THE VILLAGE INTERNATIONAL SCHOOL THODUPUZHA
SECOND MODEL EXAMINATION 2023-24
MATHEMATICS (041)

CLASS : XII TIME ALLOTED : 3 HRS.
DATE: 08-01-2024 MAXIMUM MARKS: 80

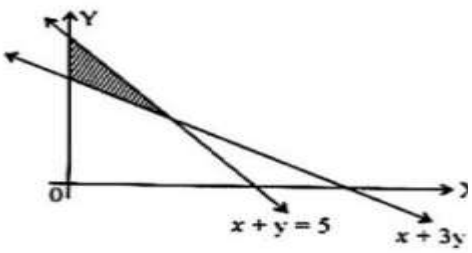
GENERAL INSTRUCTIONS:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However,
there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 3 source based/case based/passage based/integrated units of assessment of 4 marks
each with sub-parts
6. Section E has 4 Long Answer (LA)-type questions of 5 marks each.

Section –A (Multiple Choice Questions)
Each question carries 1 mark

1. For which of the values of x, the rate of increase of the function $Y = 3x^2 - 2x+7$ is 4 times the rate of increase of x?
(a) 0-33 (b) 1 (c) 0.67
(d) -1

2. In the given figure , what is the LPP shaded region known as?
a) Feasible region
b) Feasible solution
c) Optimal region
d) Objective region



3. Find the equation of a line passing through (1, 2, -3) and parallel to the line $\frac{x-2}{1} = \frac{y+1}{3} = \frac{z-1}{4}$

(a) $\frac{x-2}{-1}=\frac{y+1}{1}=\frac{z-1}{1}$ (b) $\frac{x-1}{1}=\frac{y-2}{3}=\frac{z+3}{4}$

(c) $\frac{x+1}{1}=\frac{y-2}{3}=\frac{z+3}{4}$ (d) $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z-1}{1}$

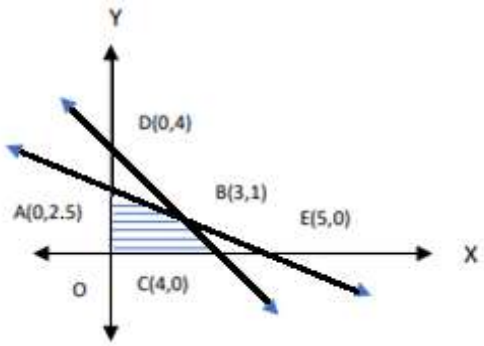
4. The value of $sin^{-1}(\cos\frac{\pi}{9})$ is

(a) $\frac{\pi}{9}$ (b) $\frac{5\pi}{9}$ (c) $\frac{-5\pi}{9}$ (d) $\frac{7\pi}{18}$

5. The total revenue in rupees received from the sale of x units of a product is given by
R(x) = $3x^2$ + 36x + 5. The Marginal revenue, when x = 15 is
(a) 116 (b) 96 (c) 90 (d) 126

6. Besides non negativity constraint the figure given below is subject to which of the following constraints.

(a) $x+2y \leq 5; x+y \leq 4$
(b) $x+2y \geq 5; x+y \leq 4$
(c) $x+2y \geq 5; x+y \geq 4$
(d) $x+2y \leq 5; x+y \geq 4$



7. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}}+4-3\frac{dy}{dx}=0$

(a) 2 (b) 1 (c) 3 (d) $\frac{2}{3}$

8. The set of points of discontinuity of the function $f(x)=x-[x]$, is

(a) Q (b) R (c) N (d) Z

9. Integrating factor of the differential equation $\frac{dy}{dx}+ytanx-\sec x=0$ is

(a) cos x (b) sec x (c) $e^{\cos x}$ (d) $e^{\sec x}$

10. The maximum value of Sin x Cos x is
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $2\sqrt{2}$

11. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|$ = 2 and $|\vec{b}|$ = 3 and $\vec{a}.\vec{b}$= 4, then $|\vec{a}-\vec{b}|$ is
(a) -1 (b) $\sqrt{8}$ (c) $\sqrt{13}$ (d) $\sqrt{5}$

12. Find the value of : $\hat{\imath}.(\hat{\jmath}\times\hat{k})+\hat{\jmath}.(\hat{k}\times\hat{\imath})+\hat{k}.(\hat{\jmath}\times i)$
(a) 0 (b) 1 (c) 2
(d) 3

13. If α, β, γ are the direction angles of a vector and cos $\alpha = \frac{14}{15}$, cos $\beta = \frac{1}{3}$, then cos $\gamma =$

(a) $\pm\frac{2}{15}$ (b) $\pm\frac{1}{5}$ (c) $\pm\frac{1}{15}$ (d) $\pm\frac{4}{15}$

14. The angle between the vectors $\vec{a}$ and $\vec{b}$ if $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 4\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

15. The rate of change of area of a circle with respect to its radius r at r = 6 cm is

(a) 10π (b) $12\,\pi$ (c) $8\,\pi$ (d) $11\,\pi$

16. If random variable X represents the number of heads when a coin is tossed twice then mathematical expectation of X is

(a) 0 (b) $\mathbf{\frac{1}{4}}$ (c) $\mathbf{\frac{1}{2}}$ (d) 1

17. For what value of x is $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0$?

(a) 4 (b) -3 (c) 2 (d) -1

18. If f(x) = $x\,tan^{-1}x$, then f '(1) is equal to

(a) $\frac{1}{2}+\frac{\pi}{4}$ (b) $-\frac{1}{2}+\frac{\pi}{4}$ (c) $-\frac{1}{2}-\frac{\pi}{4}$ (d) $\frac{1}{2}-\frac{\pi}{4}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true

19. **Assertion (A):** If $\int_0^1 (3x^2 + 2x + k)dx = 0$, then the value of k is -1.

