

Q21. An amount of Rs 5000 is put into three investments at 6%, 7% and 8% per annum respectively. The total annual income from these investments is Rs 358. If the total annual income from first two investments is Rs 70 more than the income from the third, find the amount of each investment by the matrix method.

Q22. A school wants to award its students for the values Honesty, Regularity and Hard Work with a total cash award of Rs 6,000. Three times the award money for Hard Work added to that given for Honesty amounts to Rs 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method.

Q23. 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students and the third group consists vigilant and obedient students. Double the number of students of the first group added to the number in second group gives 13, while the combined strength of first and second group is four times that of the third group. Using matrix method, find the number of students in each group.

Q24. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received Rs. 2,800 as interest. However, if trust had interchanged money in bonds they would have got Rs. 100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation.

Answers

1. $x = 3, y = 1, z = 2$ 2. $x = 2, y = 0, z = 2$ 3. $x = -1, y = -2, z = 3$

4. $x = 3, y = 2, z = -1$ 5. $x = 1,\ y = \frac{1}{2},\ z = -1$

6 (i) $x=\frac{1}{2}, y=\frac{3}{2}, z=-1$ (ii) $x=2, y=1, z=3$ (iii) $x=1, y=2, z=3$

(iv) $x=2, y=1, z=2$ (v) $x=2, y=-1, z=4$ (vi) $x=1, y=2, z=3$

7. (i) $x=1/2, y=1/3, z=1/5$ (ii) $x=2, y=3, z=4$

(iii) $x=1, y=-1, z=-2$ (iv) $x=2, y=-3, z=5$

8. $x=7, y=-10$ 9. $x = 0, y = 5, z = 3$ 10. $AB=4I, x=2, y=1, z=-1$

11. $x=1, y=2, z=-1$ 12. $x=3, y=-2, z=-1$ 13. $BA=6I$; $x=2, y=-1, z=4$

14. $x = 4, y = -3, z = 1$ 15. $AB = I ; x = 3, y = 1, z = 1$ 16. $AB = 14I$

17. $A^{-1}=-\frac{1}{111}\begin{bmatrix}2 & 17 & 14\\ 17 & -14 & 8\\ 14 & 8 & -13\end{bmatrix}$; $x=3, y=1, z=2$ 18. $A^{-1}=-\frac{1}{32}\begin{bmatrix}6 & 6 & 1\\ -2 & -2 & 5\\ 18 & -14 & 3\end{bmatrix}$; $x=2, y=1, z=1$

19. $A^{-1}=-\frac{1}{3}\begin{bmatrix}0 & -2 & 1\\ -3 & 13 & -2\\ 3 & -12 & 0\end{bmatrix}$; $x=1, y=2, z=1$ 20. $A^{-1}=\begin{bmatrix}0 & -1 & -3\\ -2 & 1 & -3\\ 3 & -2 & 7\end{bmatrix}$

21. Rs 1000, Rs 2200, Rs 1800 22. Rs 500, Rs 2000 and Rs 3500

23. 5, 3 and 2 24. Rs. 25,000