TVIS

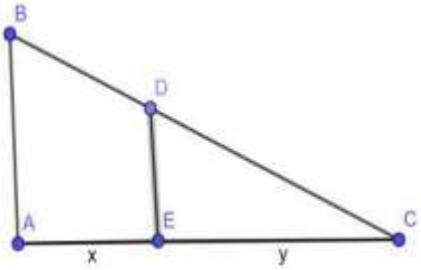
TERM 1 EXAMINATION - 2023-24

CLASS: XII **Max. Marks: 80**

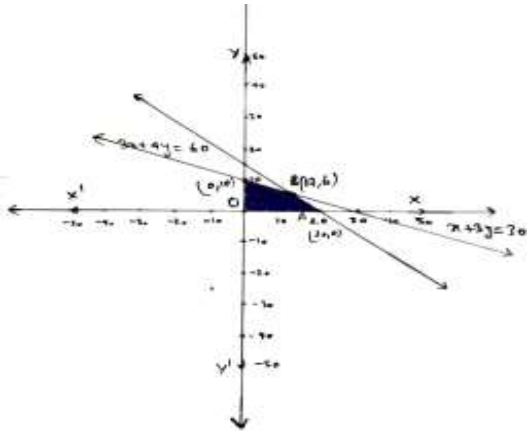
SUBJECT: MATHEMATICS **Time : 3 Hrs**

Marking Scheme

Q.NO.	**SECTION A-(Multiple choice questions 1-Mark each)**	
1	(b)	
2	(a)	
3	(a)	
4	(c)	
5	b	
6	c	
7	(c)	
8	a	
9	d	
10	(c)	
11	(d)	
12	(d)	
13	(b)	

14	(c)	
15	d	
16	(d)	
17	c	
18	(b)	
19	(a)	
20	d	
	SECTION B (VSA questions of 2-Marks each)	
21	f (x) = $\sin^{-1}\sqrt{x-1}$ $-1 \le \sqrt{x-1} \le 1 \Rightarrow 0 \le \sqrt{x-1} \le 1$ $0 \le x-1 \le 1 \Rightarrow 1 \le x \le 2,\ x \in [1,2]$ OR $\tan^{-1}\frac{1-\tan x}{1+\tan x}$ $\tan^{-1}\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}$ $\tan^{-1}\tan\left(\frac{\pi}{4}-x\right)$ $\frac{\pi}{4}-x$	
22	 Finding expression $y = \frac{2}{3}x$ Where $x =$ distance of man at any time t from street light.	

	y =length of shadow of man Getting $\frac{dy}{dt}$ = 0.2m/sec At any time t the tip of his shadow is at a distance of $(x+y)$m from street light The rate at which his shadow moving $=\left(\frac{dx}{dt}+\frac{dy}{dt}\right)$m/s=0.5m/s The rate at which his shadow lengthening= $\frac{dy}{dt}$ m/s=0.2m/s	
23	CP F = 4x + 6y (0,2) 12 (3,0) 12 (6,0) 24 (6,8) 72 (0,5) 30 Minimum occurs at (0,2) and (3,0).Minimum value is 12	
24	Writing $x=\frac{\cos y}{\cos(a+y)}$ Getting, 1 = $\frac{\sin(a+y)\cos y-\cos(a+y)\sin y}{cos^2(a+y)}\frac{dy}{dx}$ Proving $\frac{dy}{dx}=\frac{cos^2(a+y)}{\sin a}$	
25	x – 3y = 0	
	SECTION-C(Short Answer Questions of 3-marks each)	
26	R is reflexive R is not symmetric	

	R is not transitive	
27	1/3 . 5/7 . 5/8+ 2/3 . 2/7. 5/8 + 2/3 . 5/7 . 3/8 = 25/56 Or	
28	$\sin^{-1}[\,1/2(\cos^{-1} 4/5)] = \sin\frac{\theta}{2} = \sqrt{\frac{1-cos\theta}{2}} = \sqrt{\frac{1-\cos(\cos^{-1}\frac{4}{5})}{2}} = \frac{1}{\sqrt{10}}$	
30	Plotting correct figure  Shading the feasible region Finding the maximum value of Z	

Corner Point	Corresponding value of Z
(0, 0)	0
(20, 0)	1,60,000
(12, 6)	1,68,000(Max.)
(0, 10)	1,20,000

31

Let $y = x^{\sin x} + (\sin x)^x = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \ldots\ldots(1)$$

$u = x^{\sin x}$, taking log on both sides;