Reason (R): $\int x^n dx = \frac{x^{n+1}}{n+1}$

20. **Assertion (A):** The maximum profit that a company makes if profit function is given by P(x) = 41 + 24x - 8x**²**; where 'x' is the number of units and P is the profit in rupees is 57.

Reason (R): The profit is maximum at x = a, if $P'(a) = 0$ and $P''(a) < 0$.

Section –B
[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. (a) Simplify: $sin^{-1}\left[2x\sqrt{1-x^2}\right]$, x ∈ $\left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

OR

(b) Evaluate : $sin^{-1}\left(\sin\frac{3\pi}{4}\right) + cos^{-1}\left(\cos\frac{3\pi}{4}\right) + tan^{-1}(1)$

22. If y = a e ^{2x} + b e $^{-x}$, then find the value of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$.

23. (a) If three non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$.

OR

(b) Find a vector of magnitude 9, perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$, where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

24. Find the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$

25. Find the intervals in which the function f(x) = $4x^3 - 6x^2 - 72x + 30$ is strictly decreasing.

Section – C
[This section comprises of short answer type questions (SA) of 3 marks each]

26. Evaluate : $\int_0^4 |x - 1| dx$

27. (a) In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid.

OR

(b) In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspaper. A student is selected at random.

(i) Find the probability that the student reads neither Hindi nor English newspaper.

(ii) If she reads Hindi newspaper, find the probability that she reads English newspaper.

(iii) If she reads English newspaper, find the probability that she reads Hindi newspaper.

28. Evaluate: $\int \frac{(x+3)e^x}{(x+5)^3} dx$

29. (a) Solve the differential equation: $y\,dx + (x - y^2)dy = 0$

OR

(b) Find the particular solution of the differential equation $x\left(\frac{dy}{dx}\right) - y + x\,cosec\left(\frac{y}{x}\right) = 0$, given that $y = 0$ when $x = 1$.

30. Evaluate : $\int \frac{(x^2+1)}{(x-1)^2(x+3)} dx$

31. (a) Solve the following Linear Programming Problem graphically:
Minimize : Z= x + 2y,
Subject to the constraints :
x + 2y ≥ 100 , 2x – y ≤ 0, 2x + y ≤ 200, x, y ≥ 0.

OR

(b) Solve the following Linear Programming Problem graphically:
Maximize : Z = - x + 2y,
Subject to the constraints : x + y ≥ 5, x + 2 y ≥ 6, x ≥ 3, y ≥ 0.

Section –D

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

32. On his birthday Shyam decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹10 more. However, if there were 16 children more, everyone would have got ₹10 less. Let the no. of children be x and the amount distributed by Shyam for one child be y (in ₹). Based on the information given above, answer the following questions:

(i) Write the equations in terms of x and y.
(ii) Represent the matrix equation for the given information.
(iii) How many children got money from Shyam?

OR

How much amount Shyam has to spend in distributing the money to all the students of orphanage home?

33. The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation
$y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

a) Find the rate of growth of the plant.
b) What is the number of days it will take for the plant to grow to the maximum height?
c) i)What is the maximum height of the plant?

OR

ii)If the height of the plant is $\frac{7}{2}$cm, find the number of days it has been exposed to the sunlight i

34. Let X denote the number of hours a person watches television during a randomly selected day. The probability that X can take the values xi, has the following form, where 'k' is some unknown constant.

$$P(X = x_i) = \begin{cases} 0.2, & if\ x_i = 0 \\ kx_i, & if\ x_i = 1\ or\ 2 \\ k(5 - x_i), & if\ x_i = 3 \\ 0, & otherwise \end{cases}$$

(i) Find the value of k.
(ii) What is the probability that a person watches two hours of television on a selected day?
(iii) i)What is the probability that the person watches at least two hours of television on a selected day?

OR

ii)What is the probability that the person watches at most two hours of television on a selected day?

Section –E
[This section comprises of long answer type questions (LA) of 5 marks each]

35. (a) Find the product AB, if A = $\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$, B = $\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$ are two square matrices and hence solve the system of linear equations: x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7 using the product AB.

OR

(b) If A = $\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the following system of linear equations 2x – 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3

36. Define the relation R in the set N x N as follows:
For (a, b), (c, d) ∈ N x N, (a, b) R (c, d) iff ad = bc. Prove that R is an equivalence relation in N x N. Also write the equivalence class [(2,3)]

37. (a) Find the area of Δ ABC, the coordinates of whose vertices are A(2, 5), B(4, 7) and C(6, 2) using integration.

OR

(b) Make a rough sketch of the region {(x, y): $0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2$} and find the area of the region using integration.

38. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line, $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + \hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$. Also find the image of P in this line.

******END OF THE QUESTION PAPER******