$\log u = \sin x \log x$,

Differentiating with respect to x;

$$\frac{1}{u}\frac{du}{dx} = \sin x \frac{1}{x} + \log x.\cos x$$

$$\Rightarrow \frac{du}{dx} = u\left(\frac{\sin x}{x} + \log x.\cos x\right)$$

$$\Rightarrow \frac{du}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x.\cos x\right)$$

$v = (\sin x)^x$, taking log on both sides;

$\log v = x \log \sin x$,

Differentiating with respect to x;

$$\frac{1}{v}\frac{dv}{dx} = x\frac{1}{\sin x}\cos x + \log \sin x \times 1$$

$$\Rightarrow \frac{du}{dx} = v(x \cot x + \log \sin x)$$

$$\Rightarrow \frac{du}{dx} = \sin x^x (x \cot x + \log \sin x)$$

$$(1) \Rightarrow \frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x \cos x\right)$$

$$+ \sin x^x (x \cot x + \log \sin x)$$

SECTION-D(Long Answer type (LA) of 5 –marks each)

32.

Let r be the base radius x is the distance O the center of the sphere from the base and V the volume of the are

Height h of the cone = R + x

$$\therefore \quad V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}\left(R^2 - x^2\right)(R + x)$$

$$= \frac{\pi}{3}\left(R^2 + R^2 x - Rx^2 - x^2\right)$$

$$\therefore \quad \frac{dV}{dx} = \frac{\pi}{3}\left[R^2 - 2Rx - 3x^2\right]$$

$$\frac{d^2V}{dx^2} = \frac{\pi}{3}[-2R - 6x]$$

For max or min V $\frac{dV}{dx} = 0$

$$\therefore \quad R^2 - 2Rx - 3x^2 = 0$$

$$\Rightarrow (R + x)(x - 3x) = 0 \quad 2)\ x = -R, \frac{x}{3} \text{ but } x \equiv -R$$

When $x = \frac{R}{3}$ $\frac{d^2V}{dx^2} < 0$ V is max only when $x = \frac{R}{3}$

	When $x = \frac{R}{3}$ $\frac{d^2V}{dx^2} < 0$ V is max only when $x = \frac{R}{3}$ $\therefore$ Max V $= \frac{1}{3}\pi\left(R^2 - \frac{R^2}{9}\right)\left(R + \frac{R}{3}\right) = \frac{32\pi R^3}{81} = \frac{8}{27}\left(\frac{4}{3}\pi R^3\right) = \frac{8}{27}$ (volume of sphere)
33	Showing ab=ba ,hence R is symmetric Showing cb= da⇒ (c,d)R(a,b),hence R is symmetric (a,b)R(c,d)⇒ad=bc, (c,d)R(e,f)⇒cf=de af =be⇒$(a,b)R\ (e,f)$,hence R is transitive R is reflexive,symmetric,and transitive nence R is equivalence relation Or Showing $f(x) = f(y)$ but $x \neq y$ for any $x, y \in R$ Hence not one-one Showing , Range f= $1 + x^2 \geq 1$ $\forall\, x \in R$ Hence not onto Hence f(x) is neither one-one nor onto
34	P(E1) = 60/100 $P(E2)$ = 40/100 P(A/E1) = 2/100 P(A/E2) = 1/100 P(E2/A) = $\frac{P(E2).P\left(\frac{A}{E2}\right)}{P(E1).P\left(\frac{A}{E1}\right)+P(E2).P\left(\frac{A}{E2}\right)} = \frac{\frac{40}{100}.\frac{1}{100}}{\frac{60}{100}.\frac{2}{100}+\frac{40}{100}.\frac{1}{100}} = 1/4$
35	Here$\lvert A\rvert$ = -1 , A is a non singular matrix Adj A = $\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

	AX =B ,x = 1, y= 2 and z = 3	
	SECTION – E (Case Studies/Passage based questions of 4 Marks each)	
36	i) C ii) B iii) C iv) B v) a	
37	(i) Area of circle = $\frac{196\pi}{(\pi+4)^2}$ sq unit (ii) Area of square = $\left(\frac{112}{\pi+4}\right)^2$ sq units. (iii) Length of circle= $\frac{28\pi}{\pi+4}$,Length of square = $\frac{112}{\pi+4}$	
38	i) $P(E2/A) = \frac{0.2x0.3}{0.3x0.25+0.2x0.3+0.1x0.35+0.4x0.1}$ = 2/7 ii) $\frac{0.3x0.25}{0.3x0.25+0.2x0.3+0.1x0.35+0.4x0.}$ = 5/14	

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